TECHNISCHE UNIVERSITÄT DARMSTADT

Fachgebiet
Theorie Elektromagnetischer Felder

Inductive Impedance and Saw Tooth Instability

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Workshop on Broadband Impedance Measurements and Modeling

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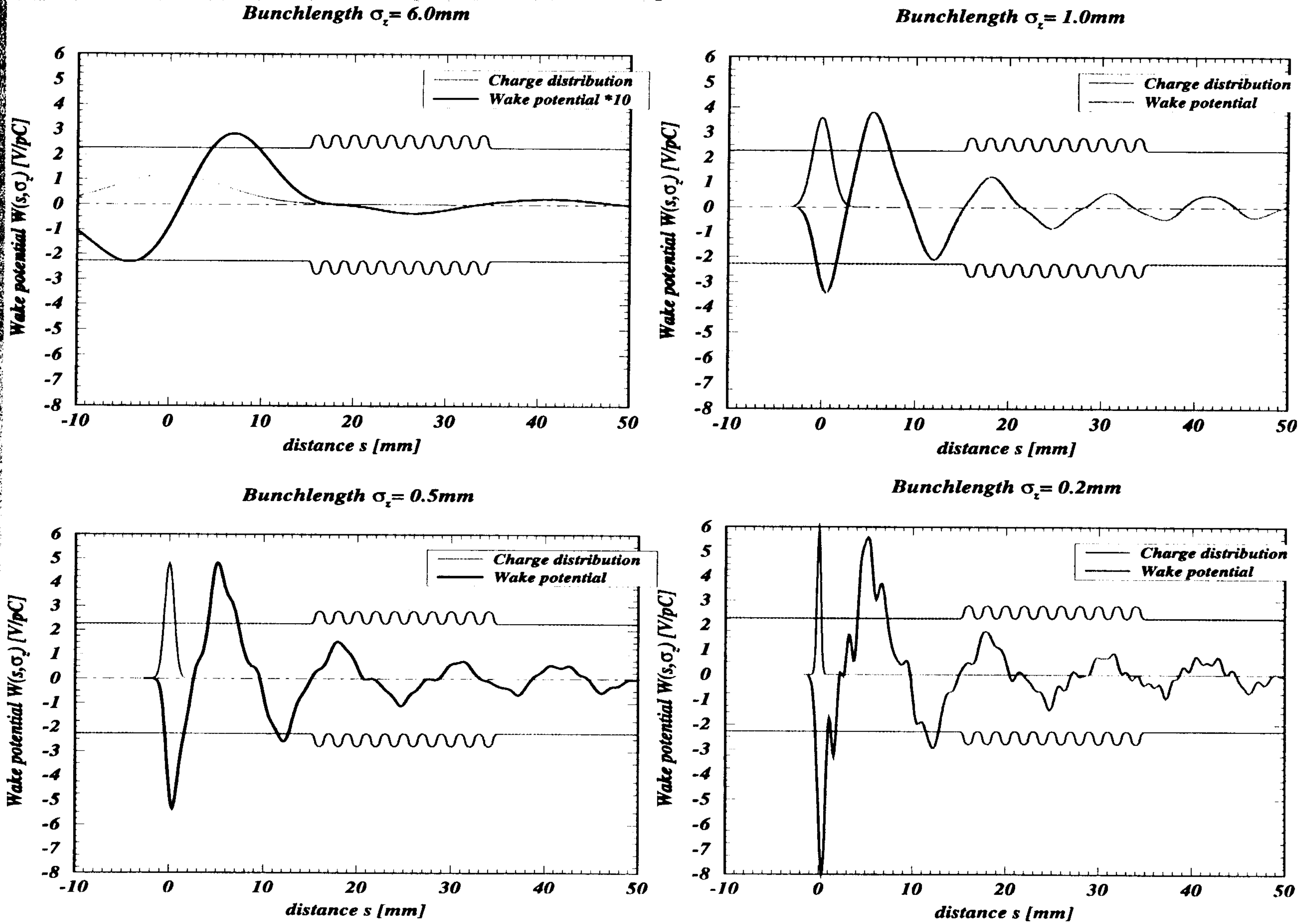
Inductive Impedance

- Inductive impedance or inductive wake potential are usually used for the description of the wake fields, resposible for the bunch „self-acceleration“, when the „head“ of the bunch loses energy and the „tail“ gets it back. So the full energy loss of the bunch is very small in comparison with the bunch energy spread.
- It usually happens when the bunch is passing by the bellows, tapers, pumping slots, masks, BPMs, or is moving inside resistive or rough vacuum chamber.

Inductive Impedance

- The inductive impedance sometimes is considered to be the low frequency part of the wake fields.
- More accurate study shows that the „inductive“ structure of the wake potentials is due to the action of the very high frequency fields.
- We present some examples:
- Bellow
- Shielded bellow
- Transition for a quadrupole

Wake potentials of a bellow



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Resonator wake field

- Sometimes these high frequency fields can be fully descibed by a simple wake function -
- Resonator wake function

$$w(s) = -W_0 \frac{1}{k_Q} \frac{d}{ds}\left[\exp(-\frac{k}{2Q}s)\sin k_Q s\right]$$

$$k_Q = k\sqrt{1-\frac{1}{4Q^2}}$$

- Resistive wall wake fields
- Thin dielectric layer inside vacuum chamber
- Surface roughness wake fields and shallow bellows

Inductive Wake Potentials

- The wake potential of the bunch for the high frequency ($k\sigma_z >> 1$; $Q >> 1$)wake function is

$$W(s) \approx qW_0 \int_{-\infty}^{s} \rho(s')\cos k(s-s')ds' \approx qW_0 \frac{\pi}{k^2}\frac{\partial}{\partial s}\rho(s)$$

- Loss factor

$$K_{loss} = \int_{-\infty}^{\infty} \rho(s)W(s)ds = 0$$

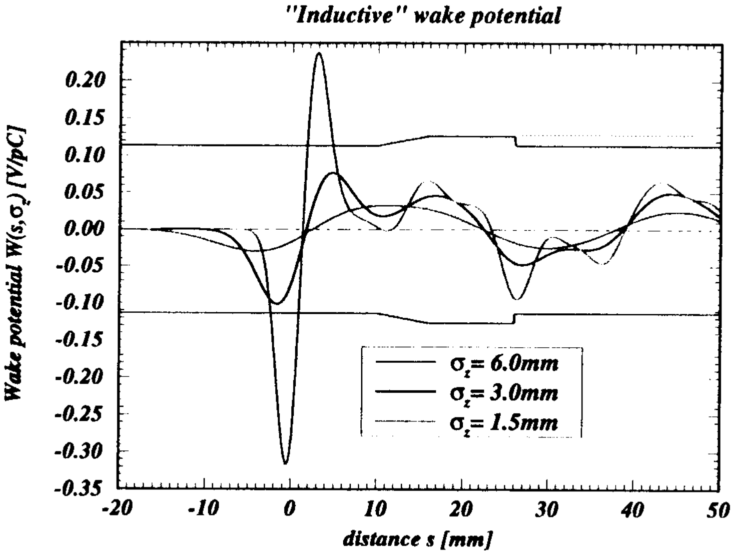
- Energy spread

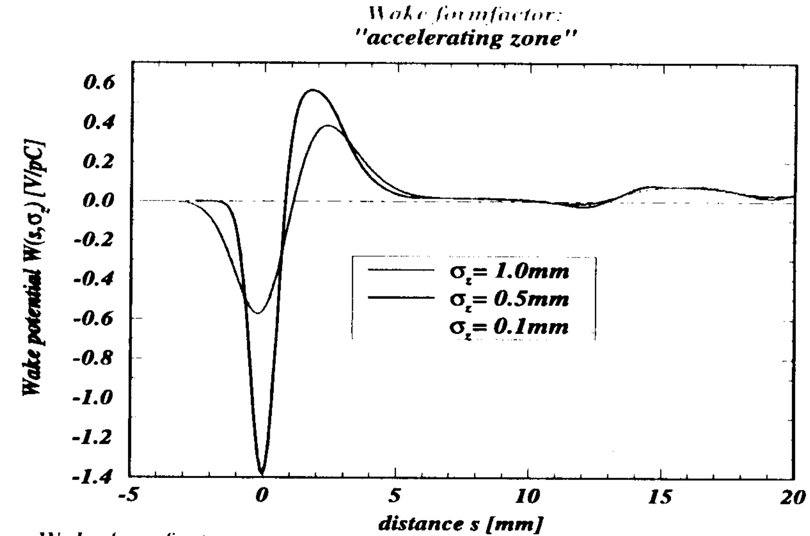
$$\Delta E_{wake} = \sqrt{\int_{-\infty}^{\infty} \rho(s)W^2(s)ds}$$

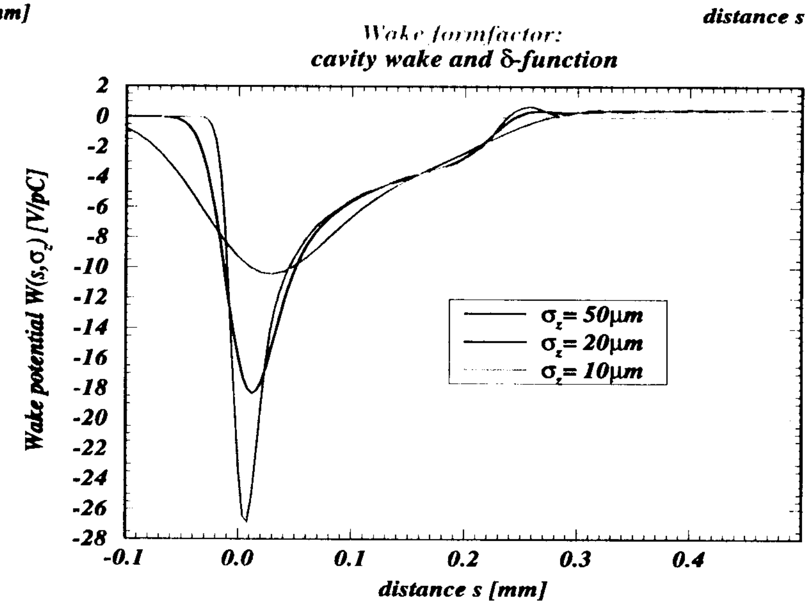
Gaussian bunch

$$\Delta E_{wake} = q\frac{W_0}{(k\sigma_z)^2}\left(\frac{\pi}{6\sqrt{3}}\right)^{1/2}$$

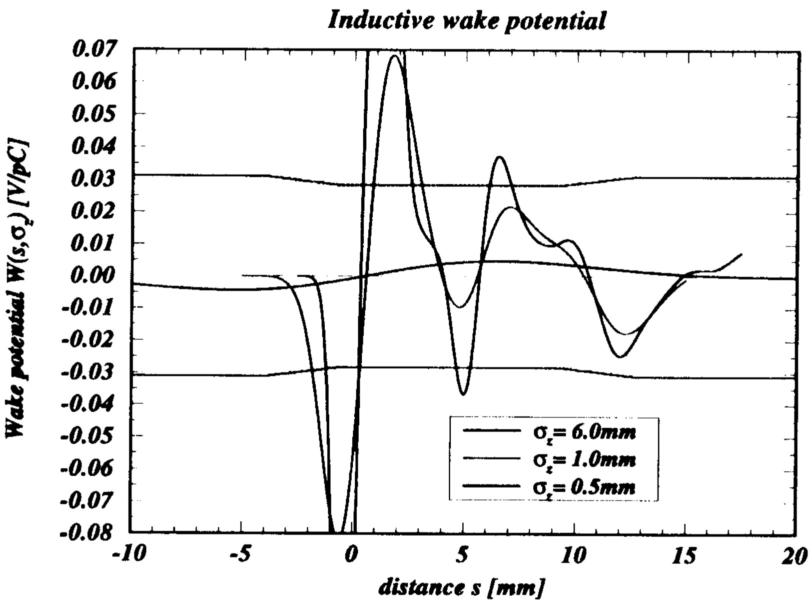
Shielded bellow

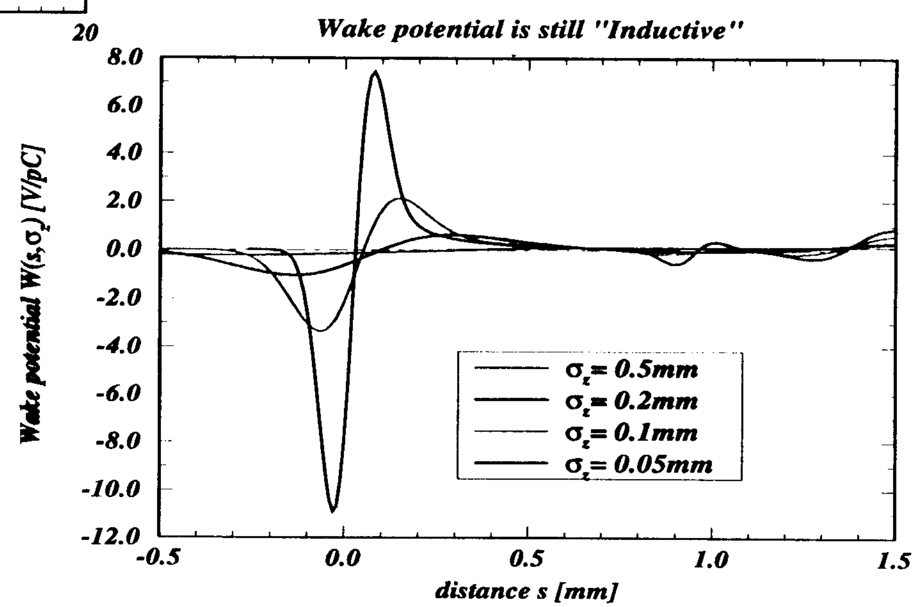






Transition for a quadrupole





Wake fields in periodic „rough" surface structure

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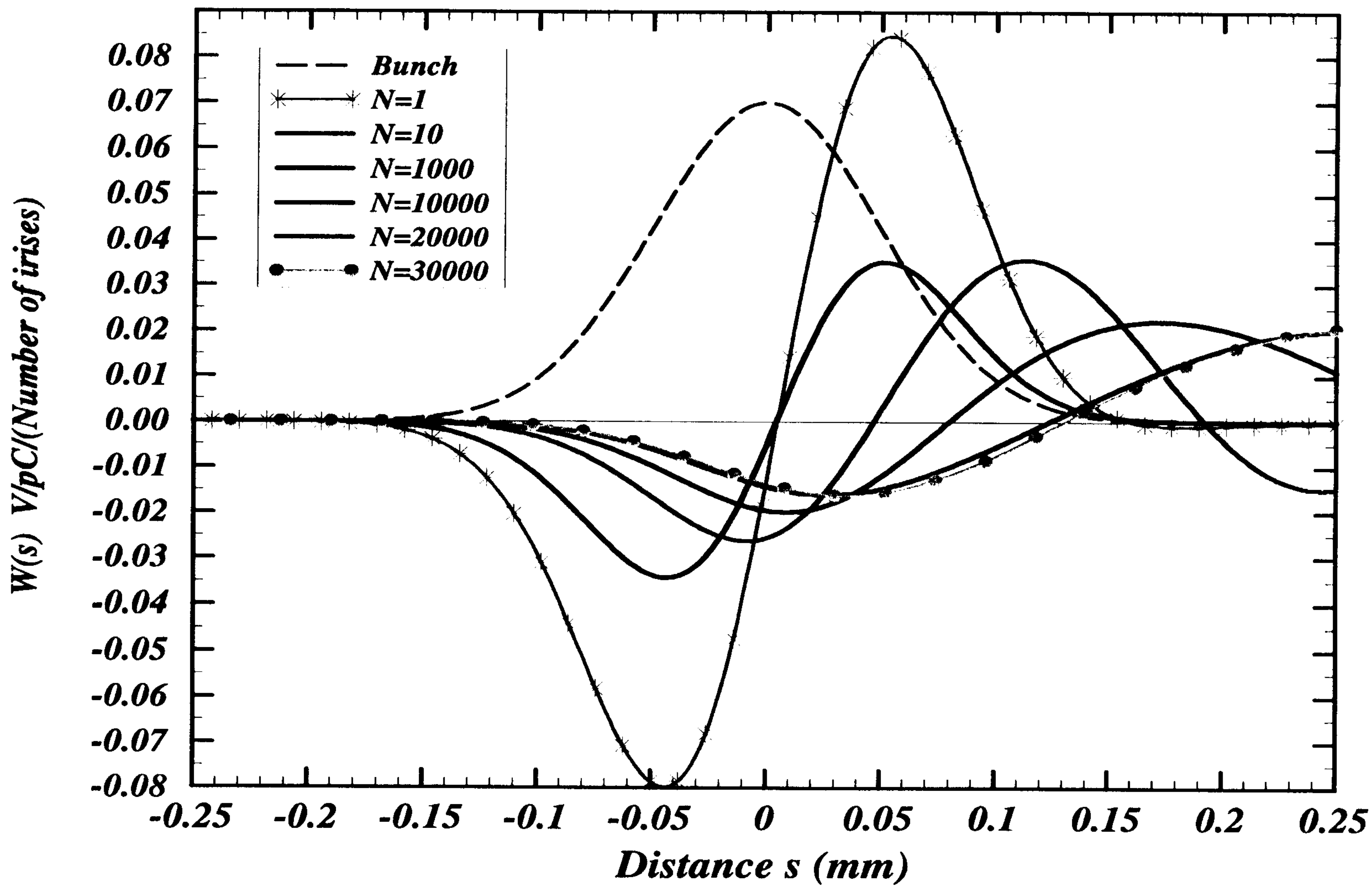
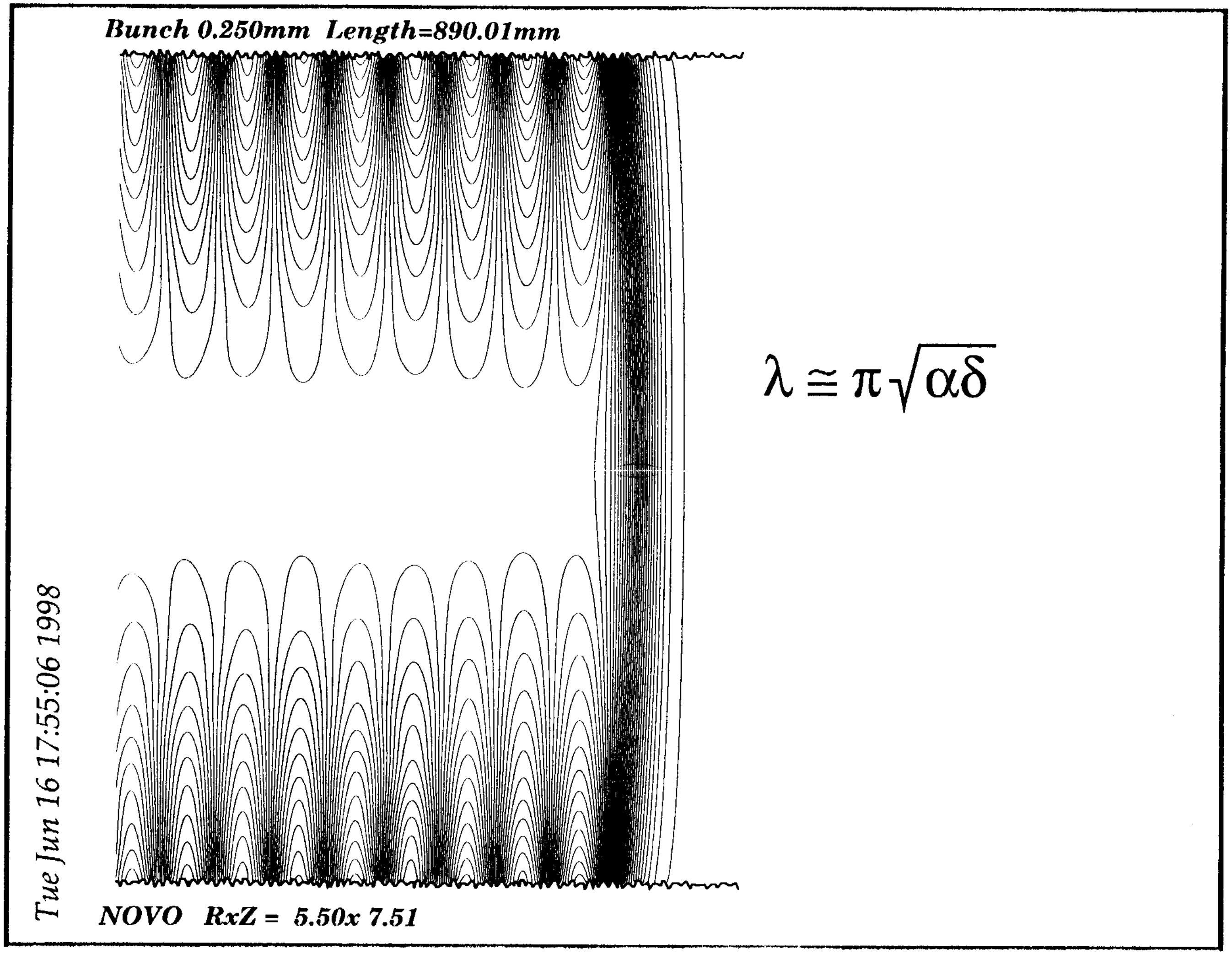
Bunch of σ=50μ ***in the tube of R=5mm with irises***
of 10*μ *by 10* μ*, period is 20*μ*. N=1,10,1000,10000,20000,30000



Surface roughness wake field



$$\lambda \cong \pi\sqrt{\alpha\delta}$$

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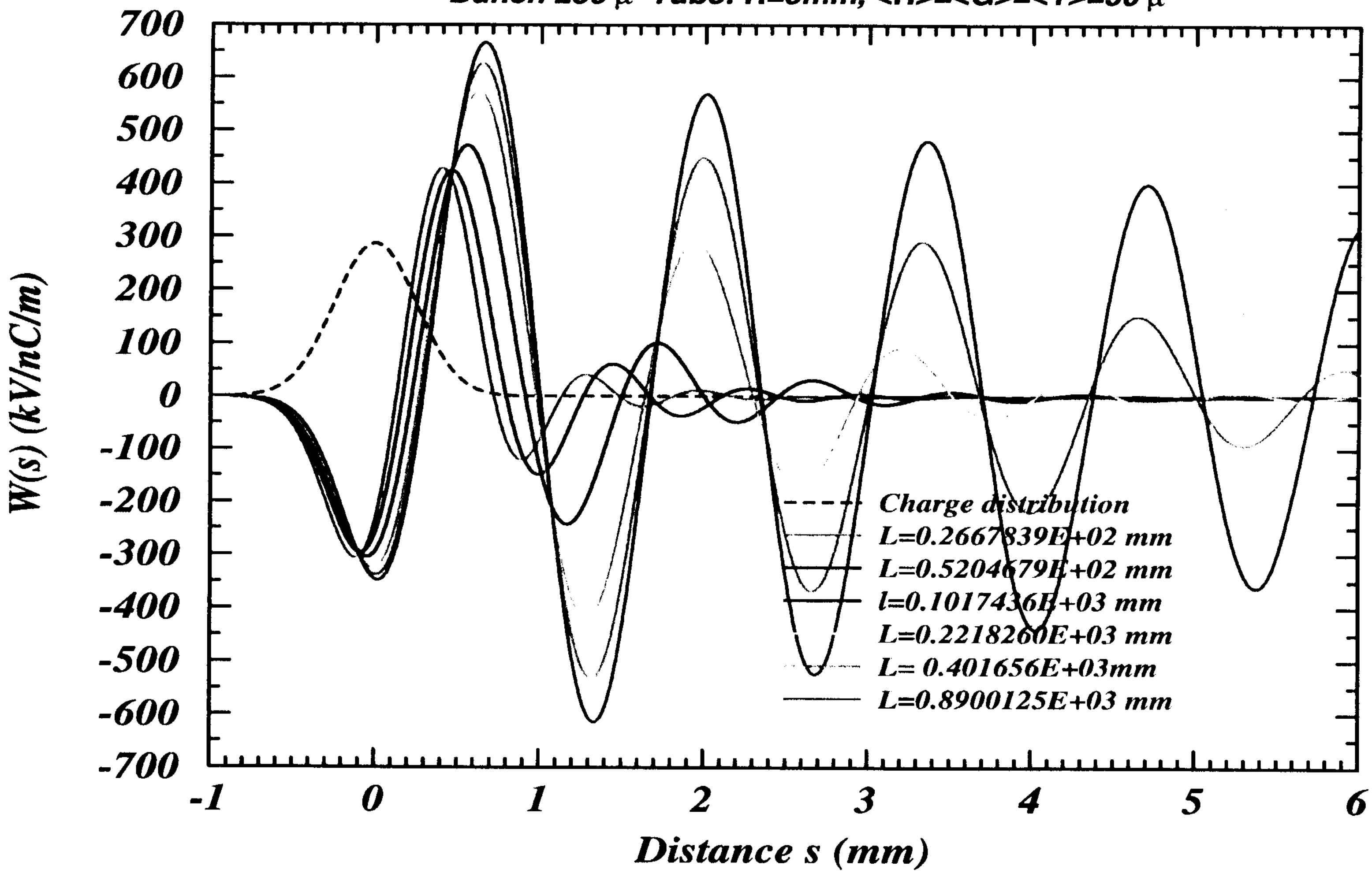
Surface roughness wake potential

Normalysed Wake Potential for different tube length

Bunch 250 μ Tube: R=5mm, <H>=<G>=<T>=50 μ



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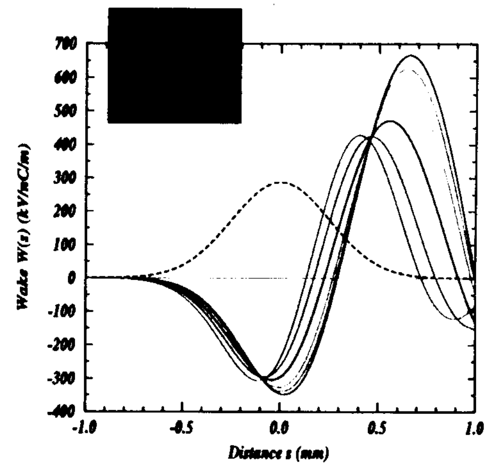
Surface roughness wake potential

- Here we have the transformation of the inductive wake potential to resistive potential with the length of the tube. The bunch has to pass the long enough distance to form this wake field.



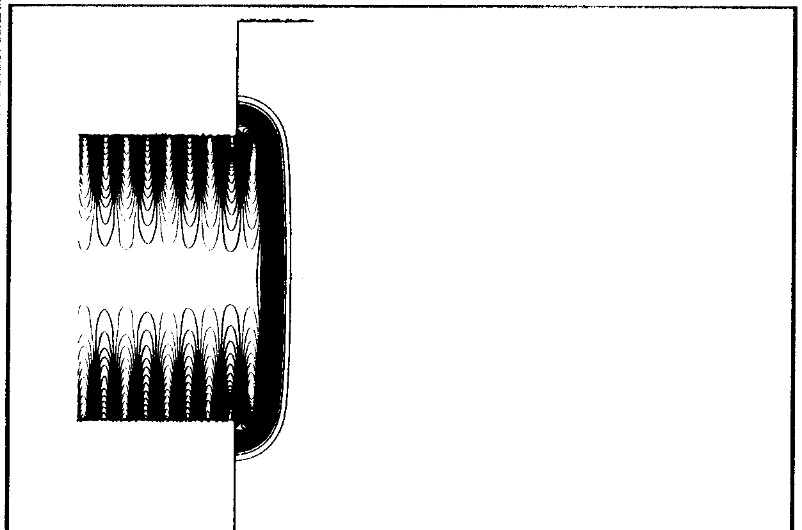
Surface roughness wakefunction is just a resonator wakefunction with amplitude

$$W_0 = \frac{Z_0 c}{\pi a^2}$$ and frequency $$k \approx \frac{2}{\sqrt{a\delta}}$$

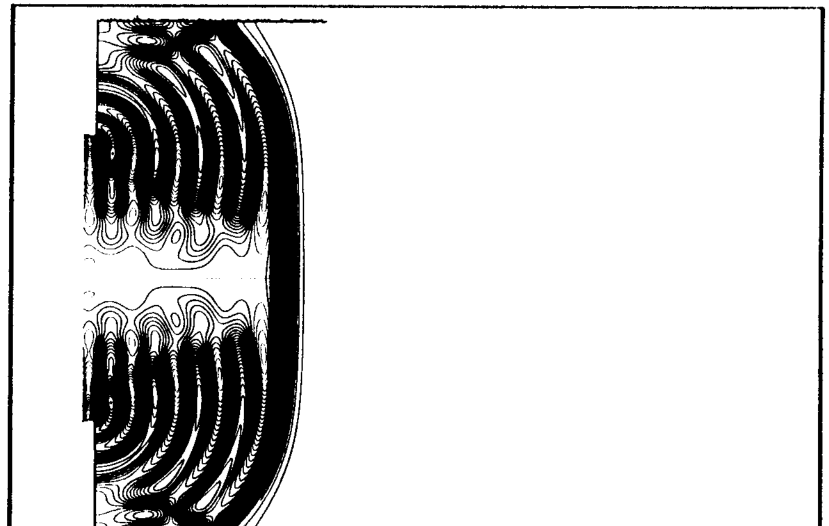
so the energy spread per unit length is

$$\Delta E_{wake} = q\frac{Z_0 c}{a\sigma_z^2}\frac{\delta}{4}\sqrt{\frac{1}{\pi 6\sqrt{3}}}$$

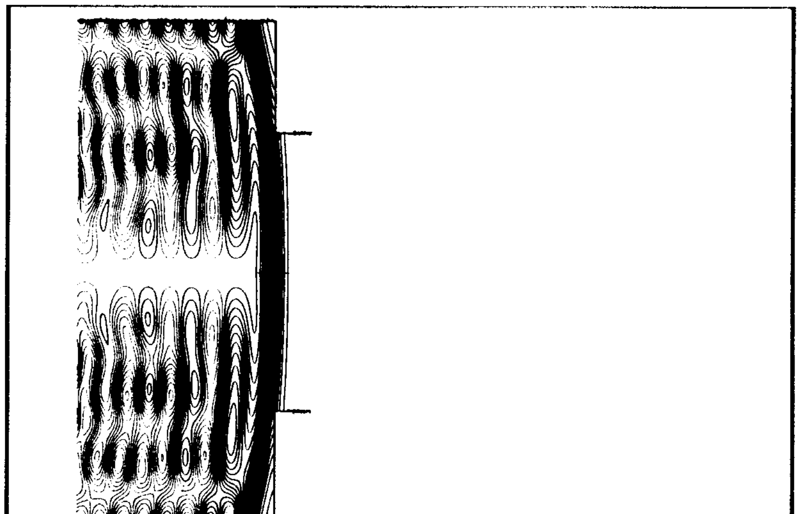
Surface roughness wake field in the chamber transition



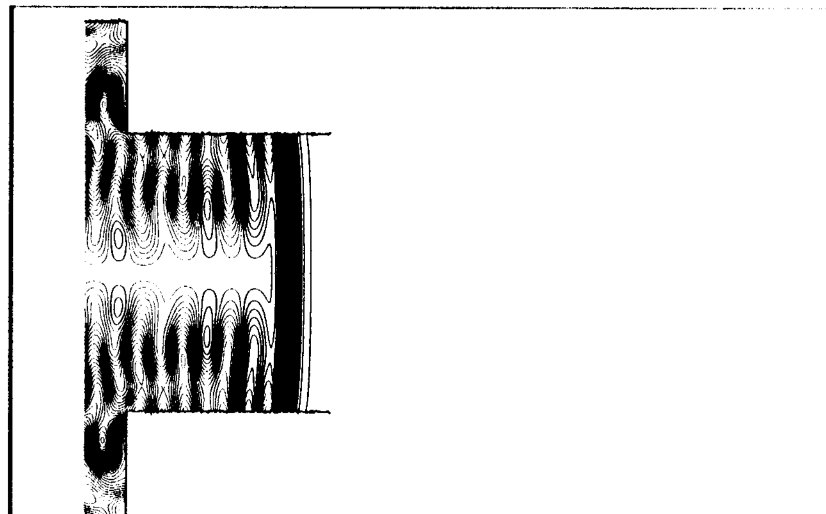
NOVO Wed Jun 16 11:27:29 1999 Bunch 0.250mm RxZ = 9.50x 7.51 Length= 892.47mm



NOVO Wed Jun 16 14:07:36 1999 Bunch 0.250mm RxZ = 9.50x 7.51 Length= 897.13mm



NOVO Thu Jun 17 12:14:41 1999 Bunch 0.250mm RxZ = 9.50x 7.51 Length= 909.56mm



NOVO Thu Jun 17 18:38:00 1999 Bunch 0.250mm RxZ = 9.50x 7.51 Length= 914.62mm

Surface roughness and resistive wall wake fields

- The main part of the resistive wake function is also described by a resonator wake function of Q=1 with the same amplitude

$$W_0 = \frac{Z_0 c}{\pi a^2}$$

- but more higher frequency

$$k_Q = \sqrt{3}\left(\frac{\sigma_c}{2ca^2}\right)^{1/3}$$

- so the surface roughness fields are stronger if the effective roughness size

$$\delta > \frac{4}{3}\sqrt[3]{a}\left(\frac{c}{2\pi\sigma_c}\right)^{2/3}$$

Estimation for the energy spread from a pumping slot

- The effective „roughness“ size of a round slot with radius h can be estimated by

$$\delta = \frac{h}{\pi}$$

- Assuming that slots are situated at the distance of 2h from each other we can estimate the number of slots on the perimeter of the tube

$$M = \frac{2\pi a}{2h}$$

- Then one slot will give the energy spread

$$\Delta E_{slot} = q \frac{Z_0 c}{\sigma_z^2} \frac{1}{2\pi^2} \frac{h^3}{a^2} \sqrt{\frac{1}{\pi 6\sqrt{3}}}$$

- Comparison with known formula shows the coherent effect of slots

Fokker-Planck Equation

- Distribution function f(t,x,p)

$$\frac{\partial}{\partial t}f+\dot{x}\frac{\partial}{\partial x}f+\dot{p}\frac{\partial}{\partial p}f=\lambda\frac{\partial}{\partial p}\left\{pf+\frac{\partial}{\partial p}f\right\}$$

$$\dot{x}=p$$

$$\dot{p}=-x+I_0W(t,x)$$

- Time is measured in synchrotron periods
- Coordinate and momentum are normalized by the length and momentum spread of a bunch of small intensity

Numerical simulation

- Original implicit finite-difference algorithm
- Correct dispersion relation (up to the meshsize wavelength)
- No numerical diffusion, distortion or modulation

- TEST: Damped synchrotron oscillations to the stable solution of Haissinski equation. High-frequency wake field $\kappa\sigma=25$
- COMPARISON with the results of multi-particle tracking simulations (R.Baartman and M.D'yachkov, 1996). Low-frequency wake field $\kappa\sigma=0.5$

Damped synchrotron oscilations to the stable „Haissinski“ solution

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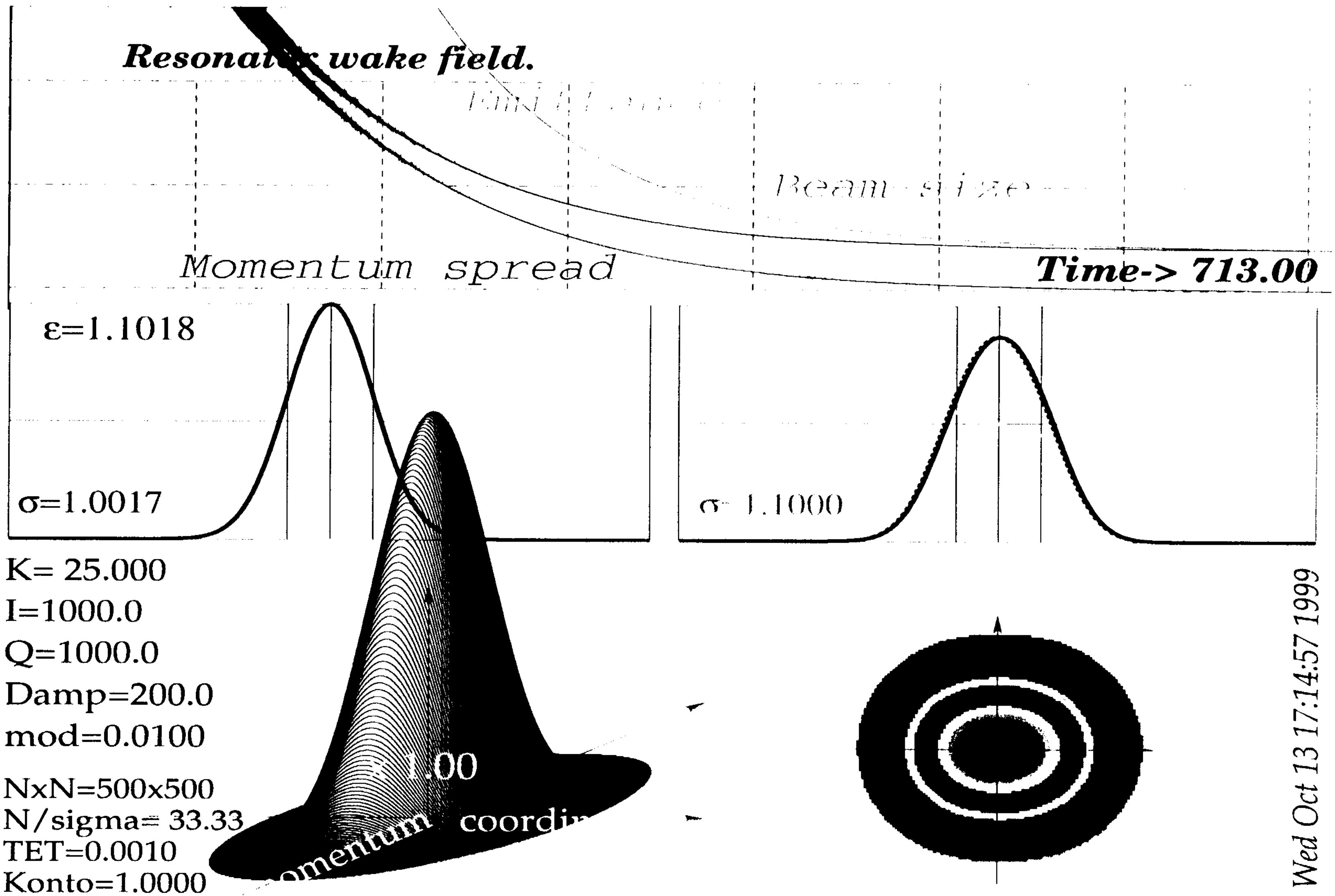

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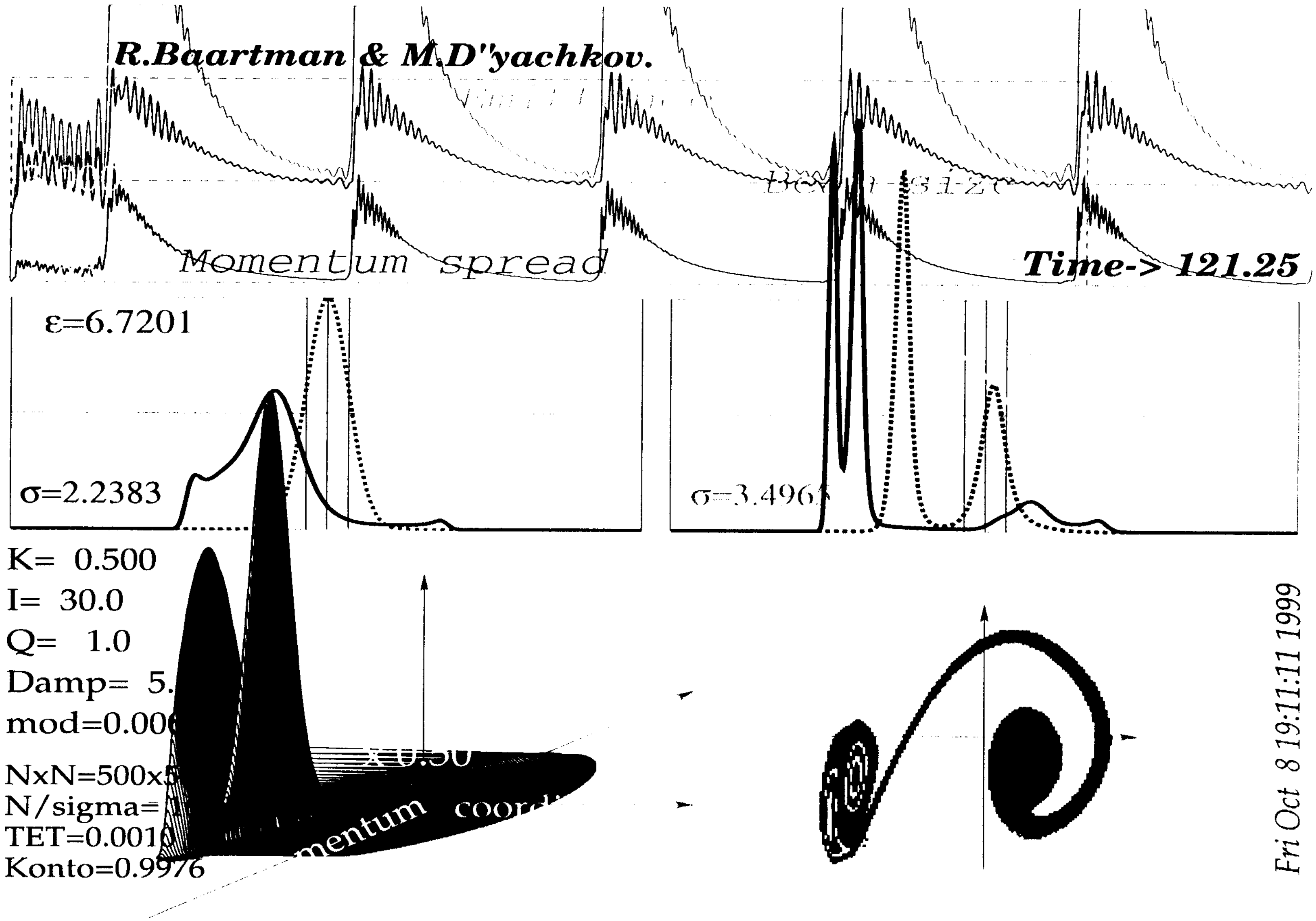
Comparison with Tracking Simulations



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Longitudinal Instability

Dimensionless intensity parameter

$$I_o = \frac{q\, W_o c}{V_{rf}\, \omega_{rf}\, \sigma_o}$$

1. Bunch lengthening. Stable solution of Haissinski equation.
   $\sigma > 1 \quad \delta p = 1$
2. Microwave instability. Small oscillations of bunch length.
   $\sigma > 1 \quad \delta p > 1$
3. Large oscillations of bunch length.
4. Sawtooth oscillations.

Microwave instability. Small oscillations.

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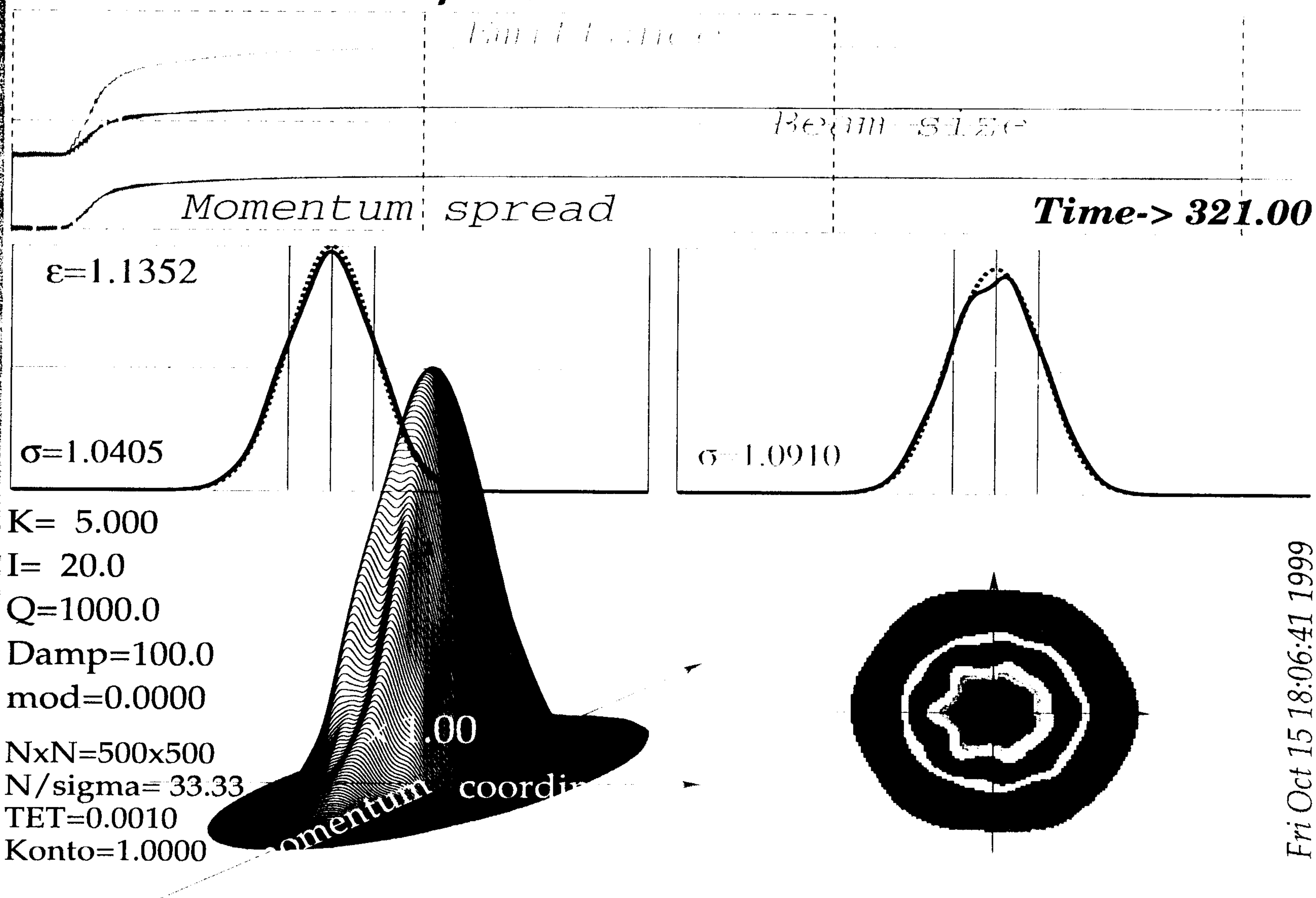
Resonator wake field.



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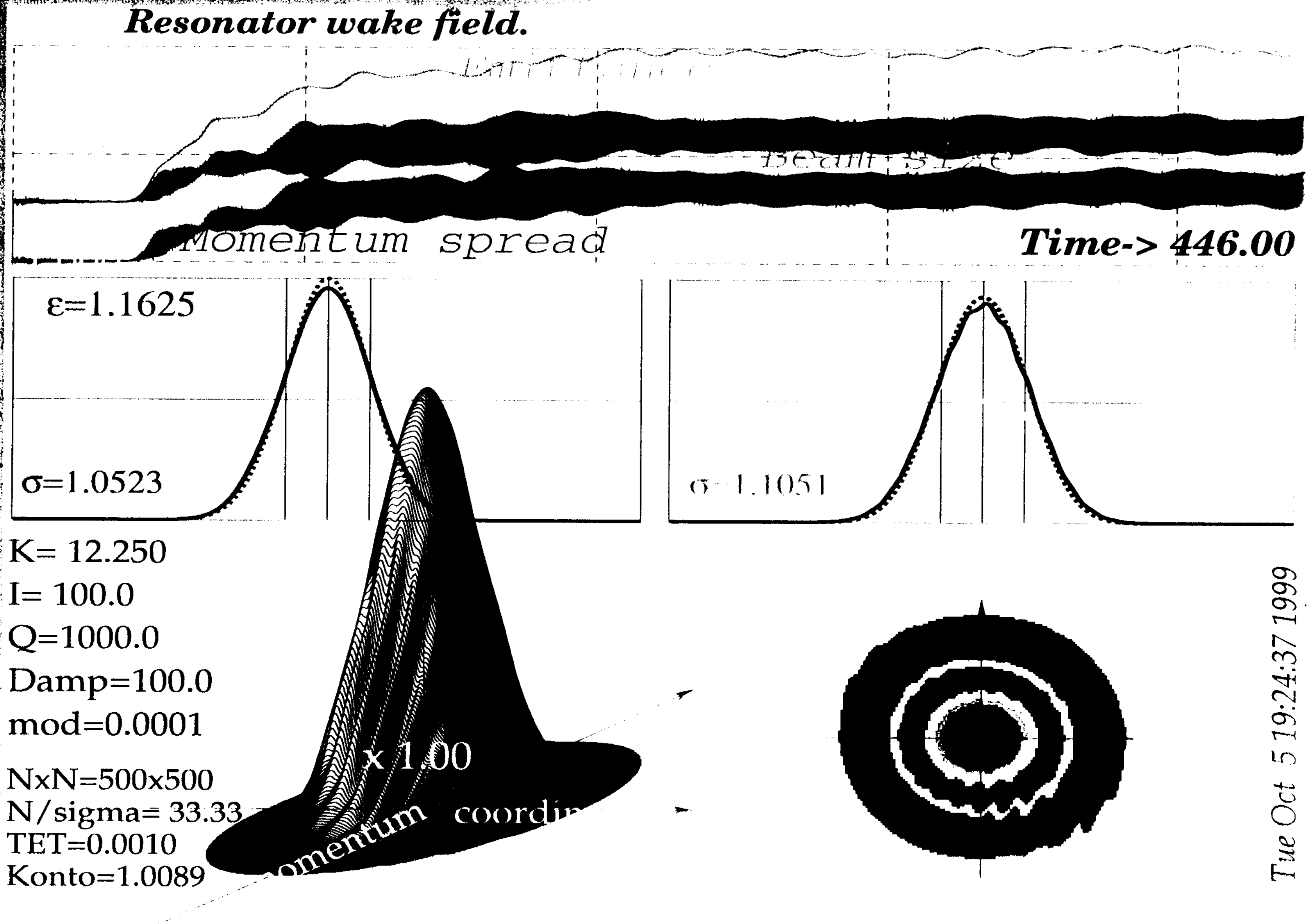
Microwave Instability. Large oscillations.



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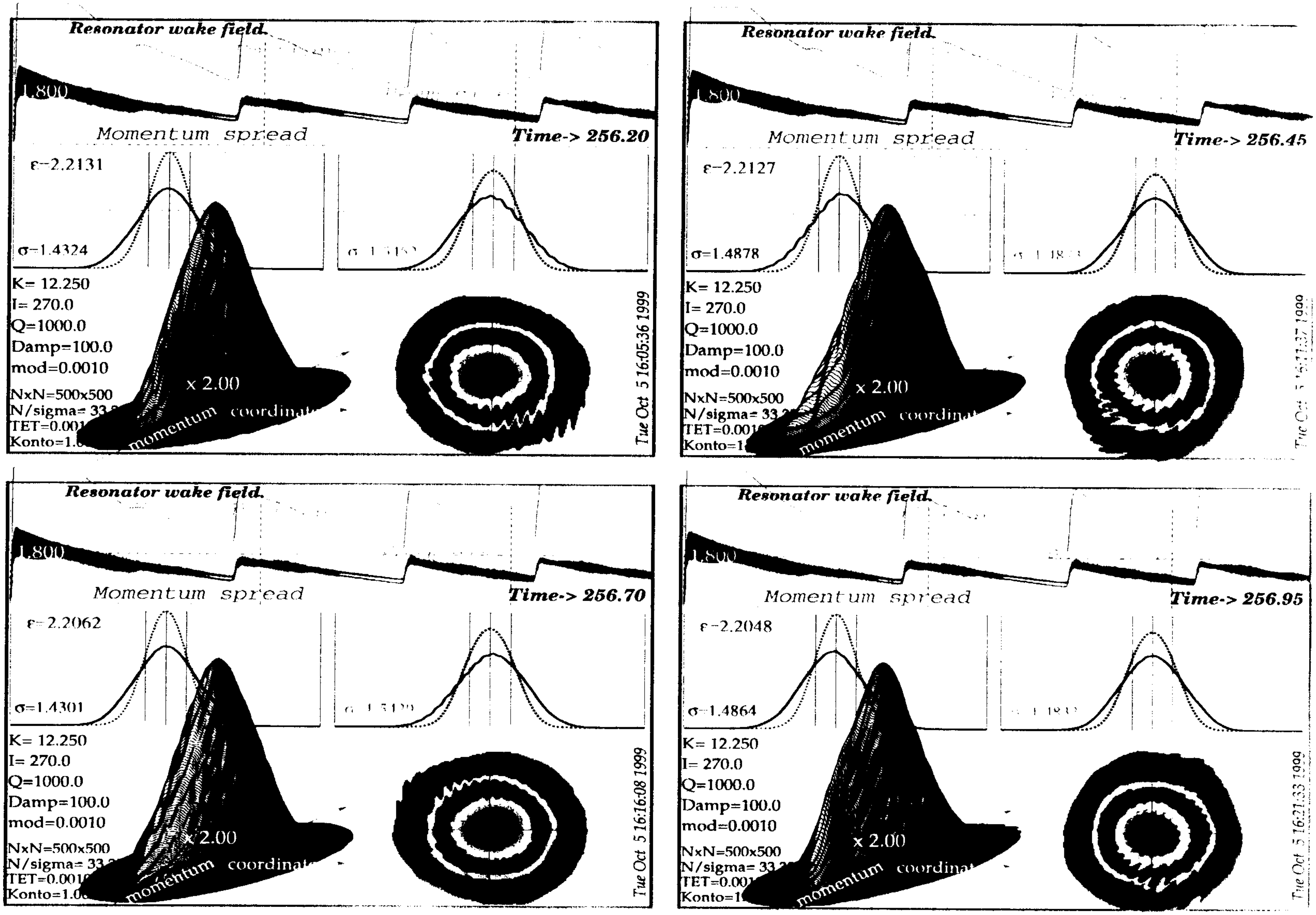
Sawtooth instability behaviour during the damping period.



Bunch shape and bunch center phase trajectory during the damping period.

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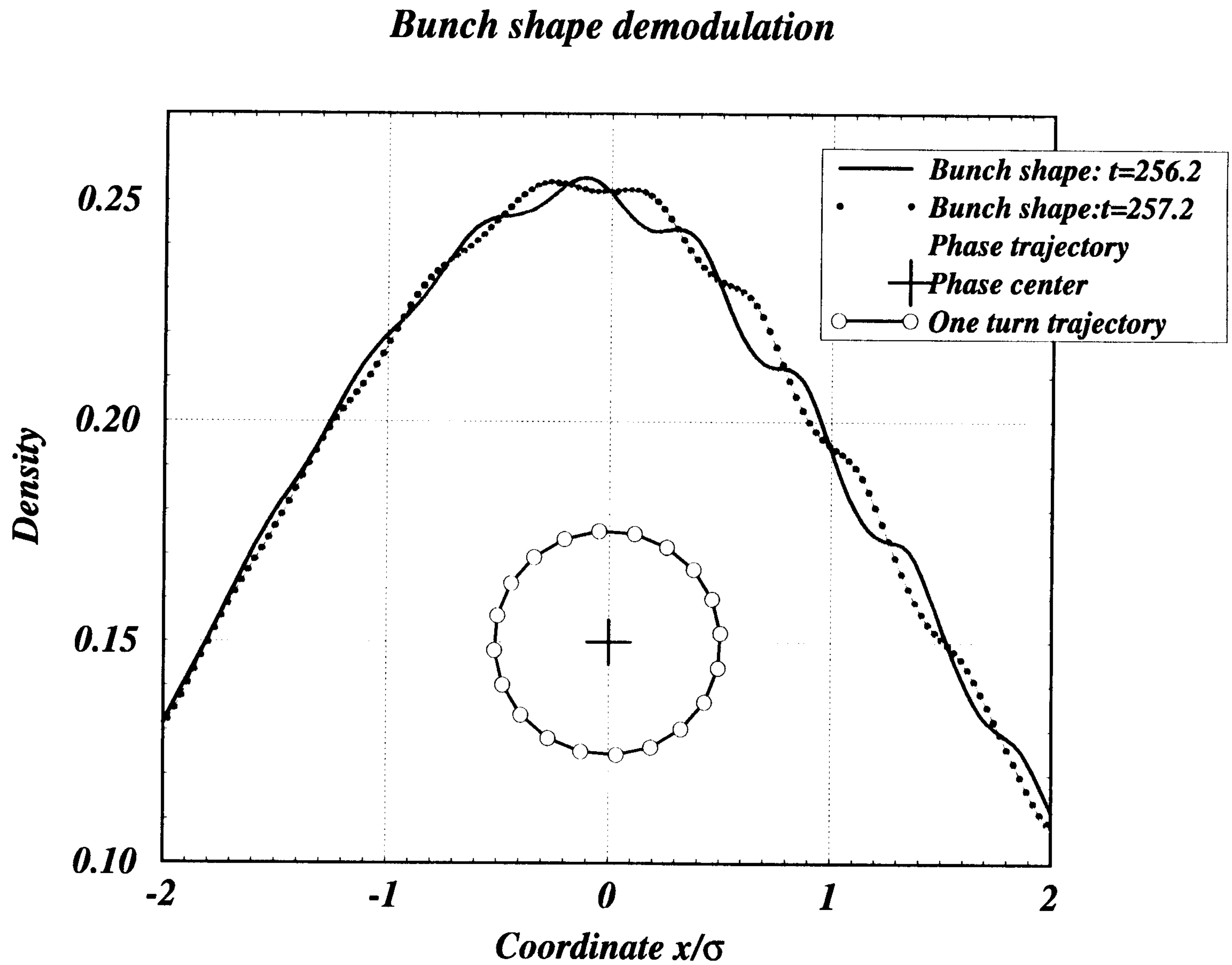

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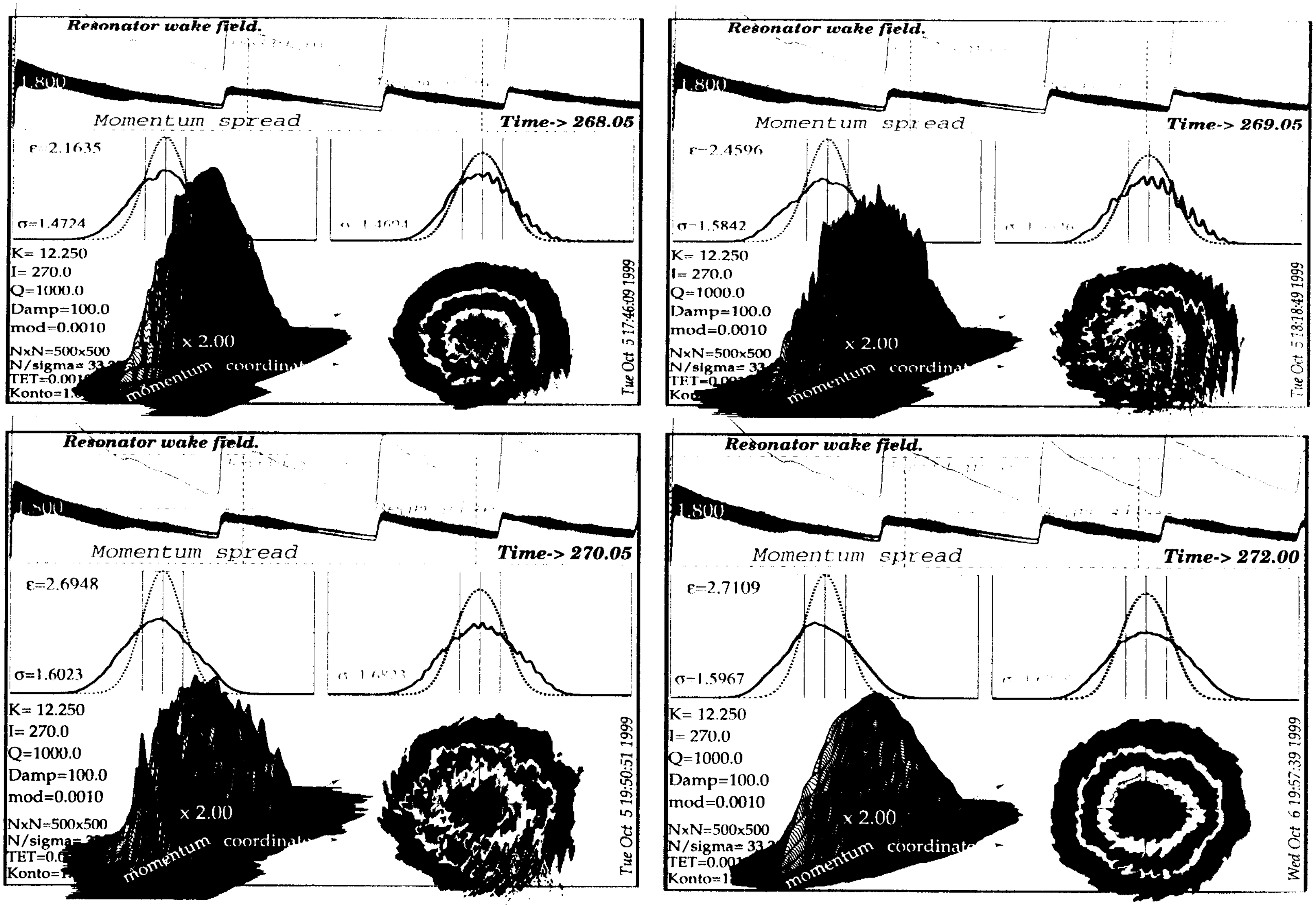
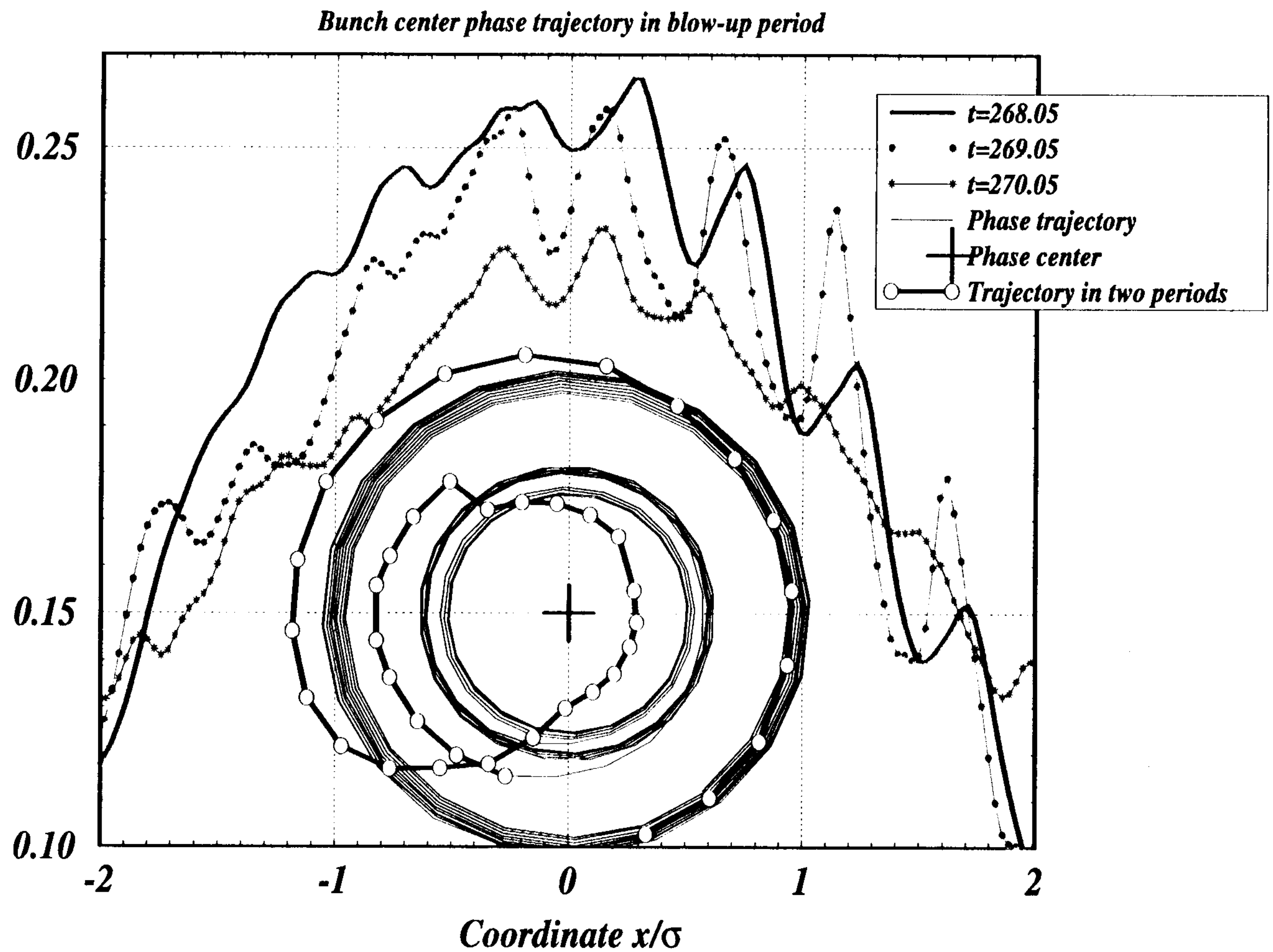
Sawtoth instability behaviour during the blow-up period



Blow-up period

- Bunch shape and phase trajectory

Bunch shape in two synchrotron periods



Stochastic bursts of radiation

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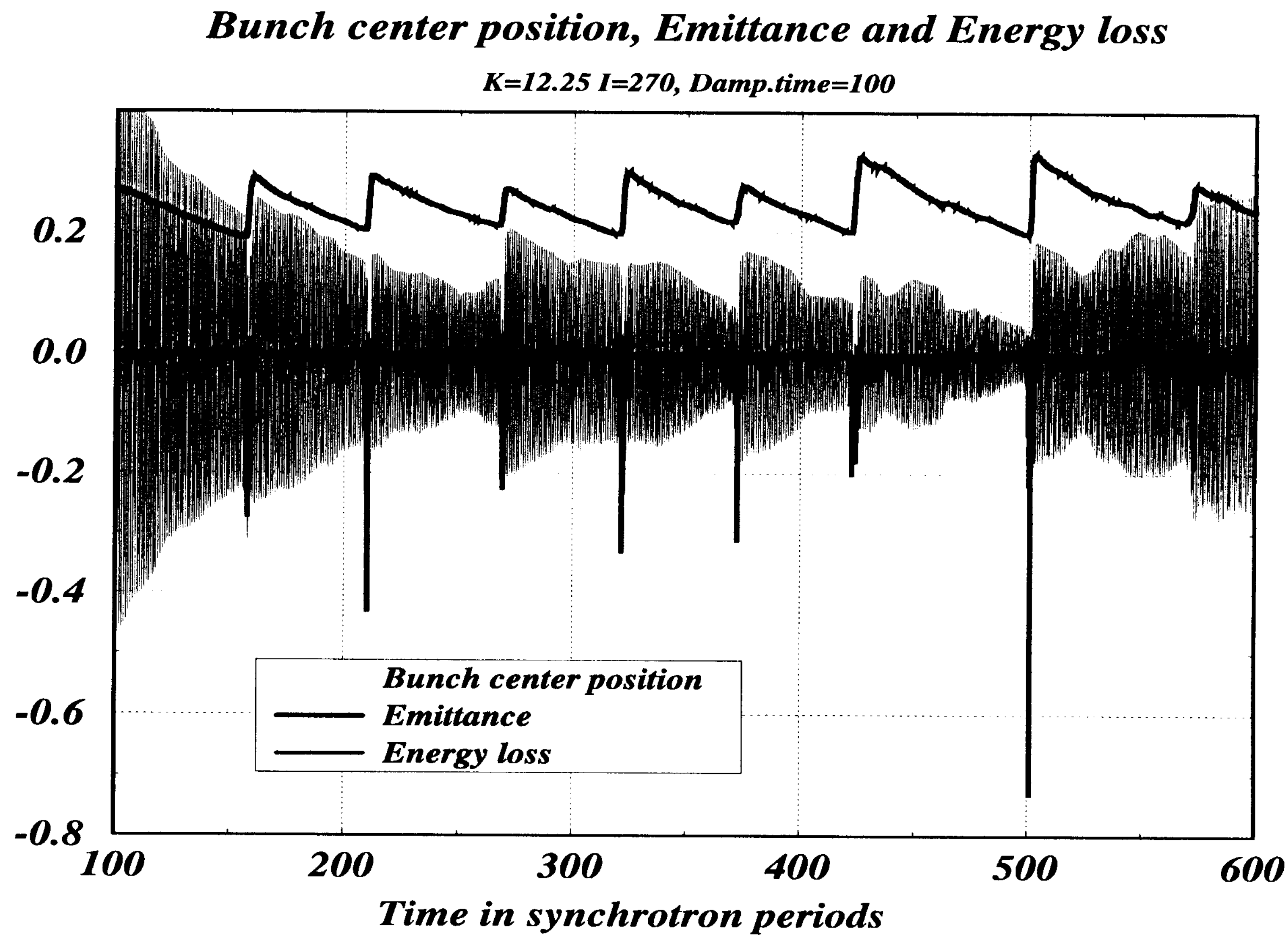
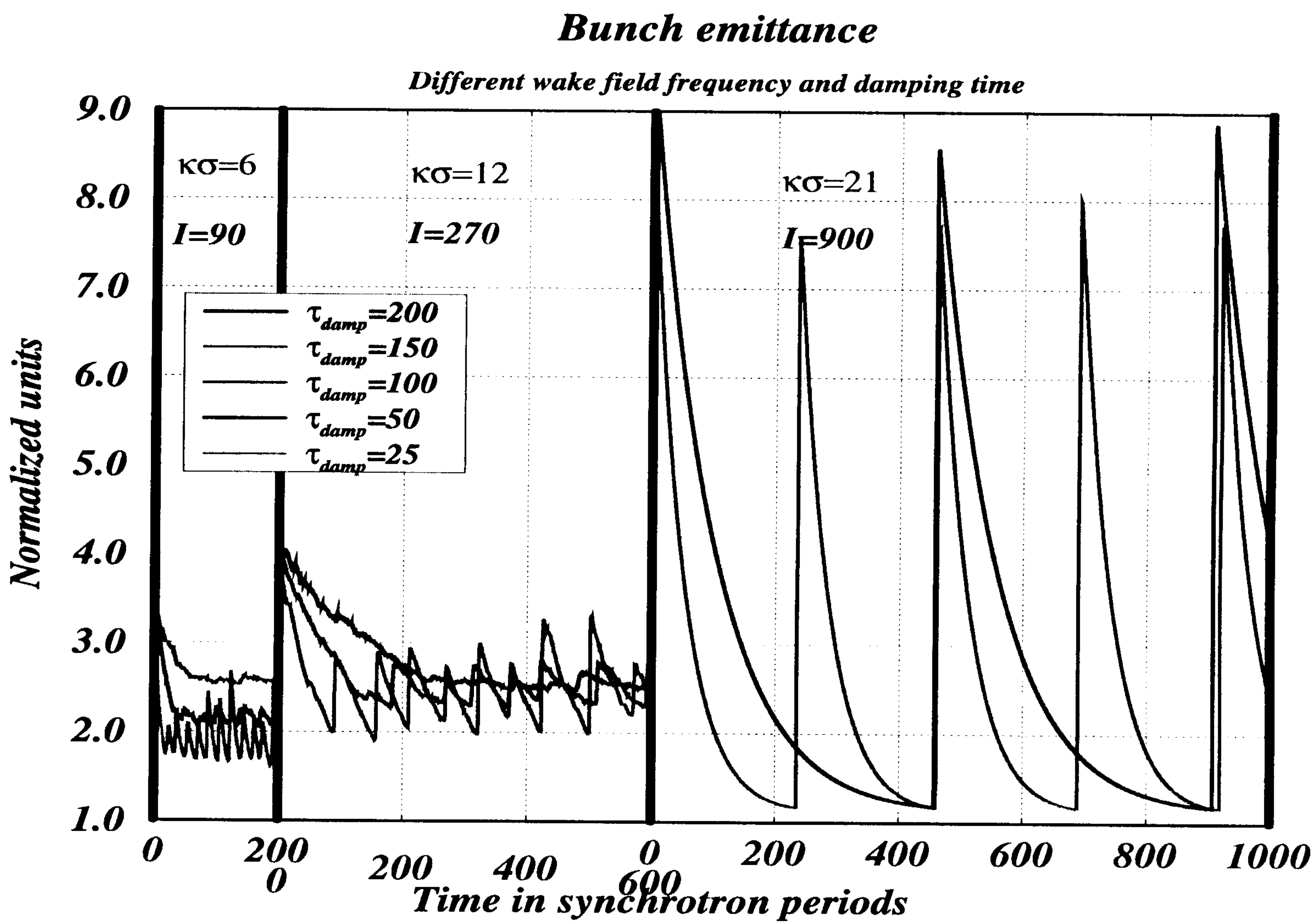


Saw Tooth Instability Behaviour

- Damping period
  - Adiabatic transition
  - Anti-resonance microbunching, 180° phase shift with each synchrotron period
- Blow-up period
  - „Metastable“ states
  - „Quick“ transition (resonance microbunching)
  - Stochastic bursts of radiation in the transition
- „Quantum“ behaviour?

Sawtooth instability and damping time



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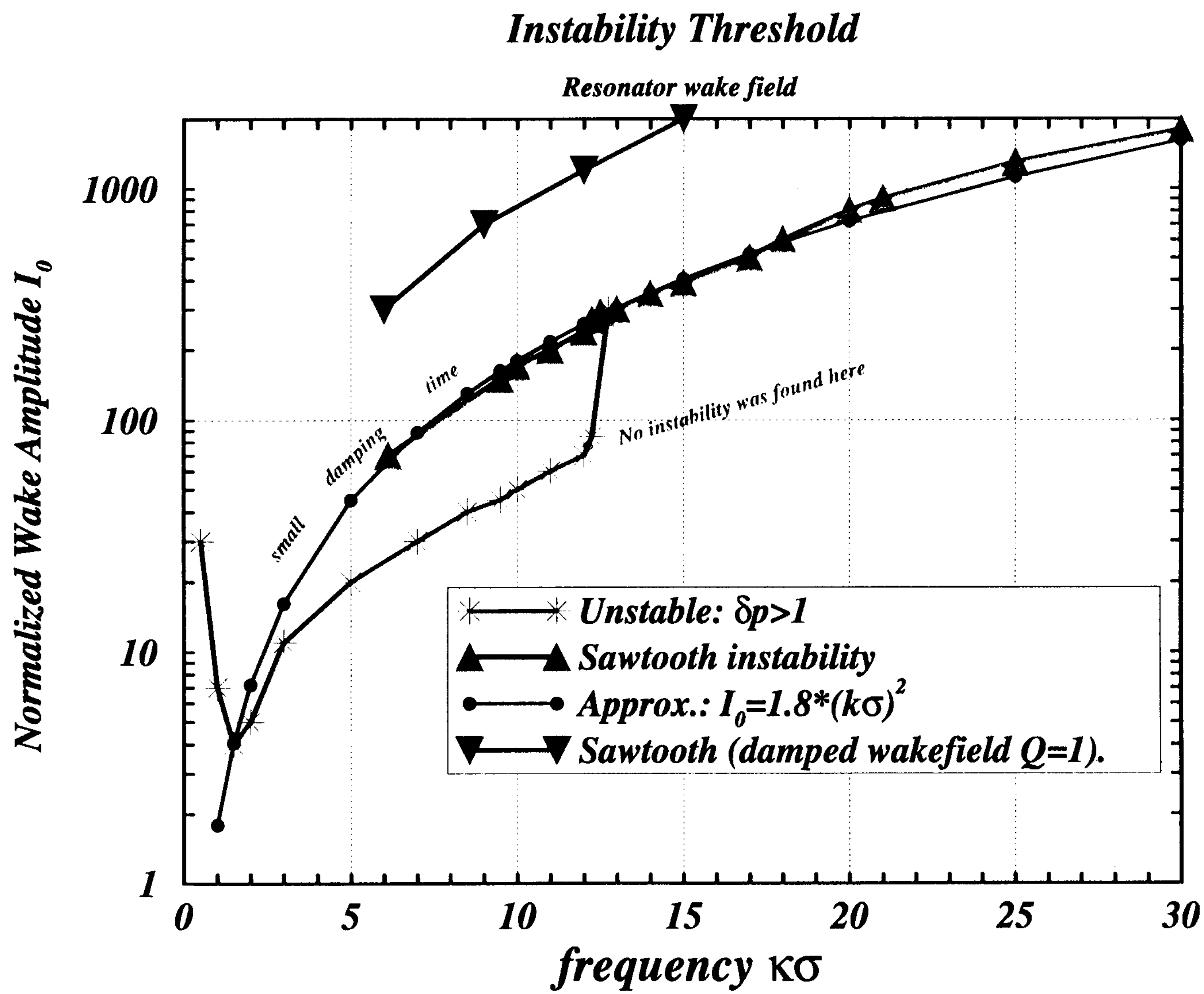
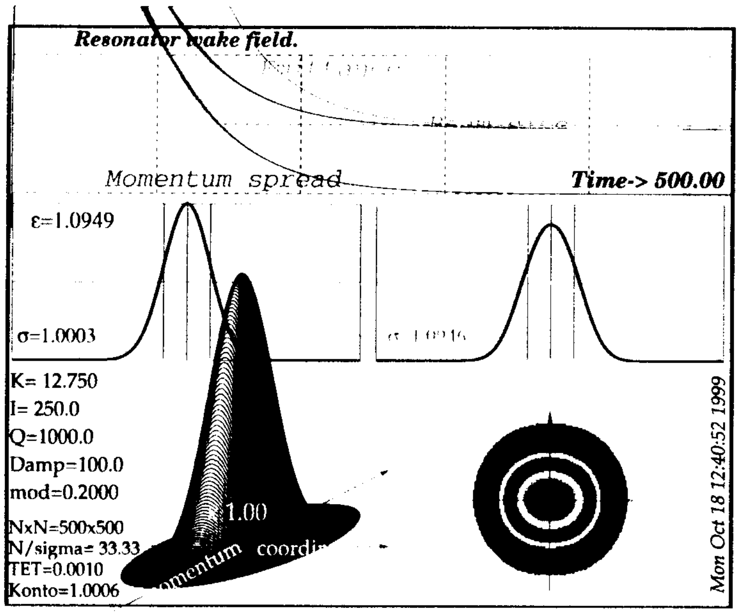
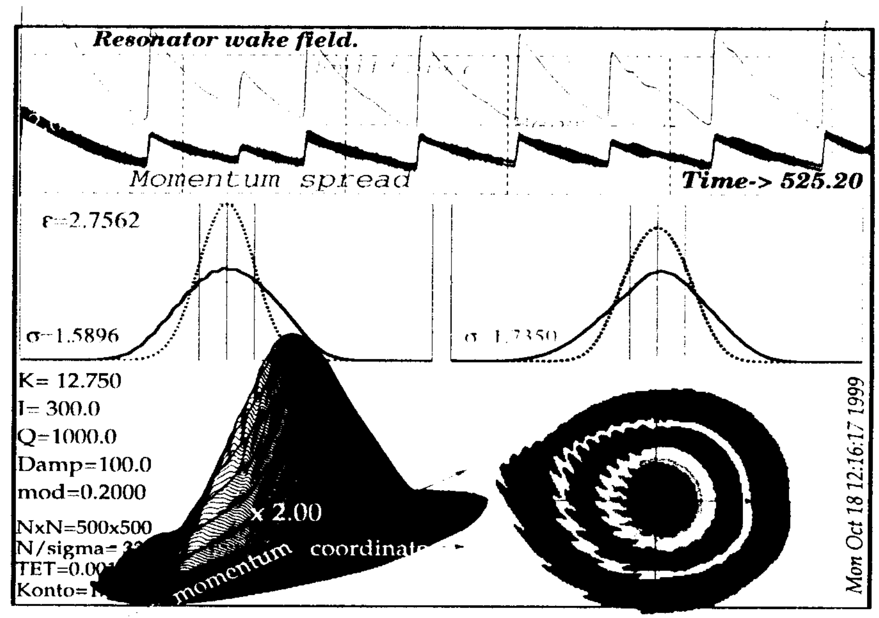
Instability threshold



No intermidiate state. Instability comes by 25% of additional charge





Estimations

- Threshold for a bunch charge (for sawtooth instability)
  - Estimation $I_{thr} \approx \left(\frac{6\sqrt{3}}{\pi}\right)^{1/2} (k\sigma_z)^2 = 1.8(k\sigma_z)^2$
  - means that
  - instability starts when the wake energy spread becomes comparable with the RF focusing

$$\Delta E_{wake} \approx \left[V_{RF} \frac{\omega_{RF}}{c} \sigma_z \right]$$

Estimation and Example

- Introducing a „roughness“ parameter for the vacuum chamber „wake“ description [redacted]

- we can give an estimation for the threshold of the bunch charge

$$q_{thr} \approx 3.6\,\eta\, h_{RF} \frac{V_{RF}}{Z_0} \frac{\sigma_z}{c} \left(\frac{\sigma_z}{R} \right)^2$$

- Example. Assuming that damping ring has the following parameters:
  - RF voltage $V_{RF} = 800kV$ Harmonic number $h_{RF} = 84$
  - Bunch length $\sigma_z = 5mm$ Effective radius of a ring $R = 5.6m$
- and „roughness“ parameter $\eta = \frac{10}{1} \frac{1}{1\%} = 10^3$

- we can get estimation for the charge $q_{thr} = 8.4nC$

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Summary

- Simulation study of the longitudinal bunch instability was performed for the very high frequency wake fields, that are usually described by the inductive impedance.
- The Fokker-Planck equation for the phase space distribution is solved numericaly by using the original implicit finite-difference method.
- It was found that in the high frequency range the bunch instability has mainly „sawtooth" character and is accompanied by the bursts of coherent radiation, stochastically distributed in time..
- This effect has to be taken into consideration in the designs of the damping rings for the strong cooling of very high intencity bunches.