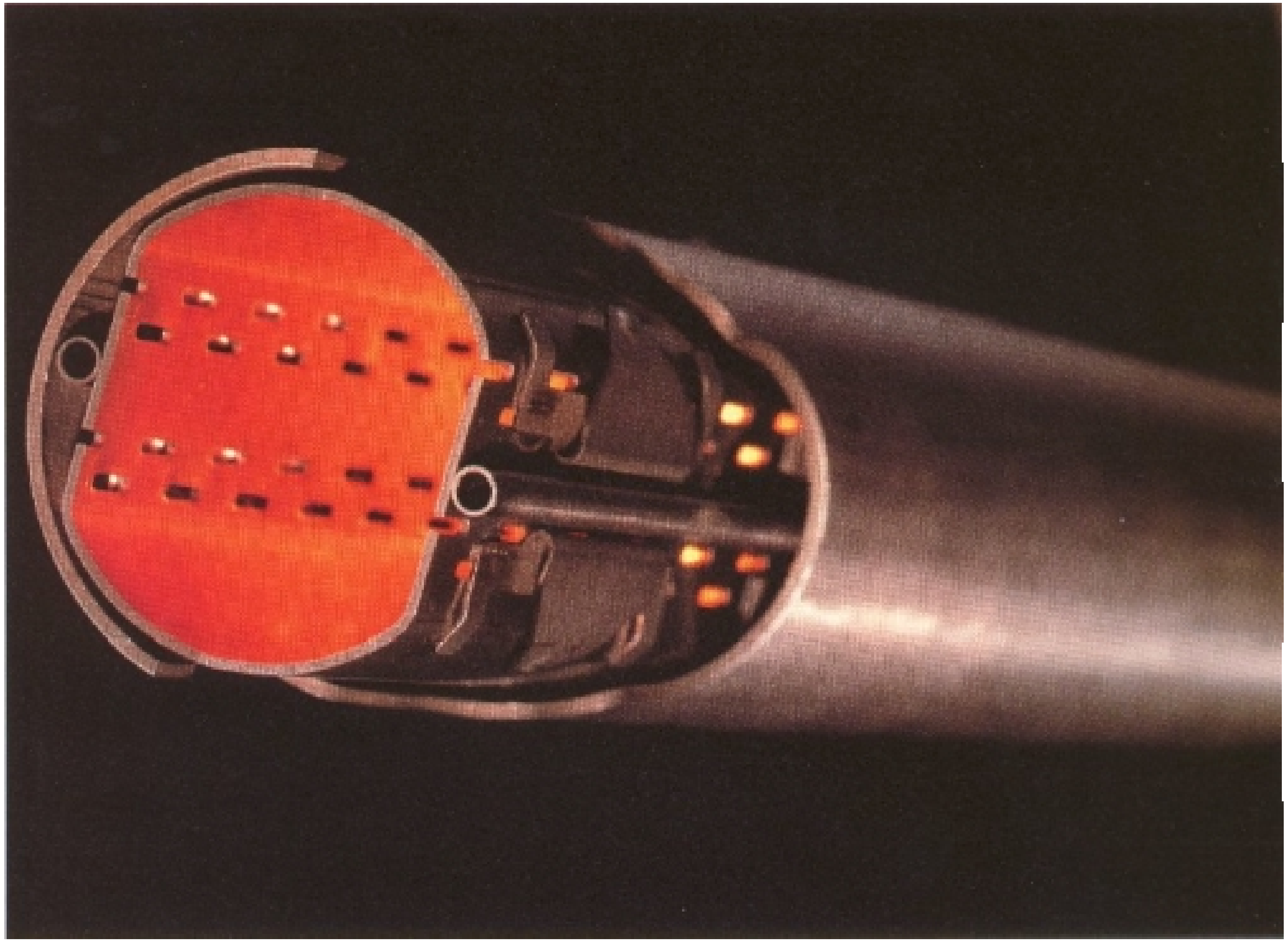
Analytical calculation of the power dissipated in the LHC liner

Stefano De Santis - LBNL
and
Andrea Mostacci - CERN

Contents

- What is the Modified Bethe's Diffraction Theory ?
- Some interesting consequences of the expressions found.
- Power dissipation in the LHC liner.

LHC Beam Pipe



Equivalent dipole (and quadrupole) moments of an aperture in the vacuum chamber

The electromagnetic field can be expressed as a sum of modes:

$$\vec{\mathbf{E}}(\vec{r},\omega)=\sum c_n\vec{\mathbf{E}}_n(\vec{r},\omega),\ \vec{\mathbf{H}}(\vec{r},\omega)=\sum c_n\vec{\mathbf{H}}_n(\vec{r},\omega)$$

The expansion coefficients can be obtained through equivalence and reciprocity theorems:

$$c_n=\frac{1}{2}\int\vec{\mathbf{H}}_n\cdot\vec{\mathbf{J}}_n dS$$

Beware: when dealing with resonating modes the above formulas are different

Expanding the magnetic field in a Taylor series about the origin

$$\vec{\mathbf{H}}_n(\vec{r})=\vec{\mathbf{H}}_n(0)+\left.\frac{u\partial\vec{\mathbf{H}}_n}{\partial u}\right|_{u=v=0}+\left.\frac{v\partial\vec{\mathbf{H}}_n}{\partial v}\right|_{u=v=0}+\ldots\approx\vec{\mathbf{H}}_n(0)+\vec{r}\cdot\nabla\vec{\mathbf{H}}_n\Big|_{u=v=0}\quad\Rightarrow$$

$$\Rightarrow\quad c_n=\frac{j\omega}{2}\left(\mu\vec{\mathbf{H}}_n\cdot\vec{\mathbf{M}}-\vec{\mathbf{E}}_n\cdot\vec{\mathbf{P}}+\frac{\mu}{2}\nabla\vec{\mathbf{H}}_n\cdot\overleftrightarrow{\mathbf{Q}}\right)$$

Bethe's Diffraction Theory (BDT)

Longitudinal impedance:

$$Z(\omega) = -\frac{1}{q}\int_{-\infty}^{+\infty} \vec{E}\cdot d\vec{\ell}$$

Loss factor:

$$k_\sigma = \frac{1}{\pi}\int_0^{+\infty} \mathrm{Re}[Z(\omega)]e^{-(\omega\sigma_z/c)^2}d\omega$$

The scattered electromagnetic field is that radiated by the equivalent dipole moments: $\vec{E} = f\left(\vec{M},\vec{P}\right)$

The equivalent dipole moments are calculated from the aperture static polarizabilities (geometric parameters) and the incident field:

$$\vec{M} = \vec{\vec{\alpha}}_m \cdot \vec{H}_0,\ \vec{P} = \varepsilon\vec{\vec{\alpha}}_e \cdot \vec{E}_0$$

Modified BDT

Bethe's diffraction theory gives a good approximation for the imaginary part of the impedance only. Real impedance and thus loss factor are null; energy is not conserved.
In the modified version of the theory [Collins] radiation reaction fields are introduced in the dipole moments calculation:

$$\vec{M} = \vec{\alpha}_m \cdot (\vec{H}_0 + \vec{H}_s), \ \vec{P} = \varepsilon\vec{\alpha}_e \cdot (\vec{E}_0 + \vec{E}_s)$$

The reaction fields dipend on the polarizability tensors, so that the dipole moments components are obtained solving a linear system:

$$[\underline{S}]\begin{bmatrix} \vec{M} \\ \vec{P} \end{bmatrix} = \begin{bmatrix} \vec{\alpha}_m \cdot \vec{H}_0 \\ \varepsilon\vec{\alpha}_e \cdot \vec{E}_0 \end{bmatrix}$$

The coefficients matrix [S] is a function of the modes chosen to represent the electromagnetic fields. Its expression is particularly simple in the low frequency approximation, when only one propagating mode is used.
The terms outside the principal diagonal represent coupling between apertures.

BDT vs. Modified BDT

- For a single hole the modified theory adds up a frequency dependant term in the dipole moments expression.
  For example at low frequency in a coaxial beam pipe:

$$M_\varphi = \frac{\alpha_{m\perp} H_{0\varphi}}{1 + j\dfrac{\alpha_{m\perp}\omega / c}{4\pi b^2 \ln(d/b)}}, \quad P_r = \frac{\varepsilon\alpha_e E_{0r}}{1 + j\dfrac{\alpha_e \omega / c}{4\pi b^2 \ln(d/b)}}$$

where the original theory would only give:

$$M_\varphi = \alpha_{m\perp} H_{0\varphi}, \; P_r = \varepsilon\alpha_e E_{0r}$$

- In the presence of multiple holes the difference is even more apparent since the modified theory takes into account the coupling between holes that the original one disregards entirely.

Differential Modified BDT

The Modified BDT allows to calculate the interaction between multiple apertures (if they are distant enough), but it cannot be used when the aperture dimensions are larger than the wavelength.
If this is the case, one can resort to use dynamic polarizabilities, that do not depend on the aperture geometry only and requires complex calculations, or subdivide the aperture in infinitesimal elements still satisfying the requirements for using the BDT and then take into account their interaction.
The dipole moments are replaced by differential dipole moments

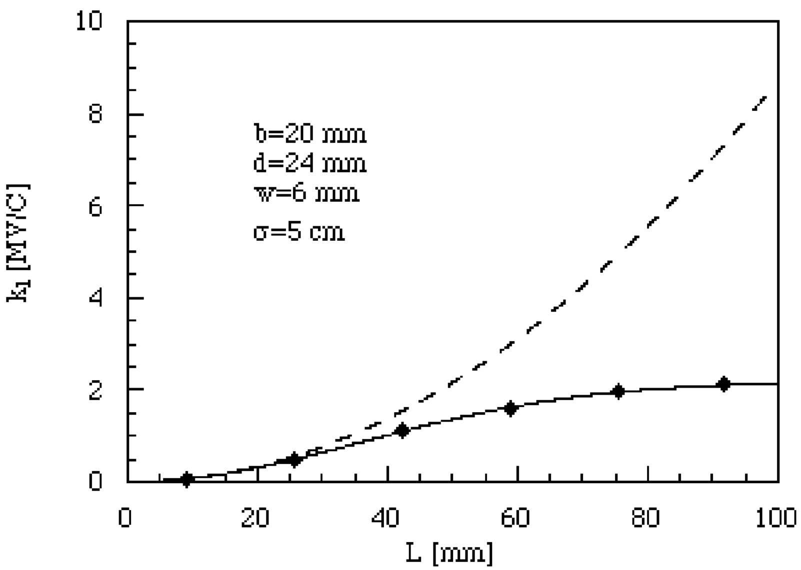
$$d\vec{M} = d\ddot{\alpha}_m \cdot (\vec{H}_0 - \vec{H}_s),\ d\vec{P} = \varepsilon d\ddot{\alpha}_e \cdot (\vec{E}_0 - \vec{E}_s)$$

and integral equations take linear system place. In the case of a long narrow slot:

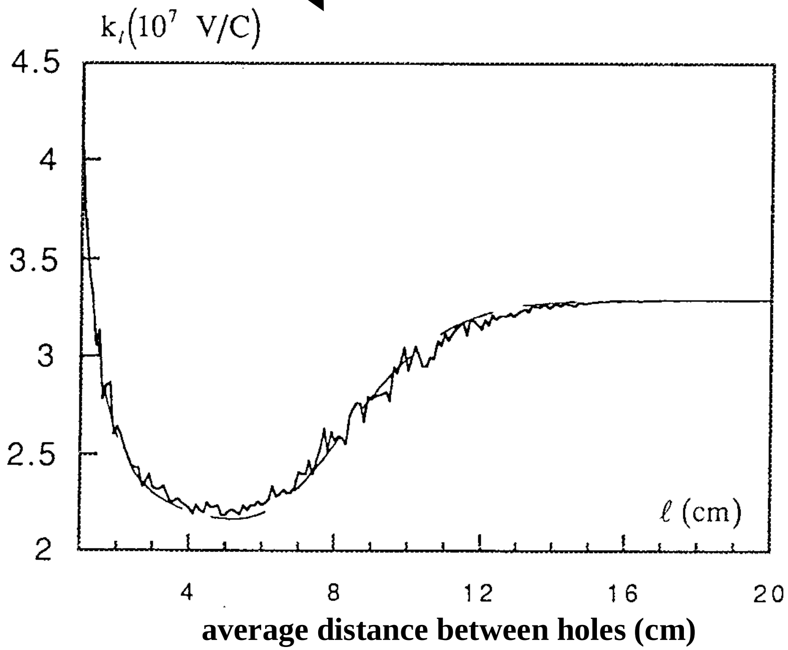
$$\frac{dM_\varphi}{dz} = \frac{\alpha_m}{L}\left[H_{0\varphi} - j\frac{\omega\mu h_{0\varphi}^2}{2}\int_{-L/2}^{L/2}\frac{dM_\varphi}{d\xi}e^{-jk_0|z-\xi|}d\xi + j\frac{\omega h_{0\varphi}e_{0r}}{2}\int_{-L/2}^{L/2}sign(\xi - z)\frac{dP_r}{d\xi}e^{-jk_0|z-\xi|}d\xi\right]$$

$$\frac{dP_r}{dz} = \frac{\varepsilon\alpha_e}{L}\left[E_{0r} - j\frac{\omega\mu e_{0r}^2}{2}\int_{-L/2}^{L/2}\frac{dP_r}{d\xi}e^{-jk_0|z-\xi|}d\xi + j\frac{\omega\mu h_{0\varphi}e_{0r}}{2}\int_{-L/2}^{L/2}sign(\xi - z)\frac{dM_\varphi}{d\xi}e^{-jk_0|z-\xi|}d\xi\right]$$

As direct consequence of the expressions calculated, one can find both expected and not-so-expected results:



Loss factor vs. length for a narrow slot (solid line), its static approximation (dashed line) compared to MAFIA simulations (black diamonds)



Loss factor for 15 round holes randomly spaced (b=20 mm, d=24 mm, R=6 mm, l=300 mm, sigma=5 cm).

The loss factor is proportional to the number of holes squared.

LHC liner - Summary

- Impedance (beam dynamics)
- Power load on cold bore (cryogenics)

A complex propagation constant has been included in our formalism.
This is suitable in the treatment of both lossy and periodic loaded structures

Main results:

- Randomization of the slots position strongly decreases the peaks of the real impedance. The imaginary part and the loss factor are unaffected.
- Dissipated power per unit length estimated around 1 mW / m.
- Identified two different regimes for the power loss per unit length:

Short device

$$P_{lin} = \frac{\sqrt{\pi}}{128\pi^4}\frac{Z_0 Q_b^2 c^2}{\sigma^3 S_b}\frac{(\alpha_m + \alpha_e)^2}{b^4 D \ln(d/b)} L$$

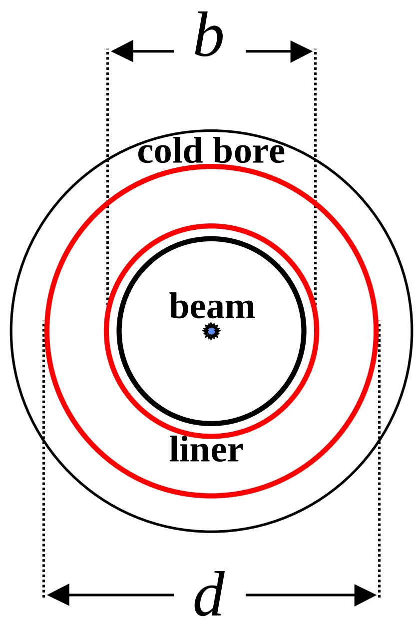
Long device

$$P_\infty = \left\{\frac{Z_0 Q_b^2 c^2}{\sigma^3 S_b}\left[\frac{\sqrt{\pi}}{8} + \Gamma(5/4)\frac{\sqrt{\sigma\omega/c}}{2\alpha D}\right]\right\}\frac{(\alpha_m + \alpha_e)^2}{b^4 D \ln(d/b)}$$

Effect of ohmic losses

Approx: distributed "dielectric" losses

$$\alpha(\omega) = \frac{\sqrt{\rho}}{2Z_0 \ln(d/b)}\left(\frac{1}{b}+\frac{1}{d}\right)\sqrt{\frac{\mu}{2}}\sqrt{\omega}$$



Analytical formulae:

•Dipole moments

•Coupling impedance

•Loss Factor $k(\sigma)$

•Power lost per unit length P

$$P = \frac{c\,Q^2 k(\sigma)}{S_b L}$$

S_b bunch spacing

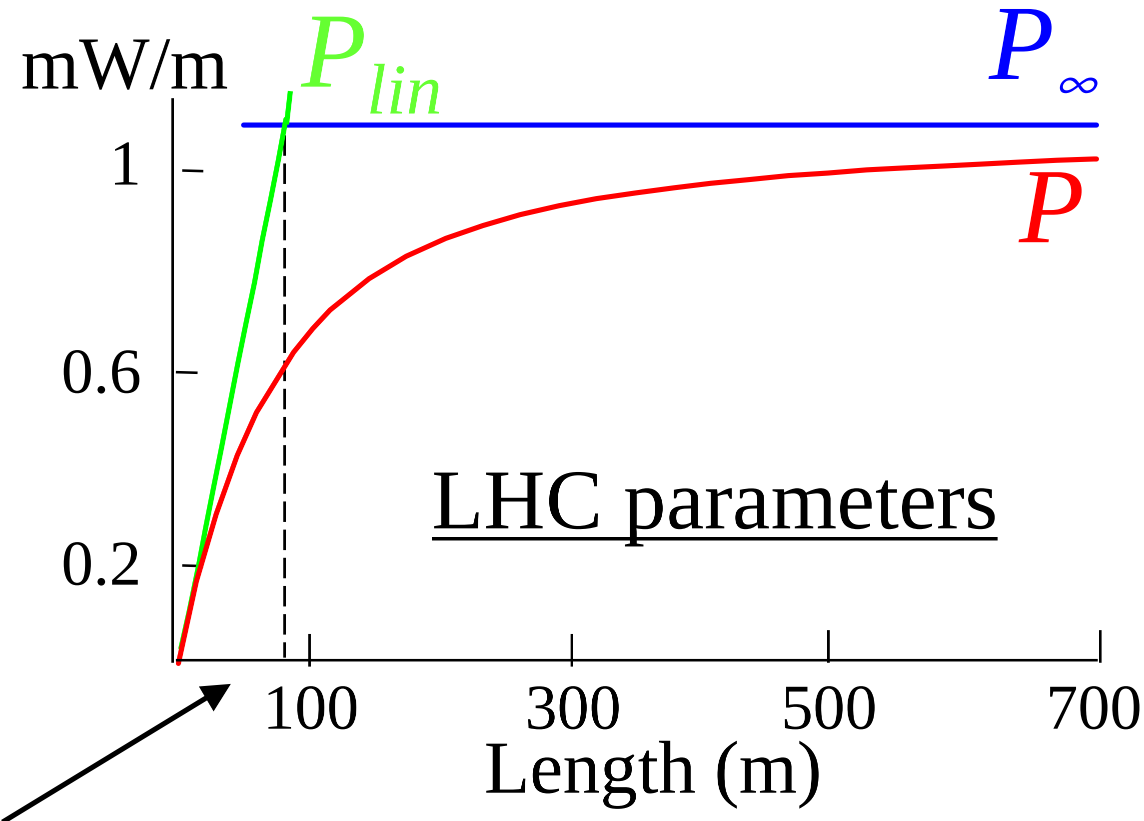
L device length

Power lost per unit length

P_{lin} no attenuation

P_∞ limit value

P "exact" formula



$$L_\alpha = \frac{4}{\sqrt{\pi}}\Gamma\left(\tfrac{5}{4}\right)\frac{1}{\alpha(\omega_c)}$$

ω_c Bunch cut-off angular frequency

Power lost per unit length

- The power lost per unit length is important in real machines (LHC) to estimate the heating of the cold bore
- L_α is ~80 m for LHC
- For small length P is still function of the length (interference effects)
- Saturation effect for long perforated chambers (the theory with no losses gives an infinite power lost)
- In LHC we remain in the linear regime (small device lengths), since the coaxial structure is interrupted by many devices (the dipole modules length is 14 m)
- The saturation value is an upper limit useful in the project
- Only a part (~ 45%) is dissipated in the external wall (r=d)

LHC beam screen

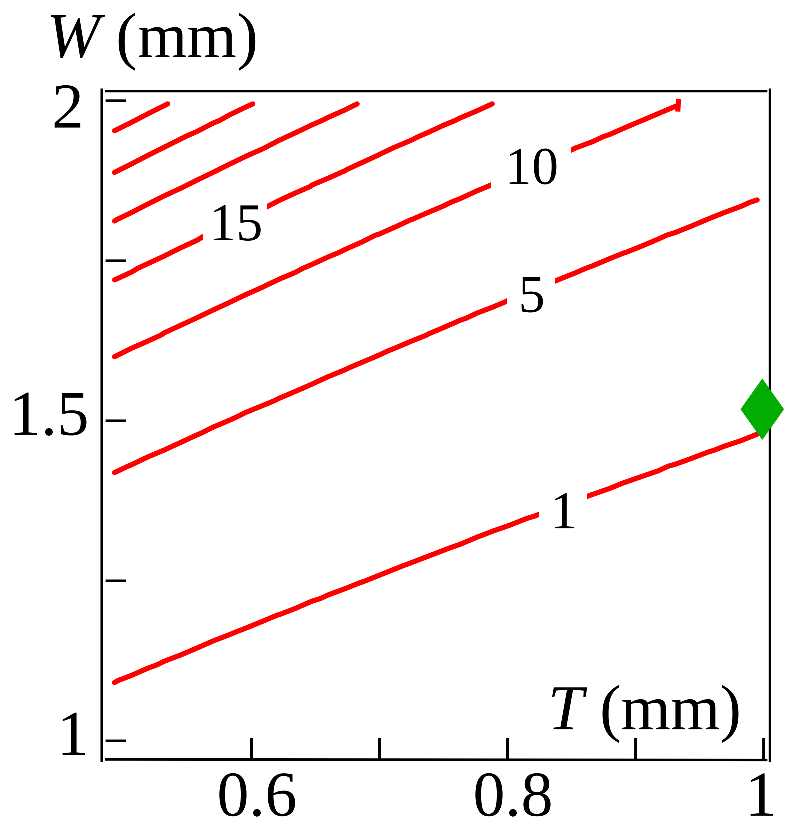
Around LHC nominal values (♦):

$$P_\infty \approx P_0 \, Exp\left(-1.75\pi \, T / W\right)$$

$$P_0 = 42 \, mW / m \left(\frac{W}{1.5 \, mm}\right)^4$$

W slot width *T* wall thickness

Curves of constant
power per unit length (mW/m)



From:
A. Mostacci and F. Ruggiero, LHC Project Note 195

Comparison with previous studies

F. Caspers, E. Jensen and F. Ruggiero, EPAC, Berlin '92

- Measurements on a 2 m long model of LHC vacuum chamber
- Simplified theory accounting only forward TEM wave

$$\left| \frac{\text{Electric Field radiated in coax. region}}{\text{Source Electric Field}} \right| \approx N \frac{\left| \alpha_m + \alpha_e \right|}{2\, b^2 \ln\left(d/b\right) \lambda}$$

- Calculated transmission coefficient is identical to what reported in EPAC '92.
- Both theoretical values are a factor 2 below measurements.

LHC beam screen

- The holes are such that the slotted surface is 4.4 % of the total surface (fixed from vacuum requirements)
- strong dependence on T and W
- The formula is a simple analytic fit to see the behavior around nominal values; it tells what happens if you reduce T or increase W
- The nominal LHC is the green point ($P_{\infty} \approx 1.1$ mW/m)
- Conclusion: NO DANGER