

Multiparticle
Longitudinal Motion
Tracking code
with Broadband
impedance

C.limborg SSRL, 1990

Aknowledgements: G. Besnier, A. Hofmann

ESRF team: Laclare, Gunzel, Nagaoka, Farvaque...

SSRL version: J. Sebek

Longitudinal Equations of motion

- Applied to each particle (τ, δ)

- $\delta_{n+1} - \delta_n = 2\pi Q_{so} (\sin(\varphi_{so} + h\omega_o \tau_n) - \sin(\varphi_{so}))$ RF

$$-\frac{2\pi I}{hV_{rf} \cos(\varphi_{so}) \omega_o^2 \sigma_{\tau o}} G_{\lambda}(\tau_n) \quad \text{Wake}$$

$$-\frac{T_o}{T_{damp}} \delta_n + 2 \sqrt{\frac{T_o}{T_{damp}}} R_n$$

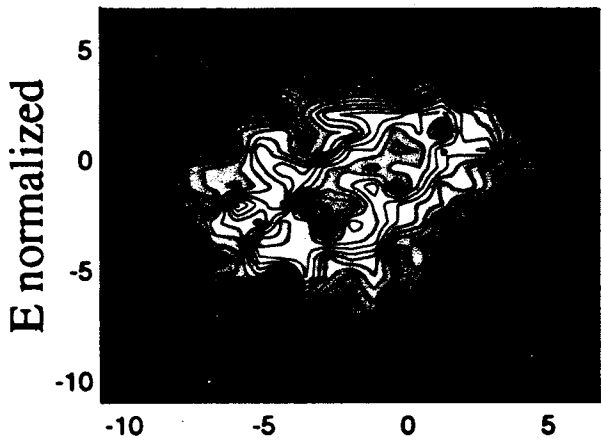
Damping Fluctuations

- $\tau_{n+1} - \tau_n = 2\pi Q_{so} \delta_{n+1}$ Variation of path length

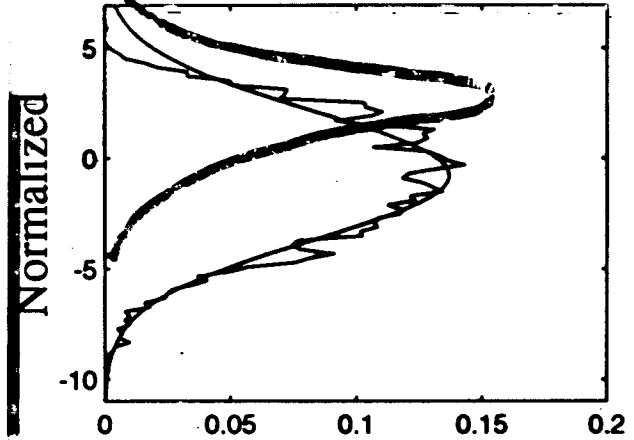
$$G_{\lambda} = G * \lambda ; \quad G = \frac{R_s \omega_r}{Q} e^{-\frac{\omega_r z}{2a}} \left[\cos(\omega_r' z) - \frac{\sin(\omega_r' z)}{\sqrt{Q^2 - 1}} \right]$$

$$R_s, Q, \omega_r$$

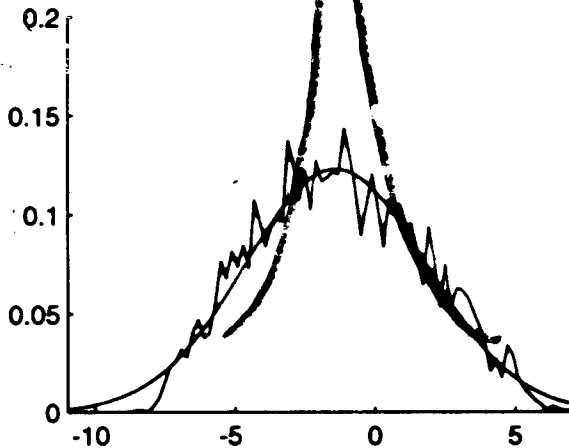
Equilibrium



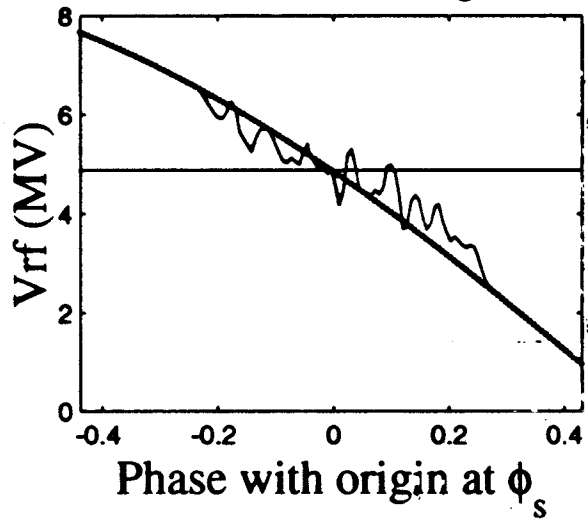
Energy Distribution



Time Distribution

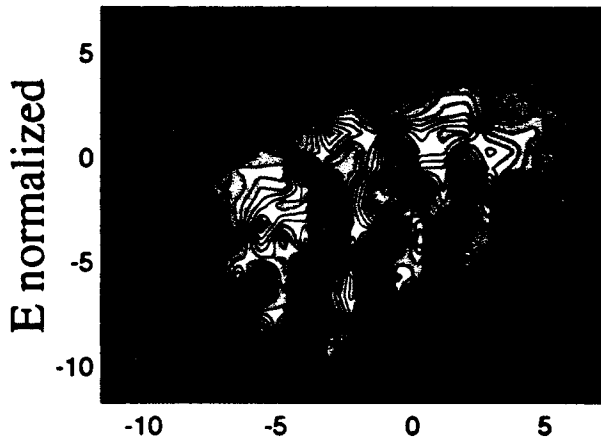


Effective Voltage

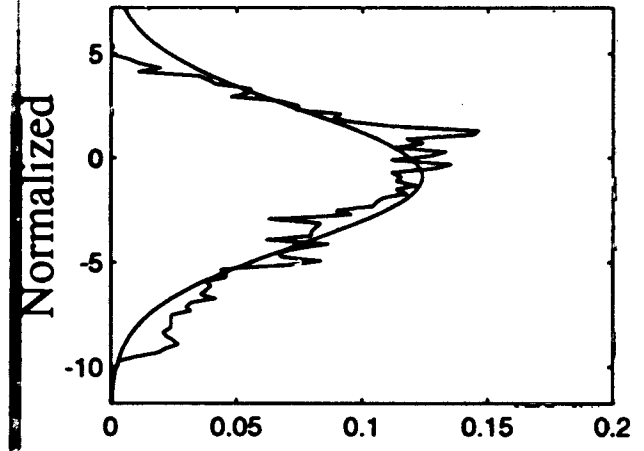


τ normalized
 with dimensionless plane
 broad at $\tau = 0$ to the
 point where the
 value is measured
 a little more and has lost
 some strength

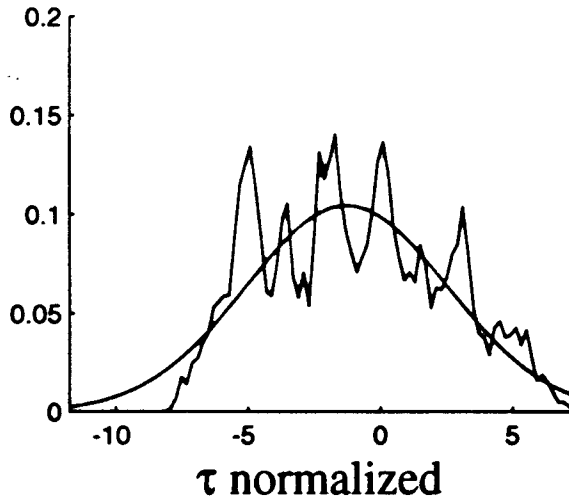
Distribution in phase Space



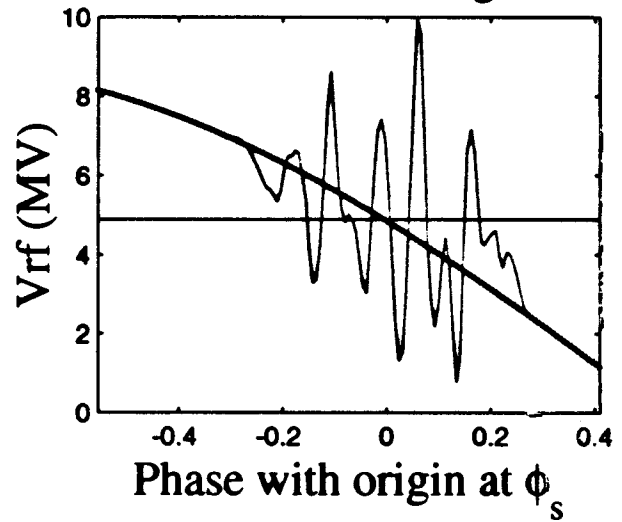
Energy Distribution



Time Distribution



Effective Voltage



With ESRF parameters

- 6 GeV, $T_0 = 2.8 \mu\text{s}$
- $\sigma_\tau = 15.5 \text{ ps}$; $\sigma_{\omega} = 10 \text{ GHz}$
- $\tau_{\text{damping}} = 3.5 \text{ ms}$ 1236 Turns
- $\tau_s = 0.487 \text{ ms}$ 173 Turns

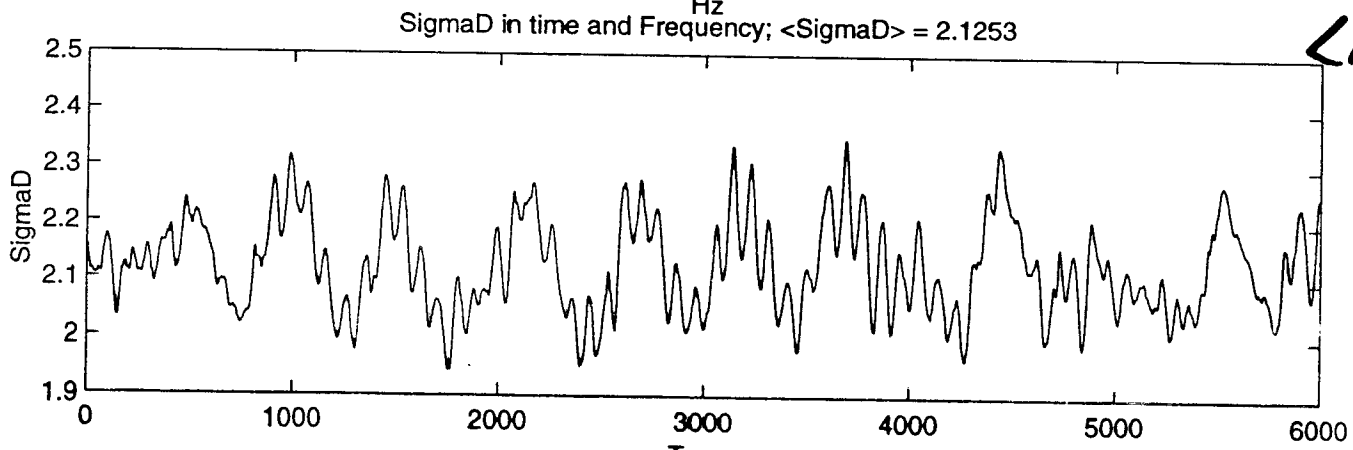
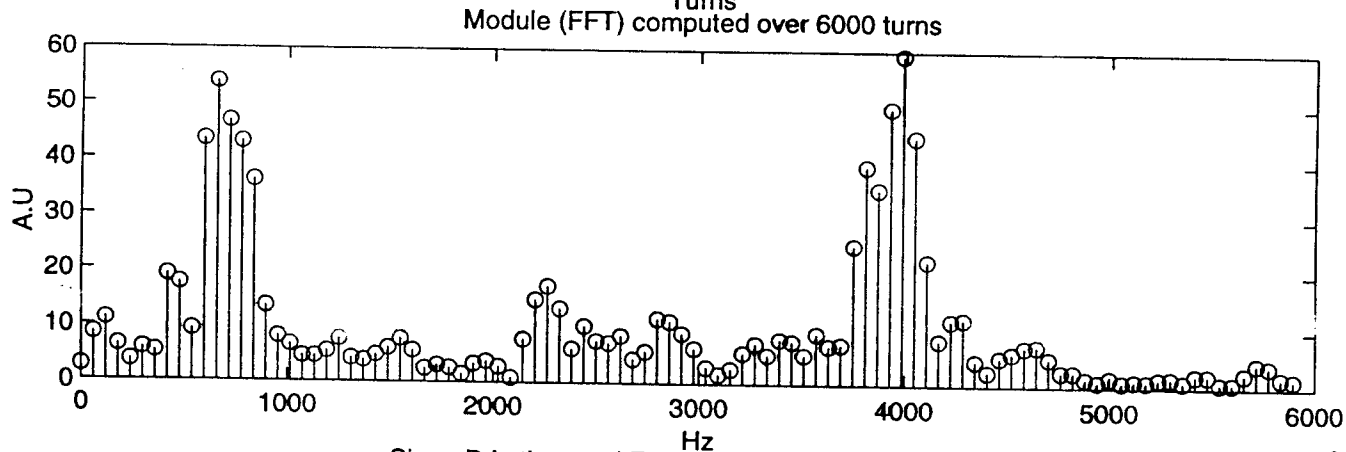
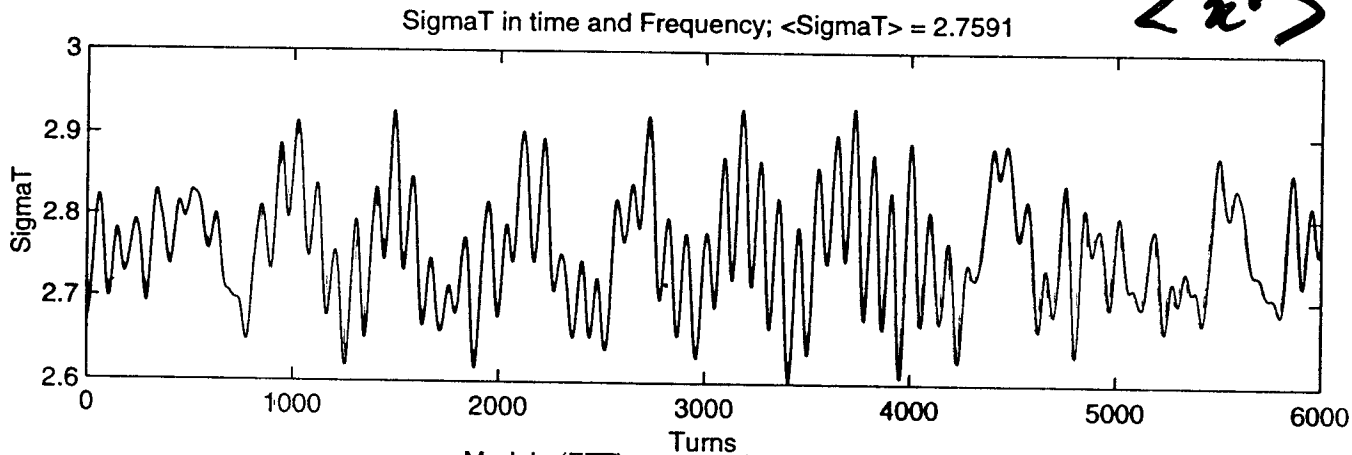
Simulation 600 Turns ($\approx 3Z_s$) 1 image / 5 turns

- total bunch $10 \sigma_\tau \approx 150 \text{ ps}$
- $30 \text{ GHz} \Leftrightarrow 33 \text{ ps} \Rightarrow 5 \lambda_r$
- $15 \text{ GHz} \Leftrightarrow 66 \text{ ps} \Rightarrow 2.5 \lambda_r$
- $7 \text{ GHz} \Leftrightarrow 141 \text{ ps} \Rightarrow 0.94 \lambda_r$
- $3.5 \text{ GHz} \Leftrightarrow 282 \text{ ps} \Rightarrow 0.47 \lambda_r$

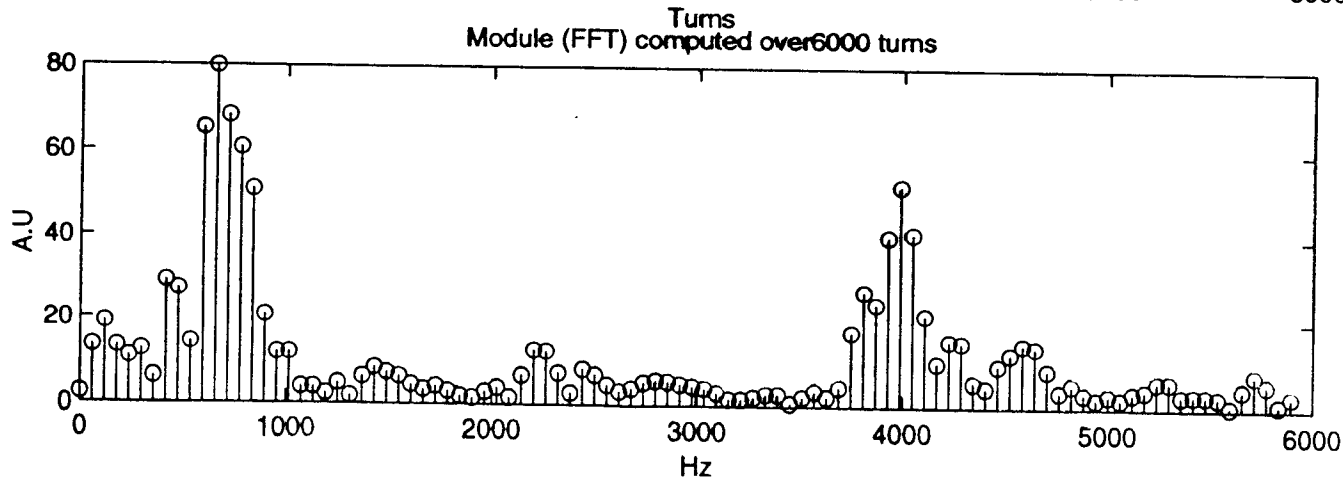
$f_r = 30 \text{ GHz}$

5 mA

$\langle x^2 \rangle$



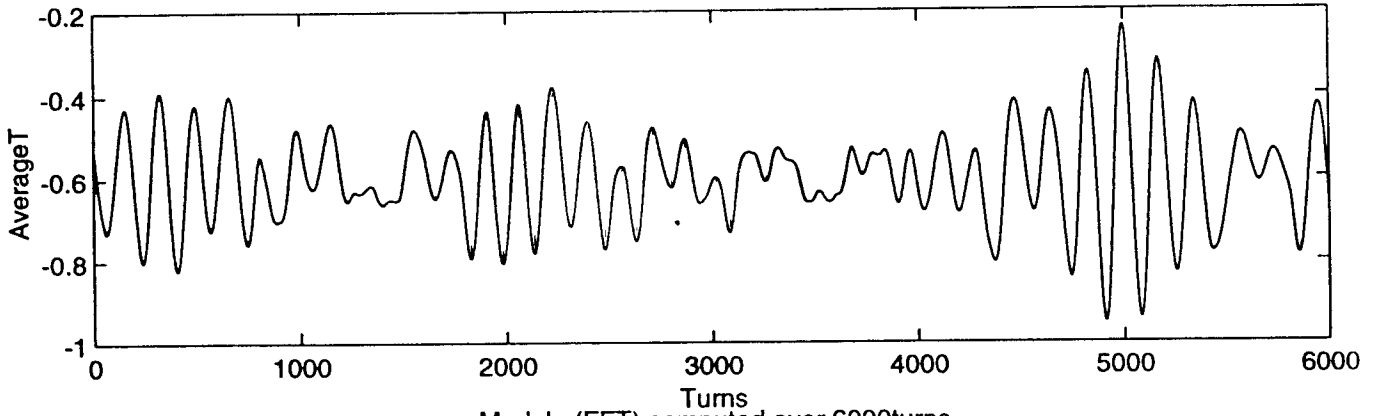
$\langle E^2 \rangle$



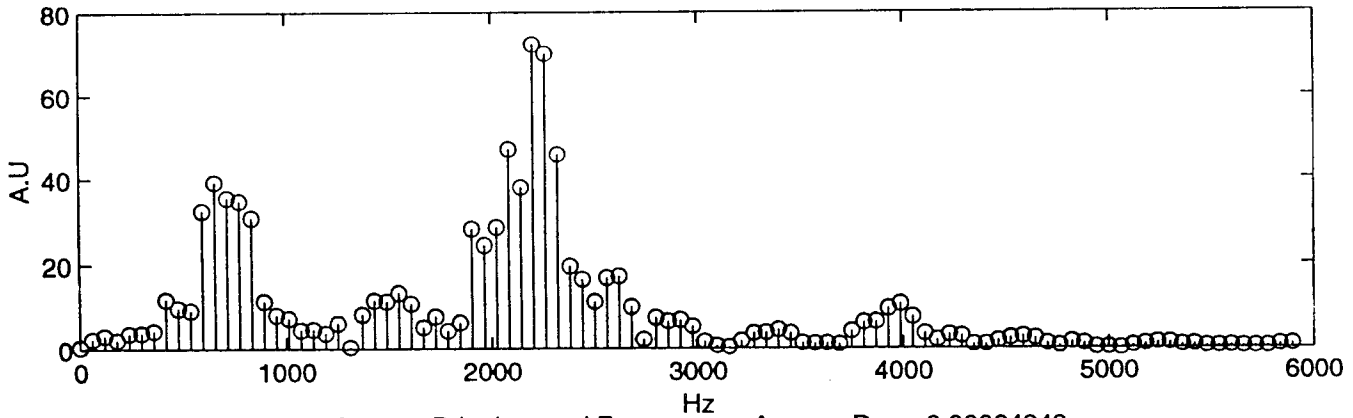
$f_r = 30 \text{ GHz}$

5 mA

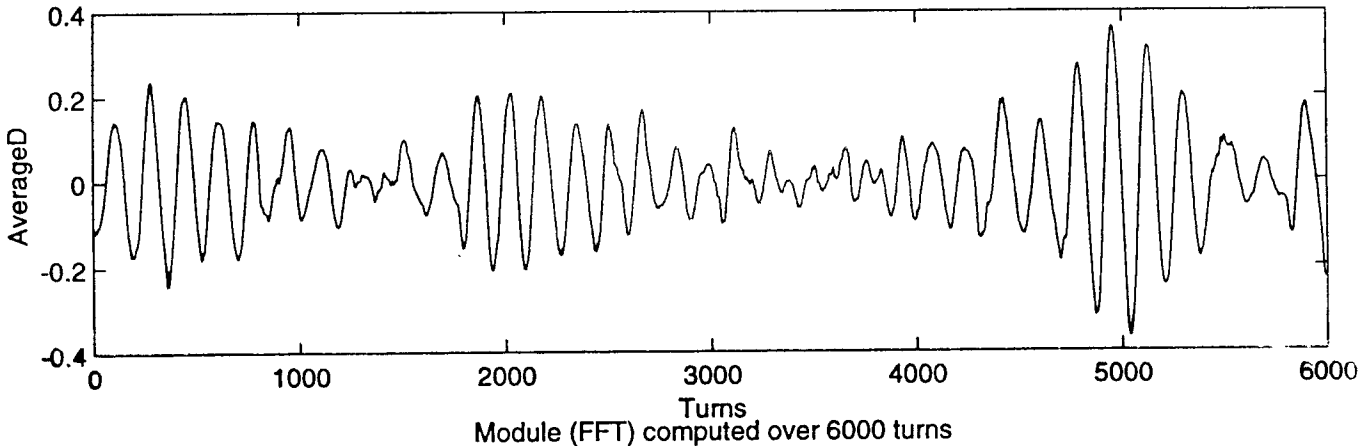
AverageT in time and Frequency; $\langle \text{AverageT} \rangle = -0.5994$



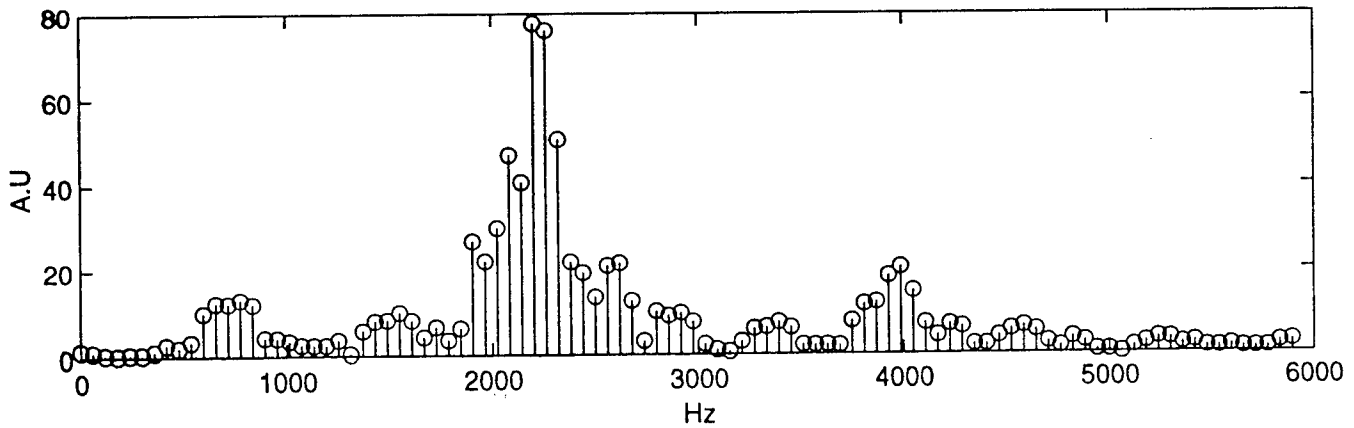
Module (FFT) computed over 6000turns

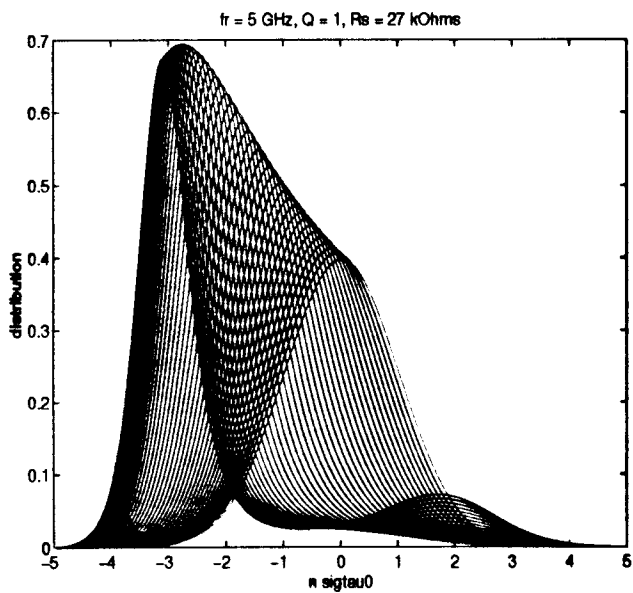


AverageD in time and Frequency; $\langle \text{AverageD} \rangle = -0.00064243$

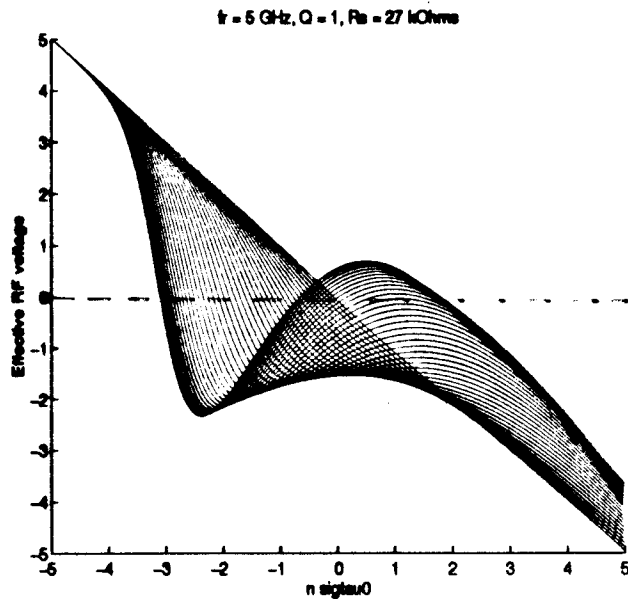


Module (FFT) computed over 6000 turns



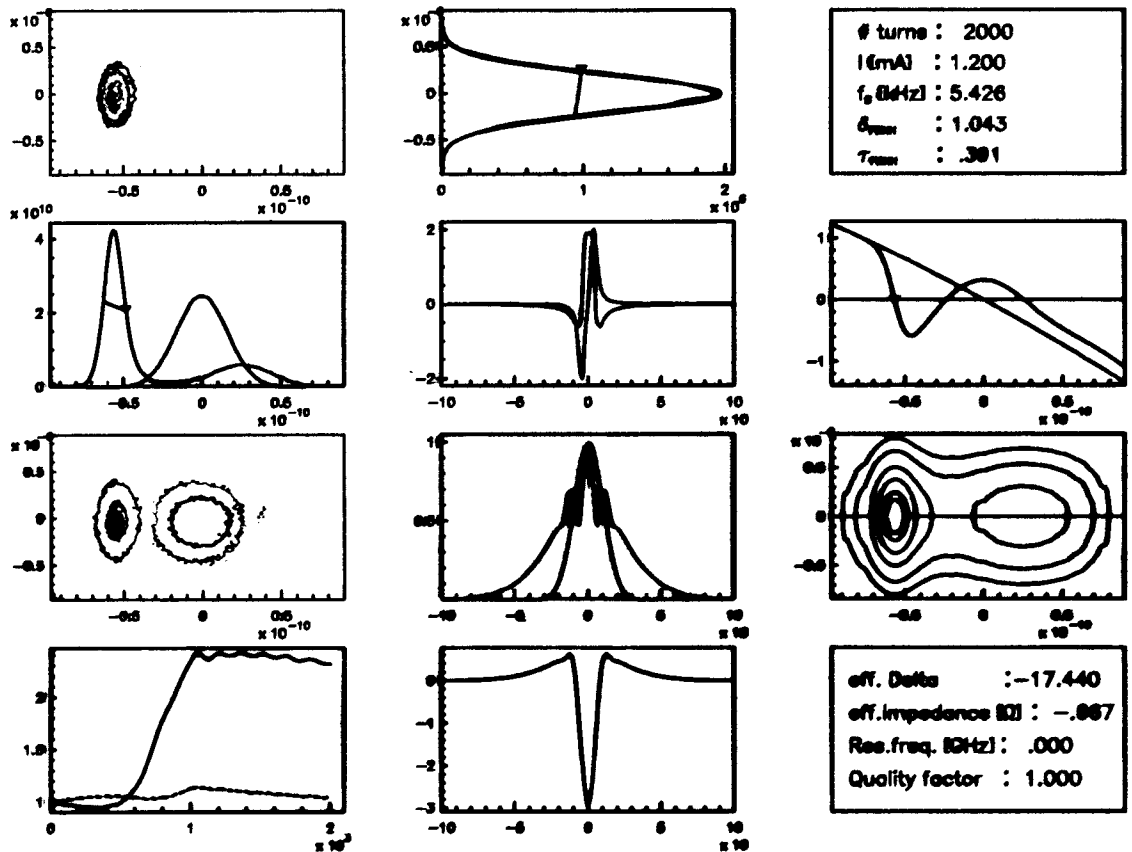


Line density



Effective RF voltage

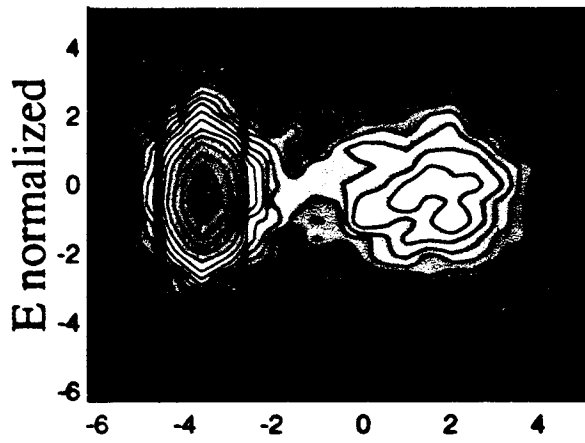
Iteration on Haissinski equation



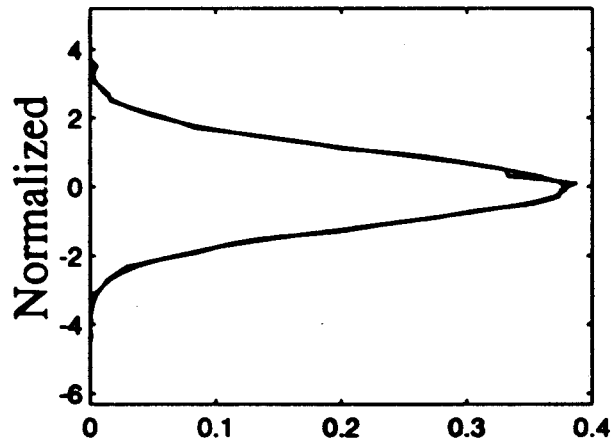
Tracking code results

Resonator ($f_r = 5 \text{ GHz}$, $Q=1$, $R_s=27\text{k}\Omega$) Comparison tracking code results and iteration on the Haissinski equation

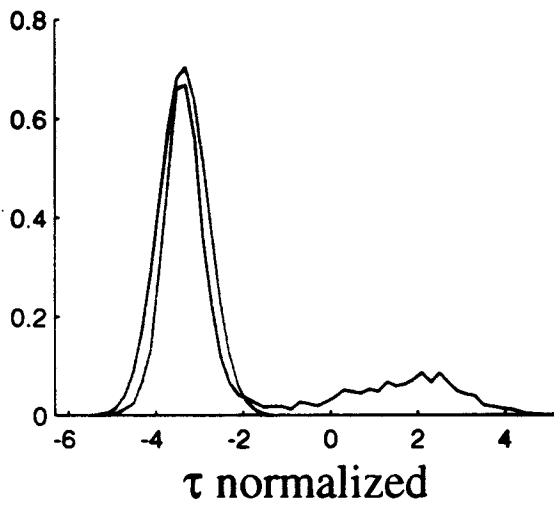
Double Bump



Energy Distribution



Time Distribution



Effective Voltage

