Stimulated Emission in the NLC-DR Wigglers

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March 1, 2000

- **Introduction**
- **NLC-DR Wiggler Parameters**
- **FEL Theory**
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Introduction

- Can the NCL Damping Wiggler be expected to produce stimulated emission?
- What, if it would ?
  - Increased Energy Spread
  - Changed Radiation Characteristics
  - Micro-Bunching

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Wiggler and Electron Beam Parameters

Wiggler Type	planar
Wiggler Period	0.27 m
Wiggler Peak Field	2.05 T
K	51.7
Wiggler Section Length	4.51 m
Number of Wiggler Sections	10
Separation Distance between Sections	0.58 m
Electron Energy	1.98 GeV
Peak Current	60 A
Relative Energy Spread	0.09%
RMS Bunch Length	4 mm
Longitudinal Distribution	Gaussian
Injected / Damped Horizontal Normalized Emittance	150 / 3 mm mrad
Injected / Damped Vertical Normalized Emittance	150 / 0.03 mm mrad
Transverse Distribution	Gaussian

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Focusing Parameters

Quadrupole Lattice	FODO
Quadrupole Length	0.15 m
QF Field Gradient	12.15 T/m (1215 G/cm)
QD Field Gradient	-2.44 T/m (-244 G/cm)
FODO Cell Length	10.18 m
Horizontal Beta Function at Wiggler Entrance	3.310 m
Horizontal Alpha Function at Wiggler Entrance	-0.195
Vertical Beta Function at Wiggler Entrance	11.403 m
Vertical Alpha Function at Wiggler Entrance	0.285
Distance between Wiggler Start and First QF Start	4.725 m
Distance between Wiggler Start and First QF End	4.875 m
Distance between Wiggler Start and First QD Start	9.235 m
Distance between Wiggler Start and First QD End	9.385 m

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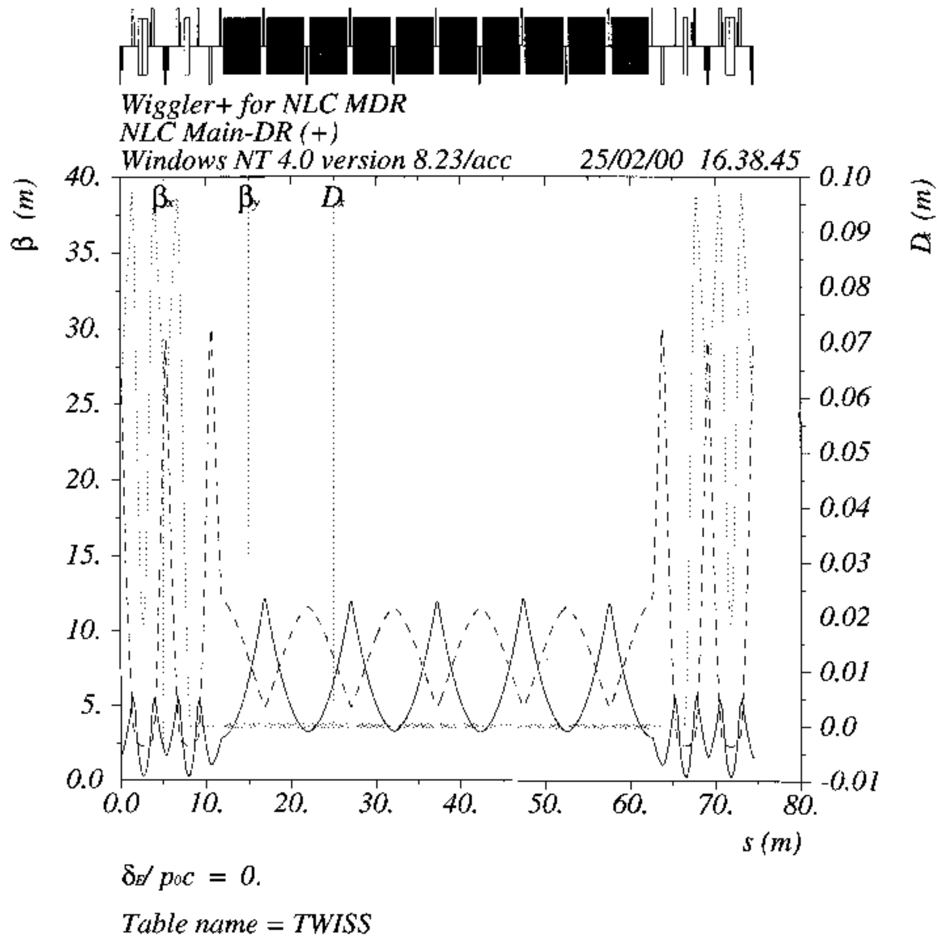
Electron Optics



Courtesy of Paul Emma

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1D FEL Theory

1D Power Gain Length:

$$L_{G1D} = \frac{\lambda_u}{4\pi\sqrt{3}\rho}$$

Pierce Parameter:

$$\rho = \frac{1}{4}\left(\frac{a_w}{4\gamma}\frac{\Omega_p}{\omega_u}F_1\right)^{\frac{2}{3}}$$

$$F_1 = J_0(\xi/2) - J_1(\xi/2)$$

Plasma Frequency:

$$\Omega_p = \sqrt{\frac{4\pi r_e c^2 n_e}{\gamma}}$$

$$\xi = \frac{a_w^2}{1 + a_w^2}$$

Undulator Parameter:

$$a_w = K/\sqrt{2}$$

$$K = \frac{B_u c}{k_u (m_e c^2 / e)}$$

Undulator Wavenumber:

$$\omega_u = k_u c$$

$$k_u = \frac{2\pi}{\lambda_u}$$

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FEL Formulae

Optical Waste Size:

$$w_o = \sqrt{2}\sqrt{\sigma_{xy}^{\;2} + 2\frac{\lambda_r L_{G1D}}{(4\pi)^2}}$$

Raleigh Range:

$$L_R = \frac{\pi\, w_o^{\;2}}{\lambda_r}$$

Electron Density:

$$n_e = \frac{I_{pk}}{2\pi\,\sigma_{xy}^{\;2}\, e\, c}$$

RMS Electron Beam Size:

$$\sigma_{xy} = \sqrt{\frac{\varepsilon_n}{\gamma}\beta_{xy}}$$

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3D FEL Theory

$$L_{G3D} = (1+\eta) L_{G1D}; \qquad P_{sat} \approx \frac{1.6\rho}{(1+\eta)^2} P_{beam};$$

$$L_{sat} \approx \log\left(\frac{P_{sat}}{\rho\,\gamma\, m_e c^2\, \Delta\omega}\right) L_{G3D}$$

Deviation from 1D Theory:

$$\begin{aligned} \eta = {} & 0.35\eta_d^{0.57} + 0.55\eta_\varepsilon^{1.6} + \\ & 3\eta_\gamma^{2} + 0.35\eta_\varepsilon^{2.9}\eta_\gamma^{2.4} + \\ & 51\eta_d^{0.95}\eta_\gamma^{3} + 5.4\,\eta_d^{0.7}\eta_\varepsilon^{1.9} + \\ & 1140\eta_d^{2.2}\,\eta_\varepsilon^{2.9}\eta_\gamma^{3.2} \end{aligned}$$

Diffraction, Emittance, Energy Spread

$$\eta_d = \frac{L_{G1D}}{L_R}, \quad \eta_\varepsilon = \left(\frac{L_{G1D}}{\beta_{xy}}\right)\left(\frac{4\pi\varepsilon_n}{\gamma_r\,\lambda_r}\right); \quad \eta_\gamma = 4\pi\left(\frac{L_{G1D}}{\lambda_u}\right)\left(\frac{\sigma_\gamma}{\gamma_r}\right)$$

Resonant Condition:

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}\left(1 + a_w^{\,2}\right)$$

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FEL Theory Prediction

Resonant Wavelength	12 microns
Emittance	3 mm mrad
1D Power Gain Length, L_G	1.02 m
Diffraction, η_D	6.100
Emittance, η_ε	0.001
Energy Spread, η_γ	0.040
3D Power Gain Length, L_G	2.3 m
3D Saturation Length, L_{sat}	50 m
3D Peak Saturation Power, P_{sat}	440 MW
RMS Electron Beam Radius	71 μm
3D Optical Mode Size (RMS)	1500 μm
Wiggle Amplitude	570 μm

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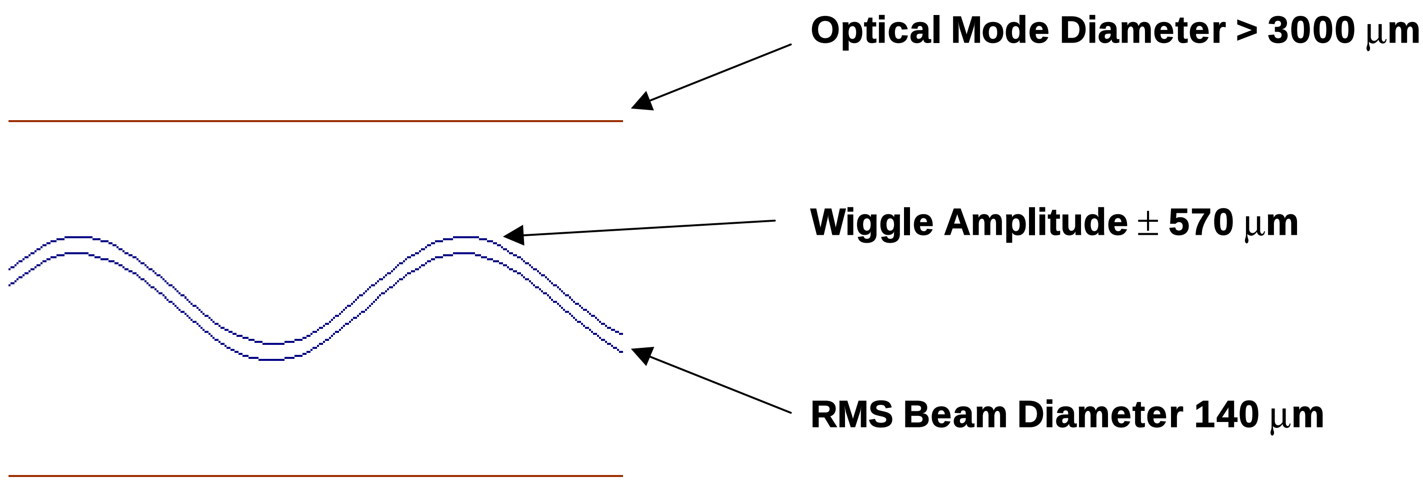
Parameter Range Violation



Wiggle Amplitude > Electron Beam Size ==> FEL Theory not Valid
Optical Mode > Electron Beam Size ==> Poor Coupling ==> Small Gain

Requires Numerical Computer Simulations to Determine Performance.

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FEL Computer Simulation Results

FEL Computer Simulation Results

- **Simulation code uses wiggler-averaged FEL equations**
- **Code should be valid even if wiggle amplitude is larger than beam size,**
- **as long as wiggle amplitude is smaller than optical mode size.**
- **Overestimate of gain possible.**

Simulation Code Ginger:

Power Gain	NONE
Bunching	0.0001
Gain Length	-

No FEL Gain ==> No Stimulated Emission

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Conclusion

- **Investigation of lasing probability for NLC Damping Ring Wiggler started.**
- **FEL Theory not applicable due to large wiggler parameter K, i.e. large wiggler amplitude**
- **Numerical FEL simulations predict no gain for single pass.**

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