Linear Collider Collaboration Tech Notes

A Coupling Correction Scheme
for the NLC Main Damping Rings

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1 Introduction

The requirement for a low extracted vertical emittance from the NLC Main Damping Rings (MDRs) leads to very tight alignment tolerances on the quadrupoles and sextupoles. The required equilibrium emittance ratio[1] is of the order 0.5%. While it may be possible to achieve this through sub-micron alignment of the quadrupoles, routine operation in such a regime has not been demonstrated at any existing comparable storage ring. Third generation light sources generally operate with an emittance ratio of the order 1%, and achieve this with the use of skew quadrupoles to correct the coupling effect of the sextupoles.

Vertical emittance comes primarily from two sources: vertical dispersion, and direct betatron coupling. Vertical dispersion is generated by vertical steering, and by dispersion coupling in the sextupoles. Assuming no roll errors in the quadrupoles, the principal source of betatron coupling is vertical orbit offset in the sextupoles. It is important to understand the relative contributions of dispersion and coupling to the vertical emittance for two reasons. First, the strategy to minimize the vertical emittance will conceivably depend on which effect is dominant. Second, the effects of intra-beam scattering depend on whether the vertical emittance comes primarily from dispersion or coupling. If the dispersion dominates, then the ratio of vertical emittance growth to horizontal emittance growth is given by[2]:

$$\frac{\Delta\varepsilon_y / \varepsilon_y}{\Delta\varepsilon_x / \varepsilon_x} \approx \frac{\left\langle \mathscr{H}_x \right\rangle_{bends}}{\left\langle \mathscr{H}_x \right\rangle_{lattice}}$$

where $\mathscr{H}_x$ is the dispersion invariant. In the present MDR lattice design, this ratio is approximately 0.64. If the betatron coupling dominates, then the ratio of fractional emittance growth is close to unity. Thus, for a damping ring lattice where the vertical

emittance comes principally from betatron coupling, IBS is potentially a more serious limitation on performance.

In this note, we begin by analyzing the relative contributions of dispersion and coupling to the vertical emittance in the present MDR lattice, under idealized conditions. We use the results of this analysis to estimate the contributions under more realistic conditions. Finally, we show that the required performance of the damping rings can be achieved under significantly eased alignment tolerances, by employing a simple correction scheme based on independent skew quadrupole fields superposed on each sextupole.

2 Generation of Vertical Emittance

We have implemented Chao's method[3] for calculating the horizontal and vertical beam distributions in the tracking code MERLIN[4].

2.1 Vertical Emittance from Vertical Dispersion

In the absence of betatron coupling, we expect the vertical emittance to be given by[5]:

$$\varepsilon_y \approx 2J_e \frac{\left\langle D_y^2 \right\rangle}{\left\langle \beta_y \right\rangle} \sigma_\delta^2 \tag{1}$$

where J_e is the longitudinal damping partition number, D_y the vertical dispersion, σ_δ the rms momentum deviation, and the brackets $\left\langle \ \right\rangle$ indicate averaging over the dipoles and wiggler. The values of the relevant parameters are given in Table 1; using these values, we find that to achieve an emittance ratio better than 0.5%, we require

$$\sqrt{\left\langle D_y^2 \right\rangle} \leq 2.5\,\mathrm{mm}$$

This assumes that there is no betatron coupling. In the presence of betatron coupling, the rms vertical dispersion will need to be somewhat lower than this value.

Table 1

Main damping ring parameters for determining vertical emittance from vertical dispersion

Normalized natural emittance	$\gamma\varepsilon_x$	2.17 mm-mrad
Longitudinal damping partition number	J_e	1.949
Mean vertical beta function	$\left\langle \beta_y \right\rangle$	7.06 m
RMS momentum deviation	σ_δ	9.09×10^{-4}

To verify the relationship (1) for the MDR lattice, we can simply turn off the sextupoles, and misalign the quadrupoles vertically, to generate some vertical dispersion. The vertical emittance generated in this way, as a function of the rms vertical dispersion, is shown in Figure 1. We see that there is good agreement with the analytical estimate from equation (1).
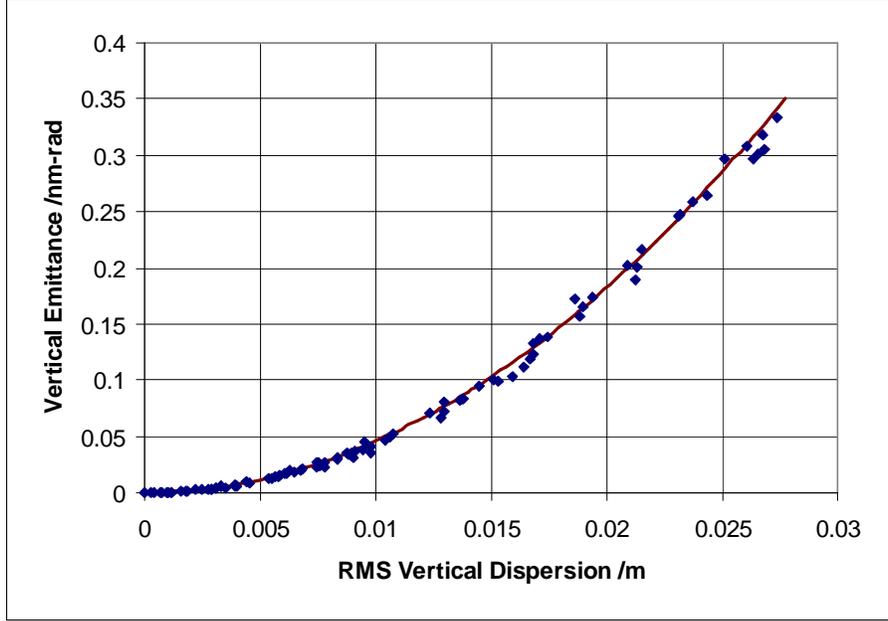
Figure 1

Relation between equilibrium vertical emittance and rms vertical dispersion. The points show the results of a simulation in the MDR lattice, with the sextupoles turned off, and dispersion generated by vertical misalignment of the quadrupoles. The line shows a fit of the form (1), using values of the parameters given in Table 1.

2.2 Vertical Emittance from Betatron Coupling

A general expression for the vertical emittance generated by betatron coupling will take into account the effects of all coupling resonances and components driving those resonances. While it is possible to derive such an expression, in practice it is not especially useful, since the driving terms depend on the precise misalignments of the sextupoles, which are not known. For the present purposes, we need only an estimate of the betatron coupling, which we can obtain from a simple expression, considering only the effects of a single lowest-order coupling resonance. If the coupling is given by a parameter g, defined by:

$$\varepsilon_y = \frac{g}{1+g}\varepsilon_0$$

where ε_0 is the natural emittance, then a simple treatment of the coupling driven by a single term κxy in the Hamiltonian, gives:

$$g = \frac{|\kappa|^2}{\frac{1}{2}\Delta^2 + |\kappa|^2} \tag{2}$$

where $\Delta = 2\pi|\nu_x - \nu_y|$ is the proximity to the second order difference resonance. In general, we can construct the global parameter κ by summing the local coupling coefficients with an appropriate phase factor:

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$$\kappa = \tfrac{1}{2} \oint \sqrt{\beta_x \beta_y} \, \kappa(s) \mathrm{e}^{\mathrm{i}\left(\mu_x - \mu_y\right)} \mathrm{d}s \qquad (3)$$

For a thin skew quadrupole of integrated strength $k_{1,s}l$ at location s_0, $\kappa(s) = k_{1,s}l \cdot \delta(s - s_0)$. Similarly, for a thin sextupole of integrated strength $k_2 l$ located at s_0, vertically displaced by a distance δy:

$$\kappa(s) = k_2 l \delta y \cdot \delta(s - s_0)$$

The coupling also affects the betatron tunes, which are given by:

$$\nu_{\pm} = \tfrac{1}{2}\left(\nu_x + \nu_y\right) \pm \tfrac{1}{4\pi} \sqrt{\Delta^2 + 4\kappa^2}$$

Equation (2) predicts the variation in the emittance ratio as the tunes are varied to cross the second-order difference resonance. The working point of the lattice, indicating the proximity to coupling resonance lines, is shown in Figure 2. A comparison between the coupling theory and a simulation of the MDR lattice is shown in Figure 3.
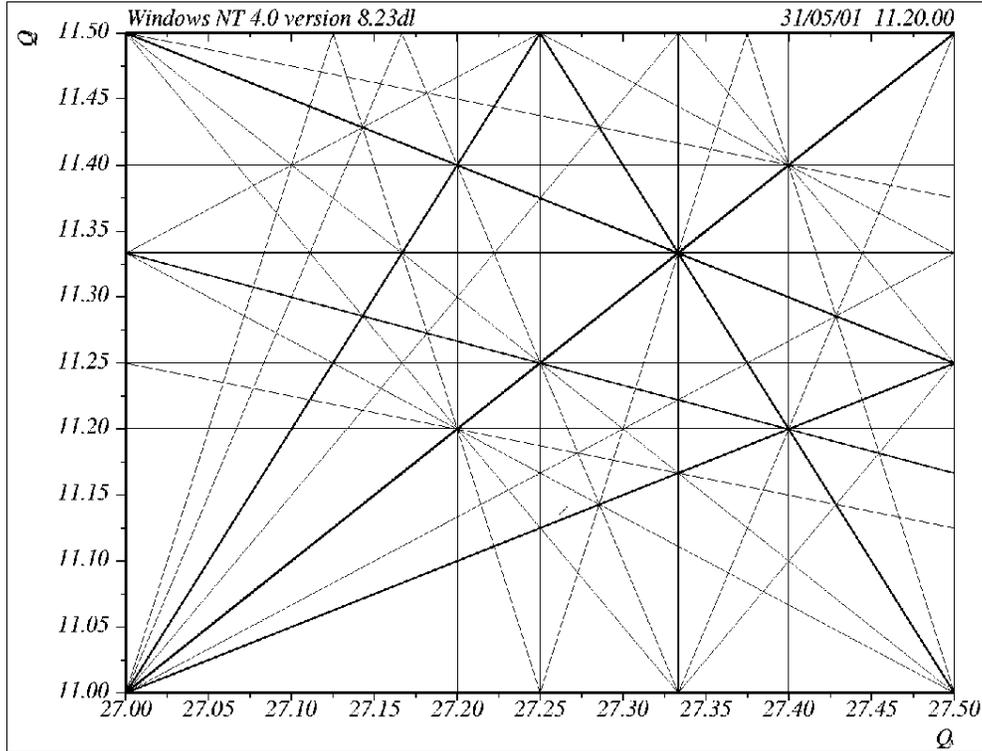

Figure 2

Working point of the main damping ring lattice in tune space. Resonance lines up to fourth order are shown; the nominal working point is (27.2626, 11.1357).

By detuning one family of quadrupoles in the arc cells, the tunes were varied across the coupling resonance. There is good agreement between the theory and the simulation, in terms of both tune variation and the coupling itself, close to the resonance; however, the nominal working point of the lattice is some distance from the coupling resonance, and here there is significant discrepancy in the coupling parameter g, as shown in Figure 4.
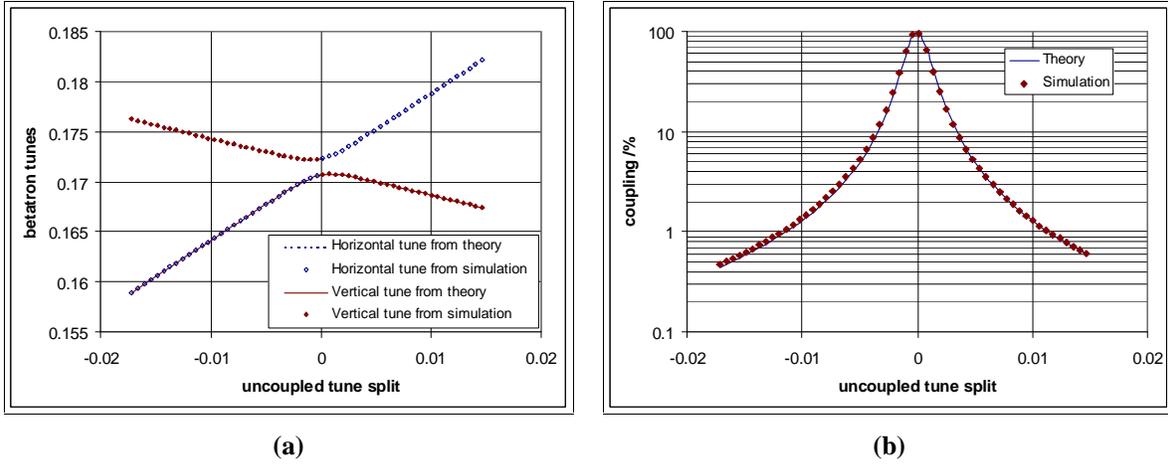
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(a)   **(b)**

Figure 3

Betatron tunes and coupling as a function of the uncoupled tune split, for a single skew quadrupole in the main damping ring lattice. The coupling was driven by a single skew quadrupole.
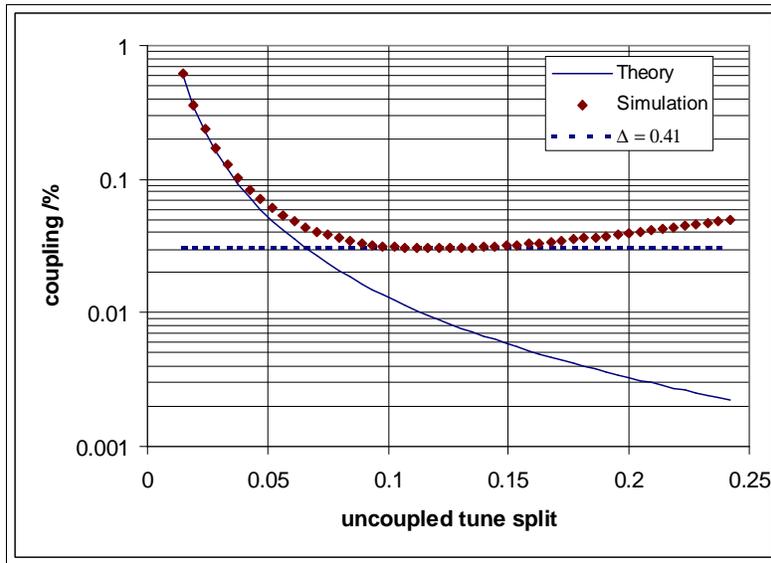

Figure 4

Coupling as a function of uncoupled tune split in the main damping ring lattice, for working points further from the second order coupling resonance. The nominal working point of the lattice has a tune split of 0.127. The coupling was driven by a single skew quadrupole.

In the region of the nominal working point, the betatron coupling from the strong second order resonance is very small, and it is likely that the effects of other resonance lines cannot be neglected; thus, we expect that equation (2) underestimates the coupling, and this is indeed the case. It appears that the selected working point actually lies close to a minimum of the coupling, along the path followed by detuning one family of arc quadrupoles; we can represent the coupling by a fixed value for the parameter Δ, which (empirically) we set to 0.41.

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2.3   Vertical Emittance from Sextupole Misalignment

Sextupoles are located in the lattice at positions of high horizontal dispersion, for the purpose of correction the chromaticity. This means that vertical misalignments of the sextupoles will generate significant vertical dispersion as well as betatron coupling. Vertical closed orbit distortion will have a similar effect since the beam passes off-axis through the sextupoles, but there is also vertical dispersion generated directly by steering in the quadrupoles.

From (3), we can calculate the coupling parameter κ in the case of sextupole misalignments from:

$$\kappa = \sum_n \kappa_n \delta y_n$$

where

$$\kappa_n = \tfrac{1}{2}\sqrt{\beta_{x,n}\beta_{y,n}}\left(k_2 l\right)_n e^{i\left(\mu_{x,n}-\mu_{y,n}\right)}$$

and the subscript n indicates the nth sextupole. Thus, calculation of κ properly requires exact knowledge of the misalignments of the sextupoles. However, if we assume that the misalignments are random and uncorrelated, we can estimate the coupling parameter from the rms misalignment:

$$\left\langle |\kappa|^2 \right\rangle = \mathrm{K}^2 \cdot \overline{\delta y^2} = \left(\sum_n |\kappa_n|^2\right)\cdot \overline{\delta y^2}$$

where the brackets $\langle\ \rangle$ indicate the average over many sets of random misalignments. The factor K^2 is a constant for the lattice, readily determined from the sextupole strengths, and the beta functions at their locations. For the main damping ring lattice, we find:

$$\mathrm{K} \approx 386\ \mathrm{m}^{-1}$$

The betatron coupling is then given by:

$$g = \frac{\mathrm{K}^2 \cdot \overline{\delta y^2}}{\tfrac{1}{2}\Delta^2 + \mathrm{K}^2 \cdot \overline{\delta y^2}}$$

For the main damping ring lattice, we use the fixed value $\Delta = 0.41$, found in Section 2.2. We can also find the vertical emittance generated by dispersion, from equation (1). Note that the beam emittance is a mean (rather than a root-mean-square) quantity:

$$2\varepsilon_y = \gamma_y \left\langle y^2 \right\rangle + 2\alpha \left\langle yy' \right\rangle + \beta \left\langle y'^2 \right\rangle$$

where the brackets $\langle\ \rangle$ here indicate an average over all particles in the beam. Thus, we find the total vertical emittance as the linear sum, rather than the sum in quadrature, of the betatron coupling and the vertical dispersion:

$$\varepsilon_y = \varepsilon_{y,\mathrm{coup}} + \varepsilon_{y,\mathrm{disp}} \qquad\qquad (4)$$

and we can readily estimate the vertical emittance we expect to see for a given set of sextupole misalignments. The results of such a calculation are shown in Figure 5. The theoretical points (and best-fit curve) are calculated from (4), using knowledge of the rms sextupole misalignment and the rms vertical dispersion. Note that the distributions of sextupole misalignments are gaussian, with a truncation at three standard deviations. The points from the simulation show the results of applying Chao's method to calculate the vertical emittance directly. Each point shows the results of averaging over 25 seeds for sets of sextupole misalignments with a given rms. This averaging greatly reduces the scatter. Apart from three unexplained points for large sextupole misalignments, there is good agreement between the theory and the simulation.
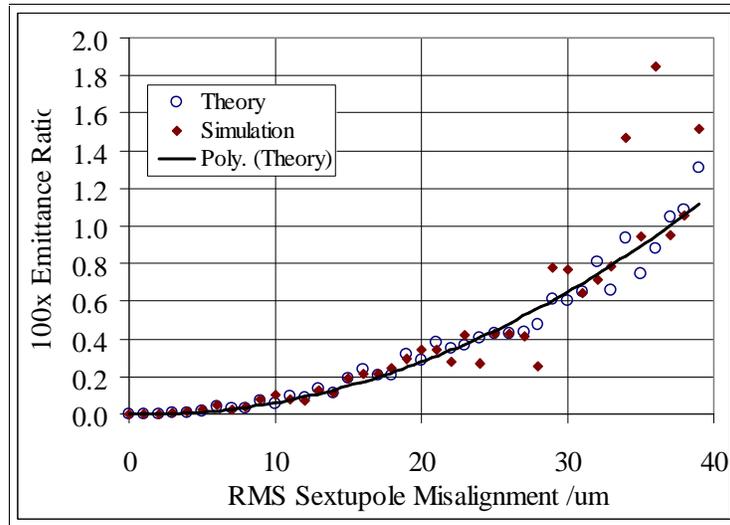

Figure 5

Emittance ratio as a function of sextupole misalignment in the main damping ring lattice. The line shows a best-fit quadratic through the theoretical points.
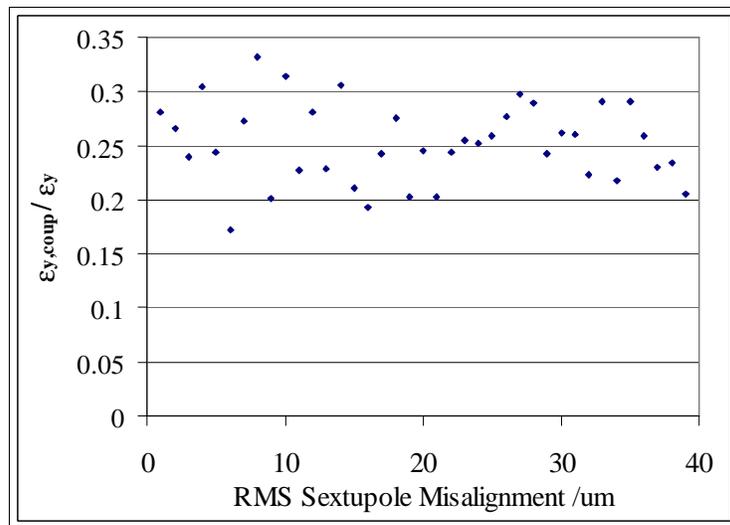

Figure 6

Contribution of betatron coupling to vertical emittance, as a fraction of the total vertical emittance, for the data shown in Figure 5.

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Although no orbit distortion has been applied, and the model is therefore incomplete, Figure 5 suggests that alignment tolerances will be tight; the desired emittance ratio of 0.5% corresponds to a sextupole alignment tolerance of less than 30 µm rms. Given the good agreement between the simulation and the predictions based on (4), we can estimate the relative contributions of direct betatron coupling and vertical dispersion to the vertical emittance, in the limit of no orbit distortion. The emittance generated from betatron coupling as a fraction of the total vertical emittance, is shown in Figure 5. There is significant scatter on the points, but the average is approximately 0.25. We do not expect this value to vary significantly for small orbit distortions.

3   Coupling Correction

We propose a coupling correction scheme based on:

   i. vertical closed orbit correction using quadrupole movers, to minimize vertical steering and beam offset in the sextupoles;

  ii. skew quadrupoles superposed on the sextupoles, to minimize the effects of residual closed orbit distortion, and the misalignments of the sextupoles.

Since the coupling from a vertically displaced sextupole matches (to first order) that from a skew quadrupole, use of independent skew quadrupoles can be highly effective in reducing both betatron coupling and vertical dispersion. In the general case, vertical dispersion from residual vertical steering means that complete correction of the vertical emittance cannot be achieved. We have already reported on the emittance that can be attained solely through use of orbit correction[1], and although the required 0.5% emittance ratio looks achievable in our model, very tight tolerances are required. For example, Quadrupole alignment of better than 1 µm rms, sextupole alignment of better than 40 µm rms and BPM resolution of 1 µm (without averaging) would be needed. Although the quadrupole alignment could be possible with high resolution BPMs, movers and beam-based alignment, these tolerances would be very demanding, and it is desirable to find ways in which they may be eased.

A straightforward approach to the problem is to use a response matrix to determine the corrector strengths needed to minimize the readings on a set of monitors. Such an approach has been demonstrated for simultaneous orbit and dispersion correction in LEP[6]. In that case, the correctors for both orbit and dispersion correction were a set of vertical steering magnets. For the NLC Main Damping Rings, we propose to use magnet movers for orbit correction, and skew quadrupoles for dispersion correction. The two sets of correctors are not strictly orthogonal, since an orbit correction will affect a dispersion correction; however, in principle, a dispersion correction using just skew quadrupoles will not affect the orbit. Thus, in the case of the Main Damping Rings, it appears the most efficient approach is first to correct the orbit, then to use the skew quadrupoles to correct the dispersion. With unlimited resolution on all monitors and correctors, the vertical emittance is limited only by the opening angle of the synchrotron radiation.

In practice, we set some limited resolution on the BPMs, and on the mover step size. Our simple correction procedure is then as follows:

i. Apply orbit correction, with the mover positions determined from the SVD of the mover/BPM response matrix with some gain, until the rms closed orbit falls below some limit.

ii. Apply coupling correction, with the skew quadrupole strengths determined from the SVD of the skew-quadrupole/dispersion response matrix with some gain, until the equilibrium vertical emittance falls below the required limit.

The relevant parameters in our model are given in Table 2. The standard BPM resolution of 5 µm is assumed for the orbit correction, which will be a continuous process during operation. The dispersion correction is more invasive, requiring a change of RF frequency, and very fine BPM resolution is needed; we assume that 0.3 µm can be achieved with averaging over several seconds of readings. The equilibrium vertical emittance of 0.0115 mm-mrad corresponds to an extracted vertical emittance of 0.02 mm-mrad. With these parameters, our simulation achieved the required emittance in 95% of cases. The tolerances indicated represent a considerable relaxation over those required in the case where skew quadrupoles are not used[1]. The BPM resolutions in particular are equivalent to those achieved, for example, at the ALS[7].

Table 2

Parameters in coupling correction model.

BPM resolution	5 µm
BPM resolution with averaging	0.3 µm
BPM-Quadrupole alignment rms	5 µm
Sextupole alignment rms	80 µm
Quadrupole mover step size	1 µm
Orbit feedback gain	0.8
Dispersion feedback gain	0.8
Max energy variation	0.15%
Max residual closed orbit distortion	200 µm
Max equilibrium vertical emittance	0.0115 mm-mrad

The correction is sensitive to the accuracy of the response matrices. We have assumed unlimited resolution in finding these matrices; in practice, one can use both direct measurements and a carefully calibrated machine model, to determine these matrices with good accuracy.

The distribution of equilibrium emittance after orbit and dispersion correction in 200 cases with different seeds for random alignment errors, is shown in Figure 7. Note that in 10 cases, the correction failed to achieve the required emittance. The distributions of residual vertical closed orbit distortion and vertical dispersion are shown in Figure 8. There is no clear correlation between either vertical closed orbit distortion or vertical dispersion, and the final vertical emittance.
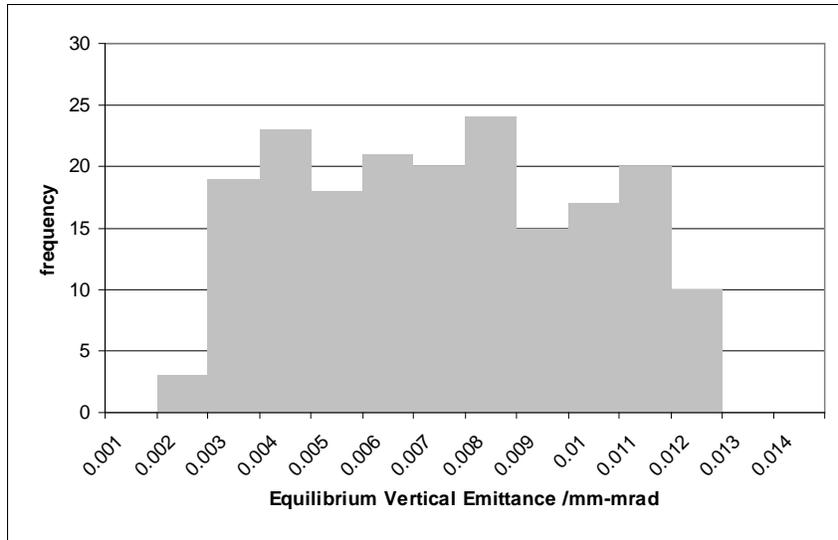
Figure 7

Distribution of equilibrium vertical emittance, after orbit and dispersion correction in 200 cases of random alignment errors.
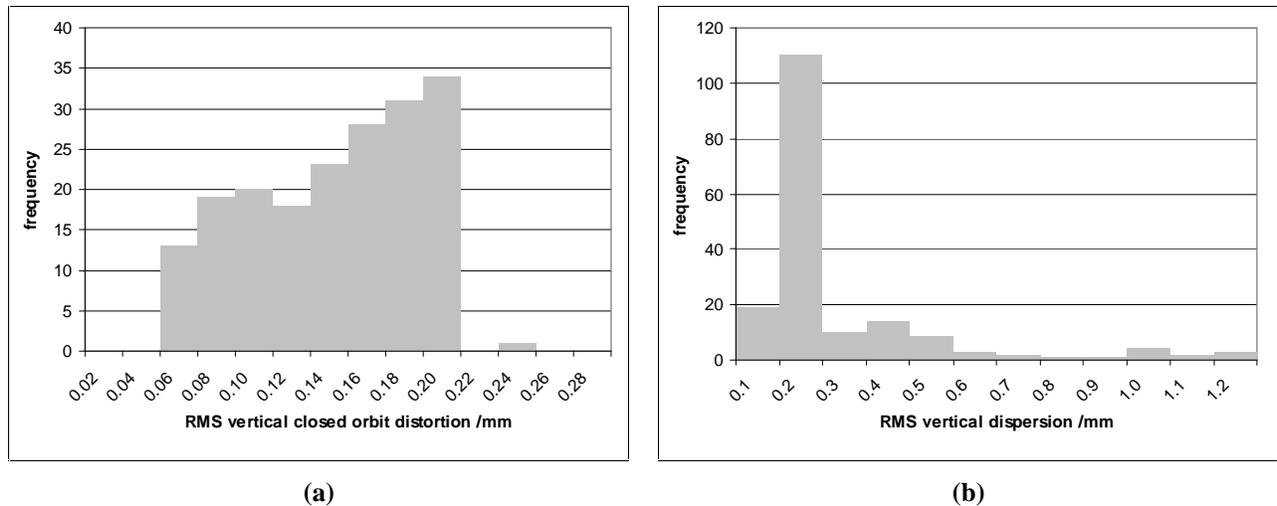

(a)

(b)

Figure 8

Distributions of (a) rms vertical closed orbit distortion and (b) rms vertical dispersion after orbit and dispersion correction.

The effect of skew correction on the dispersion can be seen in Figure 9, which shows the square of the vertical dispersion through the latticed, for the case of a single seed of quadrupole and sextupole misalignments. After orbit correction but before skew correction, the rms vertical dispersion is 12.2 mm. After skew correction, this is reduced to 0.115 mm. It is interesting to note that the skew correction is effective even in the straight sections, where there are no skew quadrupoles located (in our simple model, we positioned skew quadrupoles only at the sextupoles). The correction scheme is local in the sense that the correctors are applied at the points where the errors arise, but also has a global aspect through the use of the SVD algorithm.

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Other sources of coupling include quadrupole roll errors. Since the dipoles have a significant negative field gradient, there will also be vertical steering effects from dipole misalignments. Neither of these sources of error has yet been included in our simulations. Although it is possible they will lead to a tightening of the tolerances, it is likely that the required vertical emittance will still be achievable under realistic conditions.
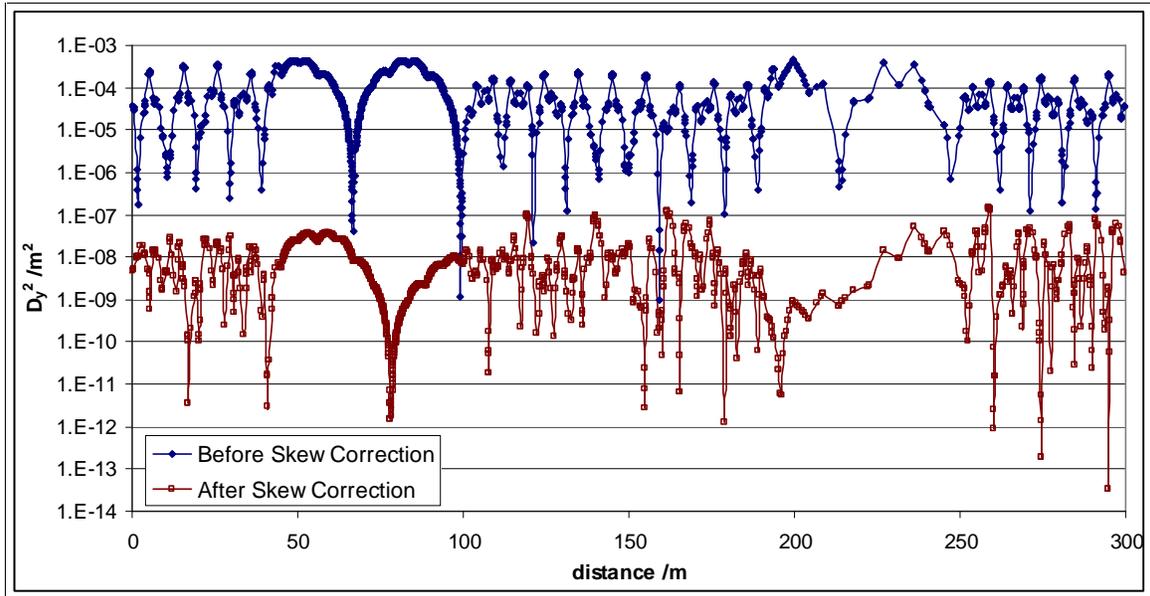

Figure 9

Square of the vertical dispersion throughout the MDR lattice, after orbit correction, and before and after skew correction. The rms vertical dispersion is reduced by the skew correction from 12.2 mm to 0.115 mm. The wiggler is located between 45 m and 100 m, and the injection/extraction straight is located between 195 m and 250 m.

4 Discussion

The vertical emittance found by simulations of the main damping rings in the presence of quadrupole and sextupole alignment errors, can be understood from estimates of the emittance generated by vertical dispersion, and betatron coupling. For small orbit distortions, the betatron coupling contributes about 25% of the equilibrium vertical emittance, the remainder coming from vertical dispersion. The working point of the lattice is appropriate from point of view of betatron coupling, being sufficiently far from coupling resonances, that little reduction of the vertical emittance can be achieved by retuning the lattice. Correction of the vertical dispersion will be more significant for achieving the very low required emittance ratio. In our simulations, this can be achieved by a fairly simple method, using skew quadrupole correctors superposed on each sextupole.

The parameters for the orbit feedback and dispersion correction system are reasonable, and do not require unrealistic precision on BPMs or magnet movers, or on sextupole alignment. We have not yet considered all sources of coupling – quadrupole roll errors have not yet been included in our model, for example. However, we expect that the

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model we have constructed will still be able to achieve the required emittance ratio, with reasonable tolerances placed on these effects.

It is important to understand the relative contributions of betatron coupling and vertical dispersion to the vertical emittance, for modeling intra-beam scattering. We note that the IBS studies that have been carried out so far, have assumed that the betatron coupling contributes 25% of the vertical emittance[8]. This is in excellent agreement with the results presented here.

5 References

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