Linear Collider Collaboration Tech Notes

Effect of Energy Spread in the Beam Train on Beam Breakup Instability

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Abstract:

It is shown herein that the energy spread of order one percent in the beam train in the NLC can suppress the emittance growth due to structure misalignments and initial jitter

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1 Introduction

In this note we consider the beam break-up instability in the train of bunches in the NLC main linac. This instability is driven by long-range wakefields in the structures. If not cured, it imposes very tight tolerances on the beam injection errors as well as structure misalignments in the lattice.

In previous studies of the instability it was usually assumed that the energy of all bunches in the train during acceleration are equal. One can intentionally introduce an energy variation between the bunches, in order to use the BNS damping mechanism or decoherence effect.

In this note, we first give an analytical estimate of the necessary energy spread in the train, and then confirm the estimate using a computer simulation with LIAR. We show that the required energy spread is in the range of 1%.

2 Analytical Estimate

We start from the autophasing condition for the BNS damping in the two-particle model for a FODO lattice [1, 2]

$$\delta = \frac{Ne^2 w l_{cell}^2}{24E}\left(1 + \frac{3}{2}\mathrm{ctn}^2\frac{\mu}{2}\right), \qquad (1)$$

where δ is the relative energy between the particles, N is the number of particles in the macroparticle, w is the transverse wake, E is the beam energy, μ is the betatron phase advance per cell in the FODO lattice, and l_{cell} is the cell length. The value of δ given by Eq. (1) would be enough for suppression of the beam break up instability between the two macroparticles with the interaction characterized by the wake w.

In order to apply Eq. (1) to the train, we first have to specify the wake function. The transverse wake is shown in Fig. 1 [3]. For a rough estimate of the effect we will use the rms value of this wake $w_{\mathrm{rms}} = 0.21$ V/pC/m/mm.
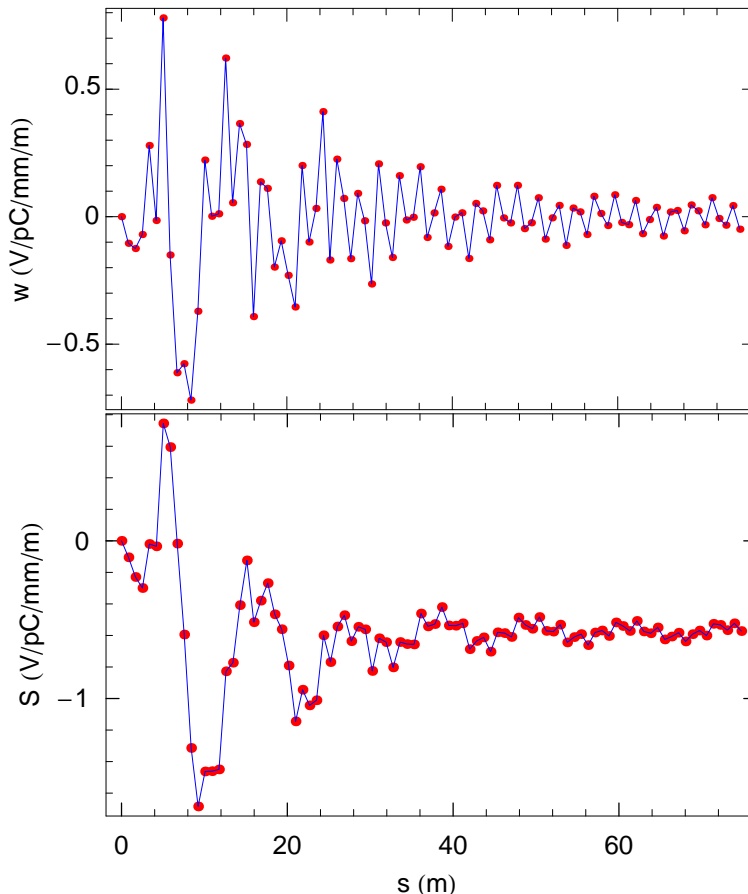
Figure 1: Transverse wake w and the corresponding sum wake S used in the simulation.

The quantity $F \equiv l_{cell}^2 \left(1 + \frac{3}{2}\text{ctn}^2\frac{\mu}{2}\right)/(24E)$ from Eq. (1) was calculated for the NLC lattice as a function of distance s and is shown in Fig. 2. For the estimate we will use the average value of $F_{av} = 0.54$ m^2/GeV.

Finally, we need to relate the quantity δ to the energy spread in the train. Since Eq. (1) was derive for two macroparticles, one can expect *that δ refers to the energy difference between the adjacent bunches in the train*. Hence, the required energy spread in the train for suppression of the instability is equal to δ multiplied by the total number of bunches in the train,

$$\delta_{\text{train}} \approx N_{\text{bunch}}\delta. \tag{2}$$

We can now estimate δ_{train} as $N_{\text{bunch}}Ne^2w_{\text{rms}}F_{av}$. For $N = 1.1 \times 10^{10}$ and
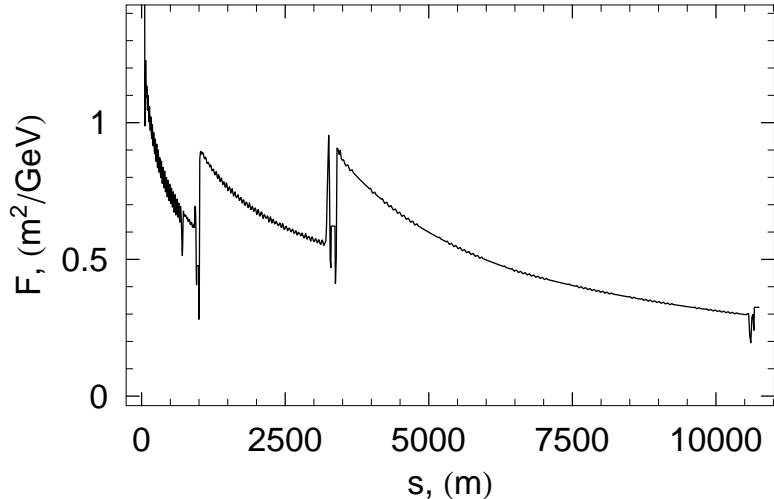
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Figure 2: Factor $F \equiv l_{cell}^2 \left(1 + \frac{3}{2}\mathrm{ctn}^2 \frac{\mu}{2}\right)/(24E)$ as a function of distance for the NLC lattice.

$N_{\mathrm{bunch}} = 95$ this gives

$$\delta_{\mathrm{train}} \approx 0.02, \tag{3}$$

which is of the same order as the energy spread in the bunch.

3 LIAR Simulations

We carried out LIAR simulations with and without energy spread in the train for the wake shown in Figure 1. In these simulations, the bunches were treated as macroparticles without internal slices.

In the first set of simulations, all the structures were randomly misaligned with rms value of 5 μm. In one case, the energy of all bunched in the train was the same. In the second case, the energy in the train varied from the head to the tail of the train as shown in Fig. 3, so that $\Delta E_k = -\alpha k$, where the k is the bunch number in the train. The slope α (and hence the rms spread) of the energy within the train linearly increased with the distance along the linac, $\alpha = \mathrm{const} \times s$, so that the relative energy spread was approximately constant (except at the beginning of the linac).

The final bunch energy at the end of the linac is shown in Fig. 3, it corresponds to the rms energy spread of 0.8%. The emittance growth in the train for the two cases is shown in Fig. 4. We see that introduction of the energy spread indeed suppresses the instability and results in much smaller emittance
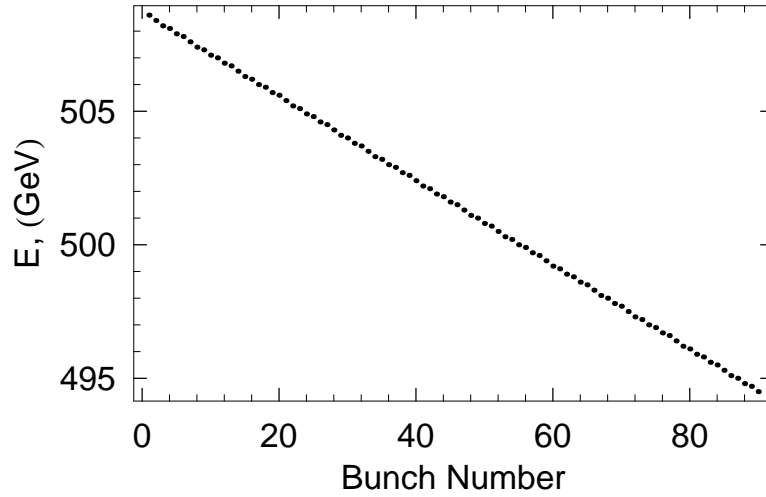
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Figure 3: Bunch energy in the train at the end of the linac.

growth. The final effective emittance of the train as a function of the rms en-
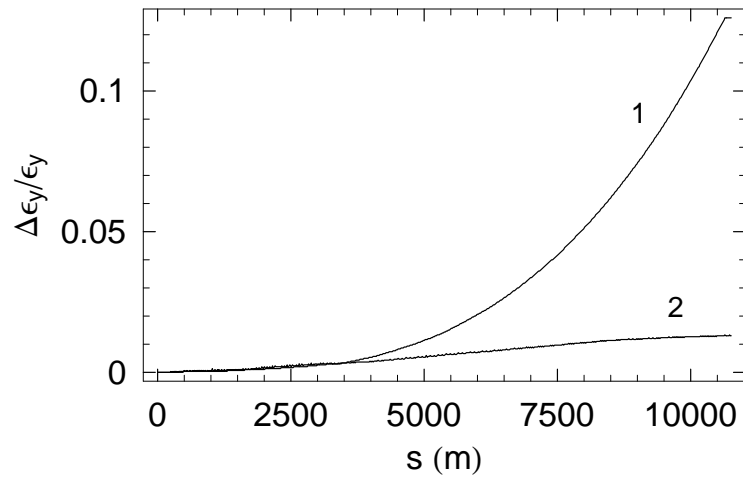

Figure 4: Emittance growth for the case without energy variation in the train (1), and with 0.8% rms energy spread (2).

ergy spread is shown in Fig. 5. Positive values of δ correspond to the BNS-like energy profile shown in Fig. 3, when the energy decreases toward the tail of the
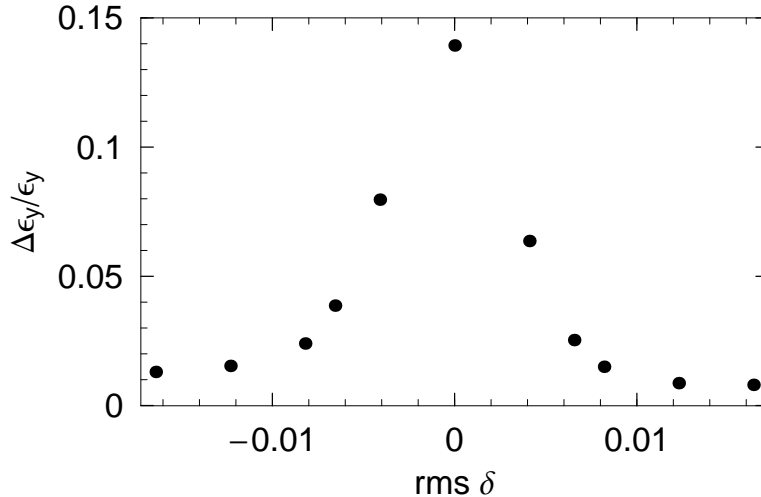
Figure 5: Relative emittance increase as a function of the energy spread for misaligned structures.

bunch. The negative values of δ, correspond to the opposite slope of the energy profile, with the energy decreasing toward the head of the train. As we see, the positive values of δ are more effective in suppression of the emittance growth.

In the second set of simulations, the train was initially offset by 1 μm (with all structures and quadrupoles perfectly aligned). The resulting emittance growth of the beam as a function of δ is shown in Fig. 6. Again, we see that the energy spread of the order of 1% results in much smaller emittance growth then for the bunches with the same energy.

4   Acknowledgements

References

[1] A. W. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators* (Wiley, New York, year1993).

[2] NLC ZDR Design Group and NLC Physics Working Group. *Physics and technology of the next linear collider: a report submitted to Snowmass '96*, SLAC-R-0485, 1996.
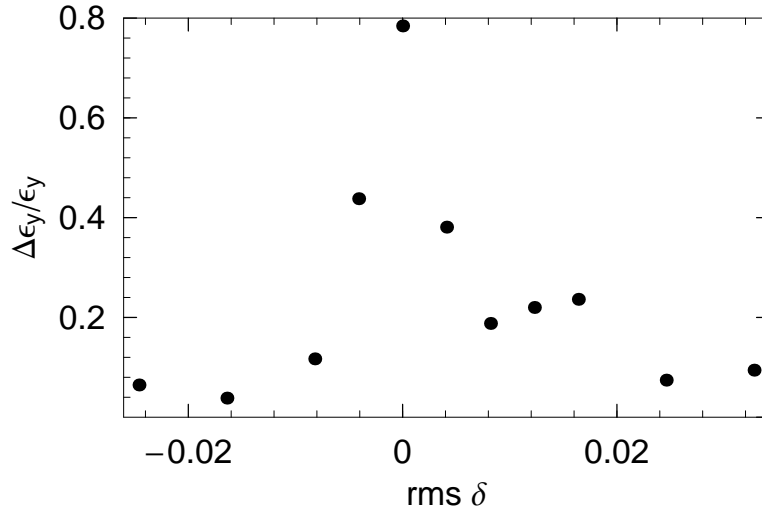
Figure 6: Relative emittance increase as a function of the energy spread for initial train offset of 1 μm.

[3] Wakefield for RDDS1 C Incl HOM Ref and 4 Cells Decoupled computed by R. Jones.