Linear Collider Collaboration Tech Notes

Transverse Deflections in a Cavity
Due to the Short-range Longitudinal Wake

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INTRODUCTION AND CONCLUSION

Consider an ultra-relativistic electron bunch passing through a (cylindrically symmetric) multi-cell linac cavity that is filled with fundamental mode rf. It is well known that this bunch—on entering the cavity—experiences a focusing kick, and—on exiting the cavity—a defocusing kick, even though the mode is cylindrically symmetric. The effects of these kicks in linacs tend to be significant only in low energy regions.

Tracking computer programs such as MAD [1] and LIAR [2] include a simple model of these kicks, one based on calculations of W.H. Panofsky [3]. According to this model the effect is represented by two thin lenses positioned at the ends of the cavity, with the strength of the lenses dependent on the accelerating gradient in the cavity. However, a beam will itself excite wakefields that modify its energy gain in a cavity, a modification that depends also on longitudinal position within the bunch. The program LIAR extends Panofsky's rf kick model to include this modification to the effective gradient experienced by different parts of the beam. In this report we investigate how the wakefields affect the rf cavity kicks. In particular, we are interested in the case when the wakefields are a significant perturbation to the problem, such as when, for example, the beam traverses an empty cavity (one with no rf).

In this report we have shown that one can calculate the transverse kicks when one knows the time-dependent variation of the longitudinal wake forces on axis. The variation in gradient due to wakefields, however, will in general differ from that due to normal rf acceleration. In particular, transients at the ends of structures, and—for constant gradient structures— an increase in gradient amplitude from beginning to end of the cavity, will mean that the model of focusing/defocusing edges, used for rf acceleration, will be inaccurate. Finally, we conclude that the method LIAR uses to treat the effect of rf focusing in the presence of wakefields on beam orbit is approximately correct.

TRANSVERSE DEFLECTIONS

Transverse deflection during the passage of a short beam through a cavity can be analyzed using an equation derived for the analysis of rf deflections in Ref. [3]. The equation relates a particle's trajectory in the cavity $r(z)$ with the gamma factor $\gamma(z)$; the variation of γ is due to acceleration, $d\gamma/dz = eE_z/mc^2$. Using the notation $p(z) = \gamma(z)r'(z)$, where the prime denotes derivative with respect to z, the equations read:

$$p' = -\frac{1}{2}r\gamma'', \qquad r' = \frac{p}{\gamma}. \tag{1}$$

As follows from the derivation, these equations assume axisymmetric fields and the paraxial approximation for the trajectory, and they are valid for arbitrary $\gamma(z)$.
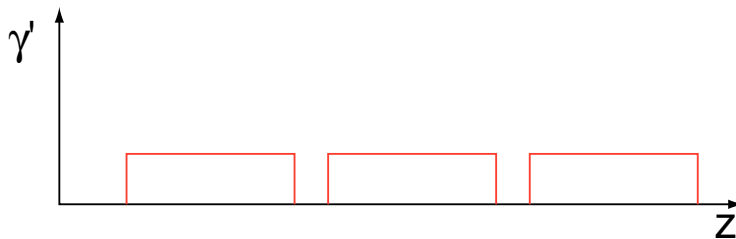

FIG. 1: Accelerating gradient in structures, to good approximation, can be considered as flat, with sharp edges at the entrance to and exit from each cavity. Shown is a sketch of $\gamma'(z)$ for three structures.

If acceleration is provided by external rf, the function $\gamma'(z)$ has a form shown in Fig. 1. (Actually, γ' is not constant inside the structure; it oscillates on the scale of the structure cell. This effect, however, is usually small.) At the entrance and exit of cavities, the second derivative can be approximated as $\gamma'' = \pm\gamma'\delta(z - z_{\text{edge}})$ and integration of the first of Eq. (1) through the edge of the cavity gives for the change of p':

$$\Delta p'|_{\text{edge}} = \mp\frac{1}{2}r\gamma', \tag{2}$$

which is the well known focusing-defocusing effect at the cavity edges.

Now, if γ' is due only to the short-range wake in the bunch, there are important differences in the behavior of γ'. First, the wakefield takes some distance—the so-called catch-up

distance—to establish itself. For the NLC structure and bunch length (rms length $\sigma_z = 100$ μm), the catch-up distance is 10–15 cells. Second, the bunch generated fields do not normally disappear abruptly at the ends of the structures and may persist for some distance beyond them. Third, there may also be relatively larger modulation in γ' due to the wake forces within a structure than is the case for rf. Hence, for the same 3 structures, an expected qualitative profile of γ' is similar to that shown in Fig. 2.
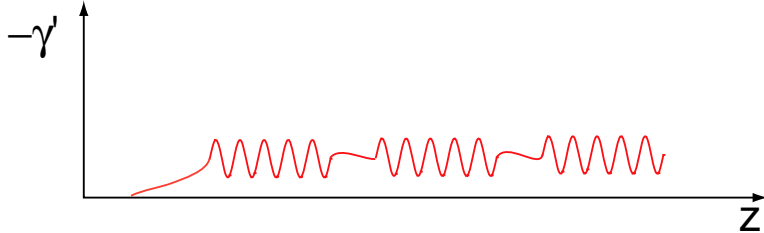

FIG. 2: The decelerating gradient due to the short range wake has a transient at the beginning of the first structure. It may oscillate from iris to iris, and not vanish to zero in the short regions between structures.

RELATION BETWEEN R' AND γ' TO FIRST ORDER

Consider a test particle in an ultra-relativistic bunch traversing a cylindrically symmetric accelerating structure. We can demonstrate that under typical conditions Eq. (1) implies that $\Delta r'(z) \equiv r'(z) - r_0'$ (subscript 0 indicates an initial parameter of the test particle) is, to first order, proportional to $\gamma'(z)$.

Let us assume that the relative change of the test particle's energy while in the structure is small compared to 1 $[\Delta\gamma/\gamma_0 \ll 1]$, and that $r_0'L/r_0 \ll 1$ (L is structure length). We can show that these assumptions are consistent with the relation $\Delta r/r_0 \ll 1$ also being true. So let us, for the moment, include this relation as an assumption. Given the assumptions, Eq. (1) can be integrated to give

$$\frac{\Delta r'}{r} \approx -\frac{1}{2}\frac{\gamma'}{\gamma} \, . \tag{3}$$

Note that the simple model of acceleration of Fig. 1 implies thin lenses at the ends of each structure, with inverse focusing lengths $f^{-1} = \pm \gamma'/(2\gamma)$. Integrating once more we obtain

$$\ln\left(\frac{r}{r_0}\right) \approx -\frac{1}{2}\ln\left(\frac{\gamma}{\gamma_0}\right) + r_0' \int_{z_0}^z \frac{dz'}{r} , \tag{4}$$

which we see is a result consistent with $\Delta r/r_0 \ll 1$. Finally, we can write

$$\frac{\Delta r}{r_0} \approx -\frac{1}{2}\frac{\Delta\gamma}{\gamma_0} + \frac{r_0'\Delta z}{r_0} . \tag{5}$$

We have shown that, for any test particle, if the relative energy change in a structure is small, and if the incoming slope is also small, $\Delta r'$ is approximately proportional to γ' and given by the simple expression Eq. (3). Note that Eq. (5) implies that how LIAR treats the effect on orbit sufficiently downstream of a structure (where E_z and thus γ' are again 0) is approximately correct. Finally, note that we have discussed the first order effect of rf focusing, *i.e.* the effects linear in $\Delta\gamma/\gamma_0$. This leads also to correction to the Twiss parameters, but the correction is to second order in $\Delta\gamma/\gamma_0$. Higher order effects in rf focusing, not considered here, can be found in the literature, *e.g.* Rosenzweig, *et al* [4]. However, for a realistic set of parameters for the NLC structure [5], the higher order effects of rf focusing are only a few percent of the first order effect we consider here.

NUMERICAL EXAMPLE

To study the time development of the longitudinal wakefield of a short bunch passing through an accelerator structure, we perform numerical calculations using TBCI [6] on a model of the so-called H60VG3 cavity (see Fig. 3) [5]. This cavity is a $5\pi/6$ (phase advance) disk-loaded structure containing 55 cells; it is detuned (nearly constant gradient) with iris radius decreasing and cell gap length increasing as one moves from beginning to end of the cavity. Our model differs slightly from the real cavity: the iris radii are not rounded and the cavity radii are taken to be a constant 10.9 mm (our short beam, however, never "sees" the outer cavity walls). The period is 10.9 mm; the iris radius varies from 5.5 mm to 3.9 mm; the gap varies from 6.3 mm to 7.55 mm. The cavity cells (total length 60 cm) are connected to beam pipes; the exit beam pipe in our model is 40 cm long, allowing the wakefields time

to catch up to the (on-axis) beam. Our driving bunch is longitudinally Gaussian with rms length 0.2 mm. Note that the nominal bunch length in the NLC linac will be 0.1 mm. The bunch length for our model was taken to be larger due to computational limitations.
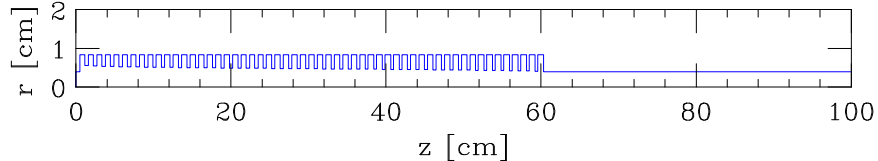

FIG. 3: Structure geometry used in the wakefield calculation: an H60VG3 cavity is is connected to beam pipes.

The longitudinal wake (the integrated longitudinal voltage per charge) for our example is shown in Fig. 4; the driving bunch shape, with head to the left, is also shown. Also shown, by the dashed curve, is the steady-state wake model applied to the dimensions of the average cell, *i.e.* the point charge wake $W_\delta(s) = Z_0 c/(\pi a^2) e^{-\sqrt{s/s_0}}$ with $s_0 = 0.41 a^{1.8} g^{1.6}/L^{2.4}$, where s the longitudinal position, $Z_0 = 377\ \Omega$, c the speed of light, a the iris radius, g the gap, and L the period [7]. For the average cell: $a = 4.7$ mm and $g = 6.9$ mm. The fact that the 2 curves in the figure are nearly identical indicates that, down to our bunch length, the effect of the transients on the wake is very small. The computed loss factor is 0.28 MV/nC, which is equivalent to 0.47 MV/nC/m of structure. We note that the wake is capacitive.

The longitudinal electric field experienced by 3 different test particles in the bunch as function of $z = ct$, with t the time, is shown in Fig. 5. Curves are shown for test particles at the center of the bunch, and at $\pm\sigma_z$ away from bunch center. The vertical lines locate the ends of the cavity. Note that the field is normalized to the bunch charge Q. Note also that the total integral of these curves gives the wakefield at the test particle locations.

We see in all 3 curves an initial transient, a region where the amplitude of the field rises gradually (due to the constant gradient nature of the cavity), and a final transient region. The results differ significantly from the behavior sketched in Fig. 1—the approximate behavior of the field due to fundamental mode rf in the structure. For example, for the bunch center test particle the initial transient regime reaches ~ 7 cm beyond the beginning of the
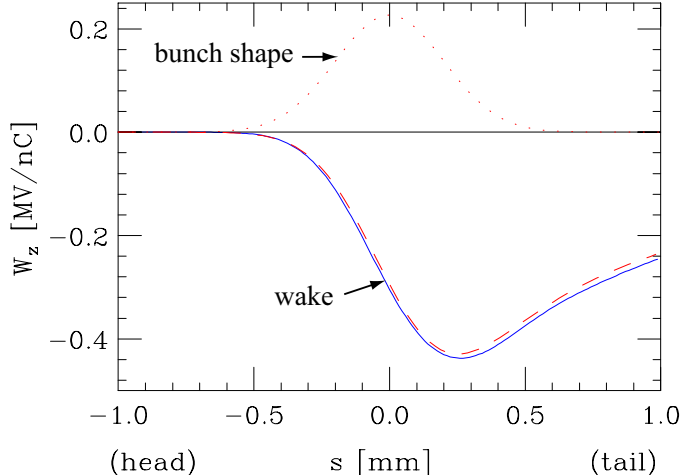
FIG. 4: The longitudinal wakefield excited by a $\sigma_z = 0.2$ mm Gaussian bunch in the H60VG3 cavity, as obtained by TBCI. The dashed curve gives the results of the steady-state wake model. The dotted curve gives the bunch shape, with the head on the left.

cavity, the intermediate region has a slope of ~ -0.3 MV/nC/m^2 and lasts ~ 53 cm; the final transient begins ~ 2 cm beyond the end of the cavity and lasts ~ 18 cm. Note that in the NLC linac, most cavities will be close to neighboring cavities (7.6 cm separation), which will modify the transients somewhat. Note also that, although for longer bunches we might expect to see oscillations in the fields within the cavity (as sketched in Fig. 2), due to the high frequencies excited by our short driving bunch, we see no such oscillations.

To obtain an idea of the strength of the rf focusing effect in the NLC, let us consider the following parameters: bunch charge $Q = 1.6$ nC, bunch length $\sigma_z = 100$ μm; beam energy $\mathcal{E} = 8$ GeV (beginning of linac), accelerating gradient $G = 50$ MV/m. From additional TBCI calculations we find that, for $\sigma_z = 100$ μm in our example structure, the loss factor increases by a factor 1.4 over the $\sigma_z = 200$ μm case. So for the design bunch length the gradient due to the wakefield, at the center of the bunch, is ~ -1.1 MV/m, or about -2% of the accelerating gradient due to the applied rf. Therefore, the cavity focusing due to the wake alone is also about 2% of that due to the rf (and in the opposite direction). The focal lengths of the effective thin lenses at the ends of the structure are very large, $f \approx \mp 16$ km (at the entrance there is defocusing, at the exit focusing). In more detail: since the distance
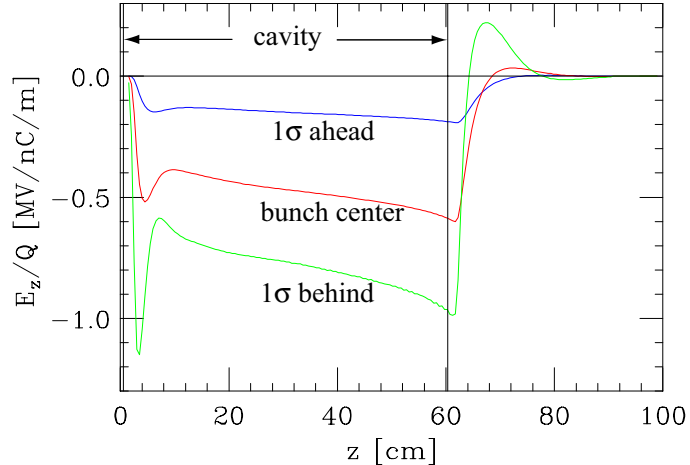
FIG. 5: The longitudinal electric field experienced by 3 different test particles as function of distance within the structure of Fig. 3. The vertical lines locate the ends of the cavity.

of the transients scales as $1/\sigma_z$, we expect that the initial transient at *e.g.* the center of the bunch to reach ~ 14 cm beyond the beginning of the cavity, and the final transient begins ~ 4 cm beyond the end of the cavity and lasts ~ 36 cm (if the cavities are separated by at least 36 cm).

Such a picture gives an idea how, when a beam traverses an empty cavity, the induced electric field—and therefore r'—differ from the case when the cavity is filled with rf, and differs with what LIAR calculates. Nevertheless, after the beam leaves the cavity the orbit as calculated by LIAR will tend to be approximately correct.

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