A New Analysis of Intrabeam Scattering Applied to KEK's ATF

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Overview

- Motivations
- New IBS analysis
- Explicit inclusion of x-y coupling
- Compare to April 2000 ATF measurements
- Conclusions

Motivations

- April 2000 ATF vertical emittance data in conflict with IBS theory.
- Standard (Bjorken-Mtingwa/Piwinski) theory is complicated and has two drawbacks:
  *How to add general coupling
  *Ambiguous Coulomb log
  (bmax = sigy, sigx, interparticle spacing???)

We hoped that by dealing with CL ambiguity and adding coupling we could understand ATF data.

→ New first principles IBS analysis for electron storage rings.

New IBS Analysis

Fokker-Planck equation → Emittance evolution equations

Advantages: Treats the general coupled case naturally

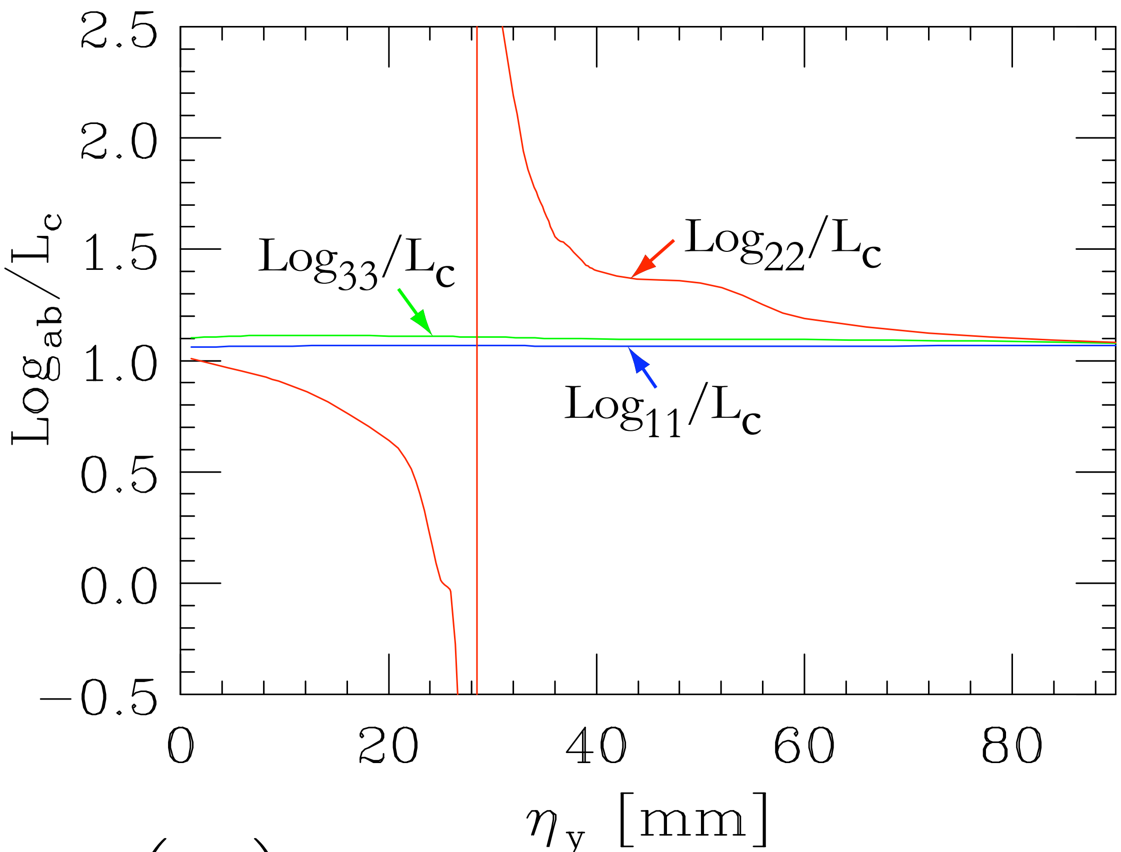
No Coulomb log

For Gaussians, we can reduce to a double integral in the general case.

$$\mathcal{A}K_{ab} = \left(\frac{d\langle p_a p_b \rangle}{dt} \right)_{\mathrm{IBS}}$$

$$2\mathrm{Log}_{ab} = \frac{-\int d\Omega \frac{h_{ab}}{h_3} \log(h_1)}{\int d\Omega \frac{h_{ab}}{h_3}} = \frac{K_{ab}}{K_{ab}^{\mathrm{BM}}}$$

Computed Coulomb Logs for ATF



K_{22} unimportant in High Energy Approximation (HEA)

HEA valid when

$$\underbrace{\left(\frac{\gamma\eta_y}{\beta_y}\right)^2}\gg 1$$

$= 1$ when $\eta_y = 1.8$ mm

$$L_c = \log\left(\frac{\sigma_y}{r_m}\right)$$

x-y Coupling

Smooth Approximation Hamiltonian :

$$H(\vec{z}) = \frac{\beta c}{2}(k_x x_\beta^2 + x'^2 + \boxed{2\kappa x_\beta y_\beta} + k_y y_\beta^2 + y'^2 - \frac{k_z}{\alpha} z_\beta^2 - \alpha\delta^2)$$

coupling term

We follow the evolution of the eigeninvariants of this Hamiltonian:

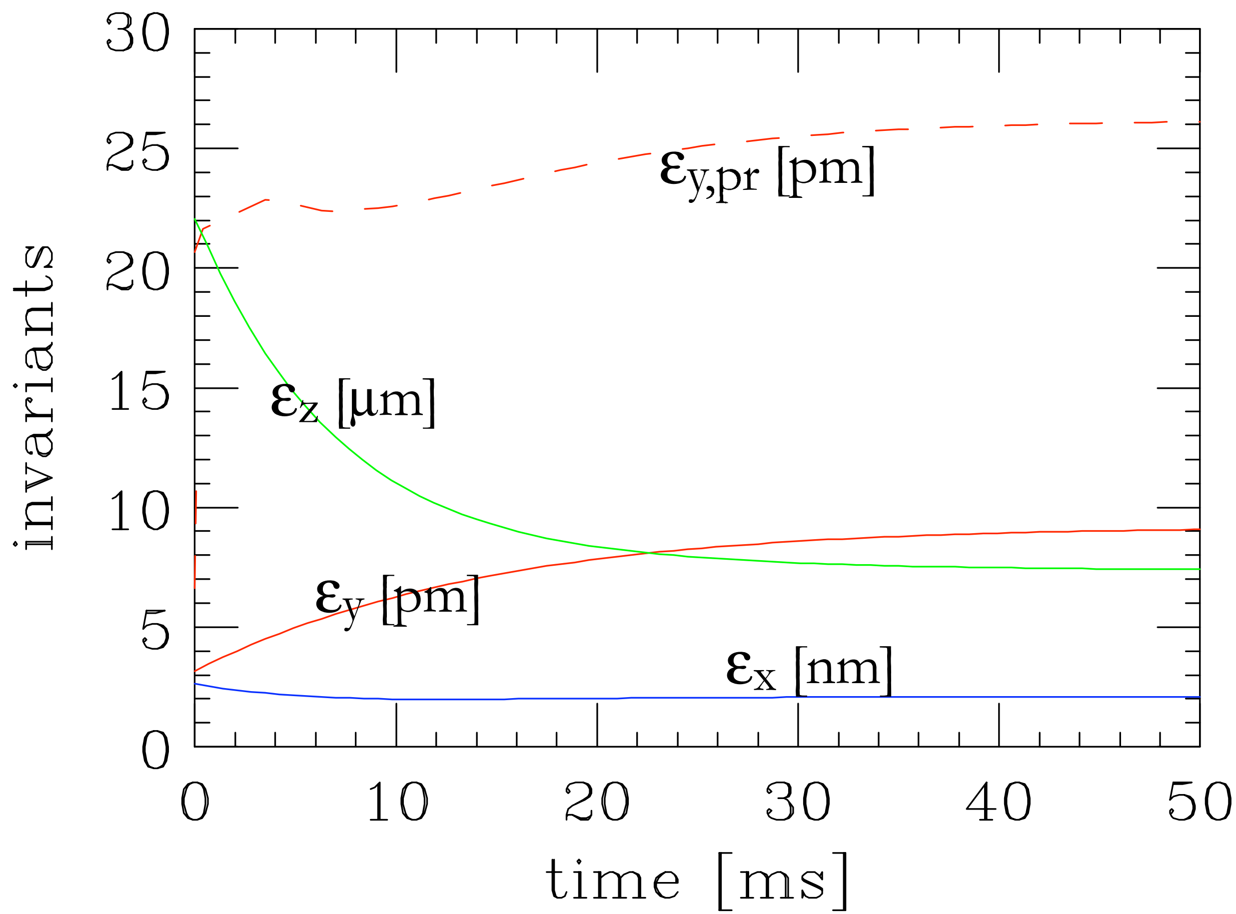
$$\kappa = \Lambda \sin\psi$$

$$k_{x,y} = k_0 \pm \Lambda \cos\psi$$

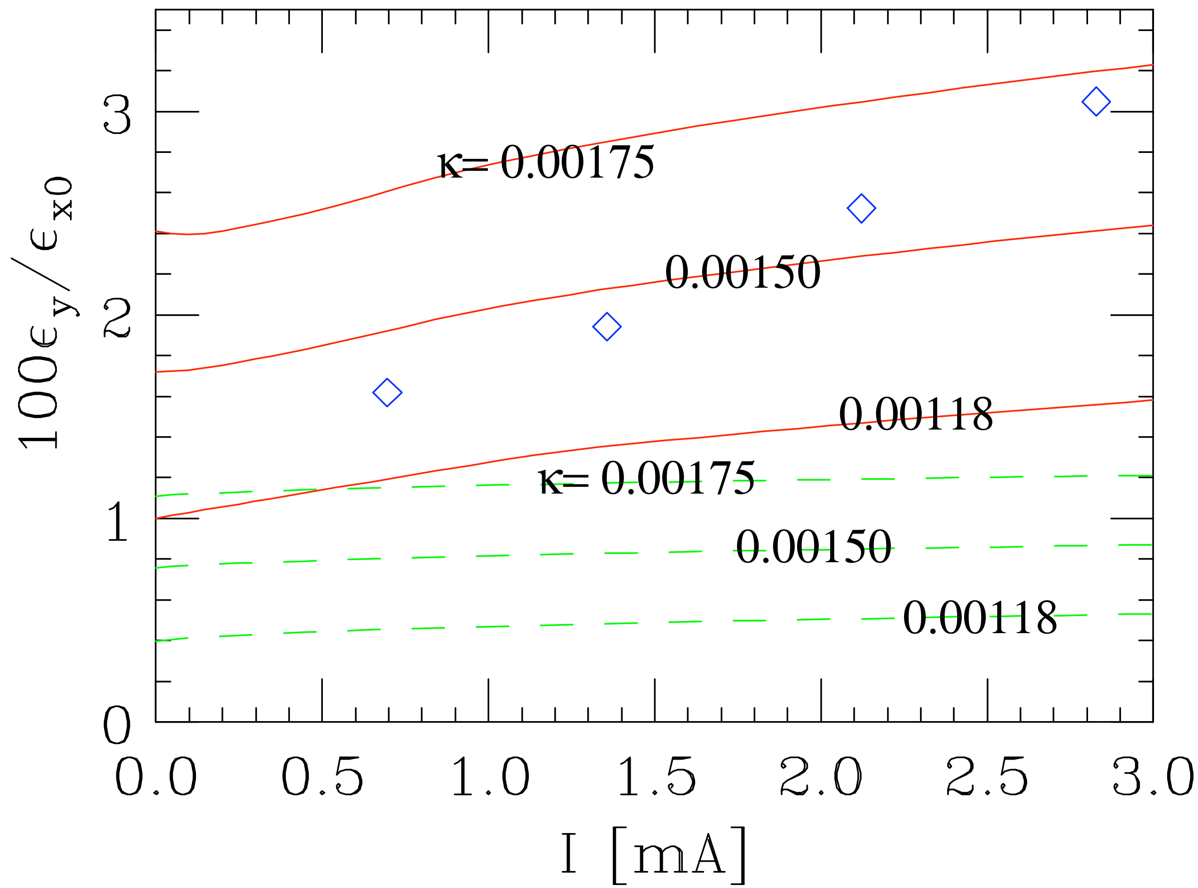
$$g_{1,2} = \frac{1}{2\sqrt{k_0 \pm \Lambda}}([(k_0 \pm \Lambda)x_\beta^2 + x'^2](1 \pm \cos\psi) +$$
$$[(k_0 \pm \Lambda)y_\beta^2 + y'^2](1 \mp \cos\psi) \pm 2[(k_0 \pm \Lambda)x_\beta y_\beta + x'y'] \sin\psi),$$

$$g_3 = \frac{z_\beta^2}{\beta_z} + \beta_z \delta^2$$

Evolution for ATF parameters



Comparison with ATF Measurements



$x_\beta y_\beta$
tilt angle
$\approx 5^\circ$

ATF Parameters

$E_0 = 1.28$ GeV, $\eta_x = 0.052$ m, $\eta_y = 0.0074$ m, $\beta_x = 3.9$ m, $\beta_y = 4.5$ m, $\rho/R = 0.260$, $\epsilon_{x0} = 1.05$ nm, $\epsilon_{y0} = 0.007$ nm, $\sigma_{z0} = 5.05$ mm, $\sigma_{\delta 0} = 5.44 \times 10^{-4}$, $N = 9 \times 10^9$. The damping times $\tau_a = 1/\alpha_a$ are $\tau_x = 18.2$ ms, $\tau_y = 29.2$ ms, and $\tau_p = 20.9$ ms. For the minimum distance cut-off we used $r_m = \frac{r_0 \beta_x}{\gamma^2 \epsilon_x} = 1.66 \times 10^{-12}$ m.

Two interesting parameters are η_y and current.

For $\eta_y = 1\ \mathrm{mm}$ $\quad \epsilon_{y0} \approx 10^{-13} \mathrm{rad-m}$ and

$$\epsilon_{y,eq} = 9.2 \times 10^{-13}\ \mathrm{m}$$

$\eta_y = 0$ $\quad \epsilon_{y0} \approx 0$ and

$$\epsilon_{y,eq} = 7.1 \times 10^{-13}\ \mathrm{m}$$

For coupled case: doubling current from 3.1 mA to 6.2 mA causes $\dfrac{100\epsilon_{y,eq}}{\epsilon_{x0}} : \ 2.4 \rightarrow 2.7$

Conclusions

- Our analysis allows more careful IBS computations --- Can we get beyond the "1/Log" accuracy?
- Our equations reduce to BM in a well defined way, allowing exploration of what the Coulomb log approximation means and when it can break down.
- We have included x-y coupling in a rigorous way
- For ATF parameters BM/CL seems good, but beware the "High Energy Approximation" for small vertical dispersion.
- The magnitude of ATF ϵ_y calculations consistent with coupling dominated region with 4-6 degree tilt angle. Current dependence does not fit. Measurements error? Non-IBS physics?

Future Work

- Explore full parameter space for ATF. Is there a realistic regime where we find a substantial difference?
- Apply to Protons and Heavy ions
- Non-Gaussian Equilibria
- Synchro-betatron coupling
- Extend beyond Smooth Approximation