"B Physics and CP VIOLATION" Course

Introduction to the operator product expansion

Lecture # 8

Dr. Mihir Worah

Friday morning, 9:30 a.m.

September 19

Training and Conference Center
INTRODUCTION TO THE

OPERATOR PRODUCT EXPANSION

i) physical picture + uses

ii) HEP [OPEN] for $AB=1$ and $AB=2$.

iii) the RGE for Wilson Coefficients

iv) loop effects in Effective Field Theories

--- x ---

next: Leading log solution to the
RGE for $AB=1$ and $AB=2$

then: Matrix Elements of $AB=1 + AB=2$
operators between physical states.

GOAL: Step thru' the Terminology +
technology used to calculate $B$
declays + mixing.
the OPE:

\[ \Theta_A(x) \Theta_B(0) = \sum_n C_n(x) \Theta_n(0) \]

practically one constructs an Effective Field Theory by "integrating out" heavy particles.

consider \( b \rightarrow c \bar{u} d \)

\[
\begin{align*}
&= \frac{2 \gamma_{\mu} (1-\gamma_5)}{p^2 - M_5^2} \int d^4x \frac{(ig_2)^2}{8} \gamma_\mu (1-\gamma_5) \gamma_\nu \\
&= \frac{2 \gamma_{\mu} (1-\gamma_5)}{p^2 - M_5^2} \int d^4x \frac{(ig_2)^2}{8} \gamma_\mu (1-\gamma_5) \gamma_\nu \left[ 1 + \frac{p^2}{M_5^2} + \ldots \right]
\end{align*}
\]
\[ \Theta_A = C_1 \delta_{\mu(1-R)} b \]
\[ \Theta_B = d \delta_{\mu(1-R)} b \]
\[ \Theta_i = C_i \delta_{\mu(1-R)} n \]
\[ C_i = Q^2 / 8M_{\pi} \]
\[ C'_i = C_i / \hbar^2 \]

\[ A : B \rightarrow F = \sum C_i(\mu) \langle F | \Theta_i(\mu) | B \rangle \]

What have we gained?

* Invarially ... the Wilson coefficients \( C_i(\mu) \) are process independent. In \( B \rightarrow D \pi \), \( B \rightarrow D \rho \) all have the same \( C_i \), the difference is confined to the matrix elements of the \( \Theta_i \).
ii) we get a clear separation of scales:

$$B \to D W = \sum C_{i}(\mu) \langle D W | \Theta_{i}(\mu) | B \rangle$$

- all the short distance physics (integrating out $W$) is in the $C_{i} \sim M_{W}$.
- all the long distance physics (hadronization) is in the matrix element $\sim m_{b}$.

* in particular, we can choose the arbitrary renormalization scale $\mu$ to our advantage.

a) since there are many scales involved, our calculations will contain large $\log(M_{W}/m_{b})$

b) physical quantities cannot depend on $\mu$: $\mu$ dependence should cancel between $C_{i}(\mu)$ and $\langle \Theta_{i}(\mu) \rangle$.

however we don't know how to calculate $\langle \Theta_{i}(\mu) \rangle$!
choosing $\mu_0 = m_b$ will lead to large $\log \left( \frac{M_0}{m_b} \right)$ in $\Theta_i$ that we have no control over.

choosing $\mu_0 = m_b$ will lead to large $\log \left( \frac{M_0}{m_b} \right)$ in $\Theta_i$ which we can calculate precisely, $\Gamma$ resum using the RGE.

iii) allows us to calculate QCD corrections and resum them almost trivially (leading logs)

e.g. $b - \bar{b}$ mixing:

can be done a 2-loop calc... extremely difficult.
The $\Delta B=1$ Operators: $Q_1 - Q_6$

(ignoring $Q_7 - Q_{16}$) $\Delta C = 0$ 
$\Delta S = 1$

**I** Current: Current

\[ Q_1 = (\bar{c}_b b_p)_{V-A} (\bar{s}_p c_p)_{V-A} \]
\[ Q_2 = (\bar{c}_b)_{V-A} (\bar{b} c)_{V-A} \]

**II** QCD Penguin:

\[ Q_3 = (\bar{c}_b)_{V-A} (\bar{q}_q q_p)_{V-A} \]
\[ Q_4 = (\bar{s}_b b_p)_{V-A} (\bar{q}_q q_p)_{V-A} \]
\[ Q_5 = (\bar{s}_b)_{V-A} (\bar{q}_q q_p)_{V+A} \]
\[ Q_6 = (\bar{s}_b b_p)_{V-A} (\bar{q}_p q_p)_{V+A} \]

The $\Delta B=2$ operator

\[ Q(\Delta B=2) = (\bar{c}_b d_p)_{V-A} (\bar{b} d)_{V-A} \]
The RGE for Wilson Coefficients.

The Callan-Symanzik Eq for Green's func:

Consider the n point Greens func

\[ \Gamma^{(n)}_{R}(p_1, ..., p_n, q, \mu) = N^\frac{n}{2} \Gamma^{(n)}_{B}(p_1, ..., p_n, q, \mu) \]

Renormalized Field
\[ \Gamma^{(n)}_{B} \text{ is independent of } \mu, \text{ the renormalization scale} \]

\[ \frac{d \Gamma^{(n)}_{R}}{d \mu} = \left[ \frac{\partial}{\partial \mu} + \frac{d g}{d \mu} \frac{1}{2} \right] \Gamma^{(n)}_{R}(p_1, ..., p_n, q, \mu) \]

\[ = \frac{n}{2} \frac{d}{d \mu} \left( \frac{d z}{d \mu} \right) \cdot N^\frac{n}{2} \Gamma^{(n)}_{B}(p_1, ..., p_n, q, \mu) \]

\[ \Psi \left[ \frac{\partial}{\partial \mu} + \frac{d g}{d \mu} - n \frac{d}{d \mu} \right] \Gamma^{(n)}_{R}(p_1, q, \mu) = 0 \]

\[ \phi = \frac{d g}{d \mu}, \quad \tau = \frac{1}{2} \frac{d}{d \mu} \left( \ln N^2 \right) \]
generalize the OPE: $\Theta_A(x) \Theta_B(0) = \sum_n \frac{\xi_n(x) \Theta_n(0)}{n}$

insert into Green's functions:

$$\Pi_{AB}^{(n)}(q, p_1, \ldots, p_n; \mu) = \sum_i C_i(q, \mu) \Pi_{i}^{(n)}(0, p_1, \ldots, p_n; \mu)$$

applying the Callan-Symanzik Eq. to the Green's function on both sides gives us the RGE's for $C_i(\mu)$.

LHS:

$$\left[ \frac{\mu}{v_1} + \frac{\partial}{\partial q^1} - n \gamma - (\delta^A + \delta^B) \right] \Pi_{AB}^{(n)}(q, p_1, \ldots, p_n; \mu) = 0$$

RHS:

$$\left[ \frac{\mu}{v_1} + \frac{\partial}{\partial q^1} - n \gamma - \delta^i \right] \Pi_{i}^{(n)}(0, p_1, \ldots, p_n, \mu) = 0$$


\[ \left[ \frac{\mu}{v_1} + \frac{\partial}{\partial q^1} + \delta^i - (\delta^A + \delta^B) \right] C_i(q, \mu) = 0 \]

\[ C_i(M, \mu) = C_i(M, M) \left[ \frac{\delta^i(M)}{\delta^i(\mu)} \right] [\delta^i - \delta^A - \delta^B]_{\mu} \]
loop Calculations in Effective Theories

consider

\[ \text{this is } \frac{1}{M^2} \]

now at one loop this contributes to \( \bar{Z} \bar{b} b \) vertex:

\[ \text{this should also be } \sim \frac{1}{M^2} \]

however using any cut \( \bar{b} b \):

\[ \int \frac{d^4 k}{k^2} \cdot \frac{1}{k^2} \sim \frac{\Lambda^2}{M^2} \to 1 \text{ for } \Lambda = M \]

\( \Rightarrow \) need to use a mass and RN scheme

like dimensional regularization or minimal subtraction

\[ \sim \frac{1}{M^2} \int d^4 k \frac{1}{k^2} \sim \frac{\Lambda^2}{M^2} \log M \text{. so it is } \frac{1}{M^2} \! ! \]
it's easy to see... however heavy particles
do not decouple. (obviously)!

they have to be integrated out by
hand!

so once you're below the particle
mass it has to disappear from the
theory completely!

\[ \beta_{\text{had}}(g_\ast) = -\frac{g_\ast^3}{(\pi \Lambda)^2} \]

\[ \frac{\Lambda}{\Lambda} \rightarrow \frac{\Lambda}{\Lambda} \]

\[ \mathcal{N}_F = \# \text{ light fermions} \]

\[ \Rightarrow \frac{4}{3} \# \text{ deg. families!} \]

\[ \delta_6(\mu) = \frac{\delta_6(M)}{1 + (\delta_6(M)/2\pi) b_0 \log(M/M)} \]