INTRODUCTION TO CP VIOLATION

Neutral B mesons

Lecture #3

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Neutral $B$ mesons

\[ B^+_H = p B^0 + q \bar{B}_0 \]

H - heavy  
L - light  
\[ B^0 = \bar{b} d ; \quad \bar{B}^0 = b \bar{d} \]
\[ B^0_s , \bar{B}^0_s ; \quad B^0_s , \bar{B}^0_s , b \bar{s} \]

$p,q$ for $B_d$ system, $B_s$ system, $K$ system all different.

For $B_d$ mesons

- many decay modes flavor defined
- possible common modes for $B_d, \bar{B}_d$ at \( 10^{-3} \) or less

$\Rightarrow$ CP eigenstates have small branching ratios

$\Gamma_H \approx \Gamma_L$ - differences small

mass difference $\frac{\Delta m}{\Delta \Gamma} \quad \text{dominates lifetime differences}$
Studying CP in the B mesons

• Does the Standard model predict it correctly?

• If not — what can we learn about beyond Standard Model physics?

⇒ step 1: observe any non-K-meson CP effect.

step 2: — observe enough separate effects to make a real test.
3 types of CP violation

\[ A_f = \langle f | H | B \rangle \]
\[ \bar{A}_f = \langle \bar{f} | H | \bar{B} \rangle \]

\[ \lambda = \frac{g}{|\rho|} \frac{\bar{A}}{A} \]

3. "Direct" CP violation = CP violation in decays

\[ \frac{\bar{A}}{A} \neq 1 \]

2. Mixing-induced CP violation = CP violation in eigenspaces

\[ \frac{g}{|\rho|} \neq 1 \]

3. CP violation due to interference between decay with and without mixing

\[ B \rightarrow f \quad \alpha \ \text{Im}(\lambda) \]

- can occur even if \[ |g/\rho| = 1 \]
  and \[ |\bar{A}/A| = 1 \].
CP violation in decay

\[ \left| \frac{A_f}{A} \right| = 1 \quad \text{or} \quad |A_f|^2 = |\bar{A}_f|^2 \]

Note: CPT requires \[ \left| \sum \frac{2}{\sqrt{3}} A_f \right|^2 = \left| \sum \frac{2}{\sqrt{3}} \bar{A}_f \right|^2 \]
but not channel by channel.

How can this occur?

Need \( >1 \) contributing term — two diagrams

- different weak phases (CKM matrix elements)
- different strong phases (rescattering)
\[ A_f = A_1 e^{i(s_1 + \phi_1)} + A_2 e^{i(s_2 + \phi_2)} \]

\[ \overline{A_f} = A_1 e^{i(s_1 - \phi_1)} + A_2 e^{i(s_2 - \phi_2)} \]

\( s \) = strong phase \quad \( \phi \) = weak phase

Then

\[ |A_f|^2 - |\overline{A_f}|^2 = 4A_1 A_2 \sin(s_1 - s_2) \sin(\phi_2 - \phi_1) \]

(* H.W. check this!)

**Problem**

Theorists cannot calculate

\( A_1, A_2, s_1, s_2 \)

All estimates are unreliable!
(2) \[ |q/p| = 1 \] CP violation included in states from mixing.

Image Asymmetry in semi-leptonic decays as seen in K system.

Expected to be very small in \( B_d \) system.

In what follows we will approximate \( |q/p| = 1 \).

Note if \( |q/p| = 1 \) it "appears" that you can choose a phase convention such that the two \( B_d \) states \( B_H \) and \( B_L \) are "CP" eigenstates.

However, this \( b \)-flavor CP definition cannot be related to light quark CP quantum numbers unless CP is conserved in all of \( H \).
\( B(t) \) = state produced as \( B^0 \) at time \( t = 0 \)

\[
B(t) = e^{-\frac{\Delta m^2 t}{2}} \left\{ (pB_0 + q\bar{B}_0) e^{i\frac{\Delta m^2 t}{2}} + (p\bar{B}_0 - qB_0) e^{-i\frac{\Delta m^2 t}{2}} \right\}
\]

\( R_B(t) \) as \( e^{-\frac{\Delta m^2 t}{2}} \left\{ \right\}

Write \( \Delta \) as \( \frac{iq}{p} \frac{\bar{A}_f}{A} \)

\[
R_B(t) = e^{-\Delta m^2 t} \left\{ 1 + \left( \frac{1}{2} \right)^2 + (1 - \frac{1}{2}) \cos \Delta m t \right\}
\]

\[ -2 \Im \Delta \sin \Delta m t \]

HW - Carry out the missing steps!

\( R_B(t) = e^{-\Delta m^2 t} \left\{ 1 + \left( \frac{1}{2} \right)^2 + (1 - \frac{1}{2}) \cos \Delta m t + 2 \Im \Delta \sin \Delta m t \right\} \)
Thus the rate asymmetry

\[ \frac{R_\beta(t) - R_{\bar{\beta}}(t)}{R_\beta(t) + R_{\bar{\beta}}(t)} = \frac{(1 - |\lambda|^2) \cos \Delta m t - 2 \text{Im} \lambda \sin \Delta m t}{1 + |\lambda|^2} \]

\[ \rightarrow - \text{Im} \lambda \sin (\Delta m t) \]

\[ |\lambda| = 1 \]

\[ \frac{1}{e} \]

\[ \text{phase of } \lambda \]

\[ \text{predicted by Standard Model for } \frac{R_\beta}{A} = 1 \text{ cases} \]

\[ 2 \times \text{phase of mixing } \frac{\rho_p}{\rho} = e \]

\[ -2i\phi_m \]

\[ \text{phase of decay } \frac{A}{A} = e \]

\[ \phi_m - \phi_D \text{ is convention independent} \]

\[ \text{each separately is NOT!} \]
General SM decays

\[ b \rightarrow s \text{ or } d, \quad \bar{q}' \quad \Rightarrow \text{"TREE"} \]

\[ b \rightarrow \text{gluon (or } \gamma \gamma) \quad \Rightarrow \text{"PENGUIN"} \]

- u, c, or t quark

Gluon momentum of order \( m_b \)
\[ \Rightarrow \alpha_s(m_b) \text{ suppression} \]

Term enhanced by large t-quark mass
\[ \Rightarrow \alpha_{em} \frac{m_t^2}{m_Z^2} \]

Electroweak penguin
\[ A_1 = A_{\text{tree}} \chi V_{qb} V_{q's} \Rightarrow \text{weak phase} \]

\[ \exists q = q' \Rightarrow \text{down type} \Rightarrow \text{No tree} \]

\[ q \neq q' \Rightarrow \text{Tree only} - \text{no possible direct CP violation in this decay.} \]

\[ A_2 = A_{\text{penguin}} : 3 \text{ terms} \]

\[ A_2 = V_{qb} V_{ts} f(mt) + V_{cb} V_{cs} f(mc) \]

\[ + V_{ub} V_{us} f(mu) \]

But unitarity gives e.g.

\[ V_{tb} V_{ts} + V_{cb} V_{cs} + V_{ub} V_{us} = 1 \]

Use this to regroup

\[ A_2 = V_{cb} V_{cs} \left[ f(mc) - f(mt) \right] + V_{ub} V_{us} \left[ f(mu) - f(mt) \right] \]

\[ \mathcal{O}(\alpha^2) \]

\[ \mathcal{O}(\alpha^3) \]
Thus decays $b \rightarrow c \bar{c} s$

Tree CKM coefficient $= V_{cb} V_{cs}^*$

Penguin CKM structure

$$\frac{\alpha_{b(t)}}{\pi} [V_{cb} V_{cs}^* \left[ f(m_c) - f(m_b) \right] + V_{wb} V_{ws}^{*} \left[ f(m_{\mu}) - f(m_{\tau}) \right]]$$

$\uparrow$

same CKM structure as tree

$\uparrow$

NEGLIGIBLE

$: Only one weak phase appears in $A$

$\bar{A}$

(up to very small corrections)

$|\bar{A}/A| = 1$

Clean prediction of asymmetry $\alpha \sin 2\beta$
\( b \rightarrow s \bar{s} s \) \quad No\ tree

Penguin only
\[
\frac{\alpha_s}{4\pi} \left\langle V_{cb} V_{us} \left[ f(m_c) - f(m_u) \right] + V_{ub} V_{us} \left[ f(m_c) - f(m_d) \right] \right\rangle
\]
\( \Rightarrow \text{ rate low} \quad \left[ \text{correction: } O(\alpha_s^2) \right] \)

but a \( \Delta \sin \beta \) also.

\( b \rightarrow u \bar{u} s \) (and \( d \bar{d} s \)) \quad B \rightarrow pK_s
\( \Delta \rightarrow \pi K_s \)

Now tree has \( V_{ub} V_{us}^* \)

so we have
\[
\frac{\alpha_s}{4\pi} A \Delta^2 \quad \text{vs} \quad A \Delta^4
\]

\( |A| \neq 1 \quad \leq \quad \text{different weak phases} \quad \text{more than 1 I-spin} \quad \text{\( \Leftrightarrow \) different I-phase} \)
Now consider \( b \to q \bar{q} d \).

Here it is convenient to write penguin term as:

\[
V_{tb} V_{td}^* f(m_t) + V_{cb} V_{cd} f(m_c) + V_{ub} V_{ud} f(m_u)
\]

\[
q = s \quad \Rightarrow \quad 0 \quad m_c = m_u
\]

\[
\ln \left( \frac{m_{u}^2 + m_{d}^2}{m_{b}^2 - 4m_{c}^2} \right) \quad \text{etc} \quad \ln \left[ 1 + 4 \left( \frac{m_{c}^2 - m_{u}^2}{m_{b}^2} \right) \right]
\]

Note: GIM suppression

\[
f(m_c) - f(m_u) \quad \text{same form as tree}
\]
Thus in all $q \bar{q}$ d cases dominant $\Delta$ penguin terms enter with different weak phase from tree terms or two competing penguin terms.

$$\left| \alpha_A \right| + 1 \Rightarrow \left| 2 \right| + 1$$

Then

$$\alpha = \frac{(1 - \left| 2 \right|^2) \cos(\Delta m t) + \text{Im} 2 \sin(\Delta m t)}{1 + \left| 2 \right|^2}$$

and $\text{Im} 2 = \left| 2 \right| \text{arg}(\phi_\mu - \phi_\tau)$

but $\phi_\tau$ not given simply in terms of CKM elements.
Ignoring for the moment penguins

\[ a(\mathcal{B} \to \phi \pi^0) = a(\mathcal{B} \to \pi^+ \pi^-) - \]

\[ = \sin c \beta \sin \Delta M t \]

this we want to measure

What can we do to get rid of the penguins?

ISO SPIN
J = spin

(\bar{u} d) \hspace{1cm} I = spin \hspace{1cm} \text{I-spin doublet} \hspace{1cm} \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}

All other quarks I = 0

gluons I = 0

\text{(But } q \bar{q} \text{ are admixtures } I = 0 \text{ and } I = 1.)

\text{Later}

Decay amplitudes can be labelled by

\bar{B}_0 \text{ decay } \Delta I \text{ of } b \rightarrow q \bar{q} \hspace{1cm} d \\
I_f \text{ of } q \bar{q}' \hspace{1cm} d \hspace{1cm} \overline{d}

For \hspace{1cm} b \rightarrow u \bar{u} d \hspace{1cm} \Delta I = \frac{1}{2} \text{ or } \frac{3}{2} \hspace{1cm} \text{ and } I_f = 0 \hspace{1cm} 1 \hspace{1cm} or \hspace{1cm} 2

\therefore \hspace{1cm} \text{In general } 4 \text{ independent I-spin amplitudes}
However

\[ g \quad \begin{array}{c}
  b \\
  c \\
  q \\
  g \\
  q
\end{array} \]

\[ g \text{ has } I = 0 \]
\[ \bar{q}q \text{ has } I = 0 \]

\[ \Delta I = \frac{1}{2} \quad 3/2 \]
\[ I_f = 0 \leftrightarrow 1 \rightarrow 2 \]

\[ \text{strong} \]

\[ \therefore \quad \text{No penguin contribution to any } I = 2 \text{ amplitude} \]

\[ \Rightarrow \quad \text{If we can isolate the asymmetry in the } I = 2 \text{ terms, then we have only 1 weak phase in } A \]

\[ \Delta = \left| \frac{A_2}{A_1} \right| \leq 1 \Rightarrow \text{Asymmetry } \alpha \sin 2\alpha \]
Requires multi-channel analysis

In \( \pi \pi \pi \) case

\[
\begin{align*}
B^+ & \rightarrow \pi^+ \pi^0 \\
B^- & \rightarrow \pi^- \pi^0 \\
B^0 & \rightarrow \pi^+ \pi^-
\end{align*}
\]

rates

rate and asymmetry

In \( \rho \rho \) case

\[
\begin{align*}
B^0 & \rightarrow \rho^+ \pi^- \\
& \rightarrow \rho^- \pi^+ \\
& \rightarrow \rho^0 \pi^0
\end{align*}
\]

\( \pi^+ \pi^- \pi^0 \)

\( \rightarrow \) interference where kinematics allows overlaps

\( \rightarrow \) enough information to separate

\( \sin 2 \alpha \) AND \( \cos 2 \alpha \)