CP VIOLATION
&
B Physics

Angular Analysis

Lecture #13

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Problem

Modes $B \Rightarrow V V \ldots$ or even higher spins.

$\ell = 0, 1, 2 \ldots$ higher $\ell$'s.

CP of final state

$= (\text{intrinsic CP of particles}) (-1)^\ell$

$\ell$ even and odd $\Rightarrow$ mixture of even and odd CP.

Asymmetry (coefficient of $\sin \Delta t$ term)

$= f_e \sin 2\phi - f_o \sin 2\phi$

$= \frac{f_e + f_o}{2} \sin 2\phi$

where $\sin 2\phi = 1 \cdot \lambda_w$

$= (1 - 2f_o) \sin 2\phi$

"Dilution" factor

- reduces asymmetry magnitude

- confuses relationship between $\alpha$ and CKM parameters.
Possible Ways Out

(i) Theorists' challenge — calculate $f_0$

- Very difficult (hopeless?) for any exclusive channel, depends on meson form factors
  \Rightarrow model dependent

- For inclusive rates e.g. $B \rightarrow$ charmless hadrons


\begin{itemize}
  \item Uses "local quark-hadron duality"
  \item Uses quark diagram kinematics to determine final hadron kinematics
  \item $f_0$ is probably cut-dependent
  \item Dependence is sensitive to assumptions
\end{itemize}

(ii) Experiments' challenge — measure $f_0$

\Rightarrow angular analysis of decays of pseudo-two-body channels
Example:  \[ B \to \pi K^* \]

- presents general ideas

see Dunietz, Quinn, Snyder, Toki and Lipkin


for tables of many channels for which such approaches could be used.

\[ B \to \pi K^* \]

\[ \pi^+ \to K^+ \pi^- \]

\[ \pi^- \to K^- \pi^+ \]

vector particle decays

via vector current to

two "massless" fermions

up to \[ m/2m \]


angular analysis of decay

pattern determines helicity of \[ \pi \]
General Warning for angular treatments — care with conventions is essential.

— picture is deceptive, each decay is analysed in rest frame of parent

If Kππ define a plane

μ⁺μ⁻ (or e⁺e⁻) are not in the same plane (in general).
Several approaches

Theorists prefer

- Transversity — projection of helicities on direction transverse to $\pi K_{13}$ plane
- Moments of angular distributions

$\Rightarrow$ pick out particular helicity contributions

$$\int dQ^2 \text{ (Rate) } Y_{em}(\theta, \phi) \text{ projection}$$

(These quantities are also useful for displaying results and checking stability of fits.)

Experiment will eventually use

- maximum likelihood fit to all data to extract helicity amplitudes
  - i.e. multiparameter fitting

Note — isospin related charged channels may help constrain the relevant parameters, should be used!

$\text{eq. } B^+ \rightarrow \bar{\nu} K^{++}$
Now some grubby details

Consider \( B^0(\ell) \to \Psi(K^{*}) \)

\[ \mathcal{J}_\Psi = 1, \quad \mathcal{J}_{K^*} = 1 \]

\[ \alpha = \sum_{\lambda = 0, \pm 1} \left( \frac{2 \mathcal{J}_{K^*} + 1}{4\pi} \right)^2 \left( \frac{2 \mathcal{J}_\Psi + 1}{4\pi} \right)^2 D(R^\Psi) \frac{D(J_{K^*})}{\lambda_0} A_\lambda(t) \]

Angular dependence

\( \alpha = \pm 1 \) helicity of \( \Psi \) and \( K^* \)

Total lepton helicity in \( \Psi \) decay

\[ \text{Rate} = \sum_{\alpha = \pm 1} \left( A_{\alpha} \right)^2 \]

U.B. Factorization of angular dependence and time dependence

\( \Rightarrow \) allows angular analysis for each time bin (with sufficient data)

\[ \text{Rate} = \left( \frac{a}{4\pi} \right)^2 \sum_{\alpha = \pm 1} \sum_{\lambda, \lambda' = \pm 1} A^\Psi_{\lambda'} A^{*}_\lambda(t) D^1(R^\Psi) D^1(R^\Psi) D^1(R^\Psi) D^1(R^\Psi) D^1(L) \]
Now we use a standard trick or two

\[ D_{m,m'}^J(R) = (-1)^{m-m'} D_{-m,m'}^J(R) \]

and

\[ J_1 + J_2 \]

\[ \sum_{J_3} (J_1, m_1, J_2, m_2, |J_3, m_3\rangle) (J_1, m_1', J_2, m_2', |J_3, m_3\rangle) \frac{J_3}{m_3} \]

\[ J_3 = \{|\bar{S}_1, -S_2|\} \]

\[ J_z \text{ for } \Psi = \pm \text{ side } = J_L \]

\[ J_z \text{ for } K \text{ side } = J_R \]

This may look like a step backwards

but it lets us do the sum over \( \alpha \) explicitly

\[ |M|^2 = \frac{3}{|M|^2} \sum_{\alpha} A_\alpha^*(l) A_\alpha(l) \sum_{\alpha'} (-1) \sum_{J_L, J_R = 0, 1, 2} \]

\[ (1 \alpha, 1-\alpha | J_L 0) (1 \alpha, 1-\alpha | J_L M'_L) D_{-m'_L, 0}^{J_L} (R_L) \]

\[ (1 0, 0 | J_R 0) (1 0, 0 | J_R M'_R) D_{-m'_R, 0}^{J_R} (R_K) \]

Note \((1 0, 1 0) = 0 \Rightarrow \bar{J}_z = 1 \) does not contribute

\[ \sum_{\alpha} (-1)^{\alpha} (1 \alpha, 1-\alpha | J_L 0) = 0 \text{ for } J_L = 0 \]

Thus \( \sum_{\alpha} \) does not contribute either.
So finally we have (after a sum)

\[
\text{Rate} = -2 \left(\frac{3}{4\pi}\right)^2 \sum_{\lambda, \lambda'} A_{\lambda} A_{\lambda'}^* \sum_{J_L, J_R = 0, 1, 2} D_{J_L, J_R}^{J_L, J_R} (R_{\lambda, \lambda'})
\]

(111 1 J_L, 0) (1 \lambda 1 - \lambda' | J_L, J_R) D_{J_L, J_R}^{J_L, J_R} (R_{\lambda, \lambda'})

Further we have

\[
D_{m, \phi} (R) = \frac{\sqrt{4\pi}}{2L+1} Y_{LM}^*(\theta, \phi)
\]

explicit angular functions

(Jackson convention used)

From here on you can proceed in 3 ways

(i) transversity analysis

(ii) moments

\[
\int d^2 \kappa \int d^2 \kappa' (\text{Rate}) \frac{Y_{LM}^*(\theta, \phi) Y_{LM}(\theta, \phi)}{a_m}
\]

angular projections

\[
= T_{s \lambda \phi} m
\]

(iii) fit for \( A_{\lambda}(t) A_{\lambda'}(t) \frac{D_{J_L, J_R}^{J_L, J_R} (\theta, \phi)}{a_m, a_{\lambda'}(t) \phi} \) D_{s \lambda \phi} m

from \( \lambda \in \{ \pm \lambda \} \)
Transversity $\leftrightarrow$ simple angular moments

with transparent physical interpretation

$C$ = projection of helicities of

$\pi^+ K^+ \pi^-$ on an axis

$\perp$ (transverse) to plane of $\pi^+ K^+ \pi^-$

here clearly $\tau_\pi = \tau_\pi = 0$

(for cases where they are nonzero further decays may be used to fix them)

Note: Reflection about plane

$R = P e^{i \pi T_T}$

but $T_T = 0 = C + \ell$

and $P = P_{\text{intrinsic}} (-1)^C$

so $R = P_{\text{intrinsic}} (-1)^C (-1)^{\tau + \ell} = P_{\text{int}} (-1)^{\ell}$

Also $CR = CP e^{i \pi T_T} = CP$

$13.9$

$13.9$
Transversity projections $B \to \Upsilon K$
(simpler moments)
- let $\Theta$ = angle between $e^-$ ($\bar{\nu}^-$) and transverse axis to $\Upsilon K$ plane.

$S(\Theta, t) = \text{Rate} = P_+(E) R_+(\Theta) + P_-(E) R_-(\Theta)$

- $P_+(t)$ = time dependent $CP$ even rate
- $P_-(t)$ = time dependent $CP$ odd rate

$P_+(\Theta) = \frac{3}{48} \left( 1 + \cos^2 \Theta \right)$

$P_-(\Theta) = \frac{3}{4} \sin^2 \Theta$

Thus integrating $\int_0^1 d\cos \Theta \cdot P(\Theta, t) = M_0$

$P_2(\cos \Theta) = \frac{1}{2} (3 \cos^2 \Theta - 1)$

$\int_0^1 d\cos \Theta \cdot P_2(\cos \Theta) \cdot P(\Theta, t) = M_2$

$M_0 = P_+(t) + P_-(t)$

$M_2 = \frac{P_+(t)}{10} - \frac{2}{5} P_-(t)$