A PLASM MODEL FOR RF BREAKDOWN IN ACCELERATOR STRUCTURES

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Inspiration:

Jens Knobloch's thesis
(Cornell University, 1997)

Gordon Bowden -- referred me to
"Handbook of Vacuum Arc Science and Technology"
ABSTRACT

A plasma model is presented for the formation of "cathode spots" and subsequent crater development near field emission sites on a copper surface in the presence of a strong dc electric field. Adding to previously published models, we propose that the two-stream plasma instability relates the plasma density to its dimensions. Arguments are presented that suggest that the formation and dynamics of such plasma spots are essentially the same phenomenon for both dc and rf fields. A consequence for accelerating structures is that, when such plasma spots are present, they erode the copper surface at a rate on the order of 100 micrograms per Coulomb of ion bombardment current. This erosion rate can have a measurable effect on the rf properties of an accelerating structure after a high-gradient processing time on the order of 100 hours.
CATHODE SPOTS AND PLASMA FORMATION

A dc cathode spot is defined as "an ensemble of a heated surface area and a dense plasma cloud in front of the surface." Such spots manifest themselves as "small luminous spots that move over the surface. Instead of resting at the location where the surface is already hot, the cathode spots displace themselves chaotically to cold surface areas that must be heated again." (Quotes from "Handbook of Vacuum arc science and technology"). The residence time in one location for a cathode spot is on the order of 10-50 ns.

We propose here that rf cathode spots have essentially the same form and dynamics as their dc counterpart, just as rf and dc field emission are essentially the same physical phenomenon. In both dc and rf fields, such a plasma blob can form very rapidly (within a few rf cycles) near a field emission site, if a source of gas is also present locally. Once plasma formation has started, ion bombardment of the surface stimulates the release of more gas in a regenerative process. See Jens Knobloch’s simulations in “Advanced Thermometry Studies of Superconducting Radio-Frequency Cavities,” Ph.D. Dissertation, Cornell University, 1997.
Figure 6.25: Ion bombardment of the rf surface once a plasma has been established near the emitter. The ions are accelerated by the potential created between the plasma and the rf surface. Applied field = 30 MV/m. Note that the aspect ratio is not 1:1.

Figure 6.12: Position plots of the electrons at the beginning of the third rf period. (a) The entire simulation region — note that the aspect ratio is not 1:1. (b) Magnified view of the $3 \times 3 \, \mu m^2$ region closest to the emitter. Clearly visible is the injection of emission current from the rf surface despite the fact that the applied electric field is zero.
Figure 24. Two subsequent pictures showing a change of a Cu spot. Laser absorption technique as in Fig. 22. Cathode top, anode bottom. Exposure time of the frames is 0.4 ns, interframe time 3 ns, current 80 A.

Figure 25. Probability for the occurrence of discernible spot changes as a function of interframe time $\Delta t$ (ns). Spot changes can occur in a few nanoseconds.
Figure 22. Spot picture obtained by a laser absorption technique. Cu, 20 A, Cathode top, anode bottom. Exposure time: 0.4 ns. The plasma structures have a size ≤ 10 μm.

Note Anode Spot
Figure 1. Crater forms caused by type 1 and 2 spots on molybdenum (prepared by melting in situ). The arc was driven by a magnetic field.\(^{14}\) (a) Clean surface, spot type 2: chain of overlapping craters; (b) surface having a thin oxide film produced by short exposure of the cathode to air, resulting in arc spot type 1: chains of dispersed craters with some transitions to the overlapping type (arrows); (c) detail from (b).
Shallow craters similar to Fig. 10 can often be found in the vicinity of crater chains. Figure 11 gives an example. Probably still more of them are buried below the large erosion structures. Thus, the effect is not due to the special arrangement in Ref. 62, but is of general importance. It might also be that the craters started with a thin and deep hole that subsequently was filled with liquid metal. Schwirzke[63][64] discussed such early crater structures in detail, suggesting that electron return currents essentially contribute to their formation (unipolar arc system). In Ref. 65, such a crater has been published to demonstrate the effect of ion bombardment.

Figure 11. Small craters similar to Fig. 10 near a chain of broad craters on Mo (arrows).

Spot Substructure. With type 2 spots, division occurs as with type 1, but the splitting currents are believed to be much higher. Using optical diagnostic, Djakov and Holmes[37] found spot splitting currents for Cu near 10^4 A. Anders et al. measured spot on slit, got ~ 30 A/slot for Cu.
ION CURRENT DENSITY AT A METAL SURFACE

When a plasma is in contact with a metallic surface, a cathode sheath forms which is essentially a **space-charge-limited diode** which can deliver an intense ion bombardment current to the surface. From the Child-Langmuir Law,

\[
J_{\text{ion}} = 9.3 \times 10^{-14} \frac{V_{s}^{3/2}}{\lambda_{D}^{2}} \text{ A/cm}^2
\]

where \( V_{s} = 20 \text{V} \) and \( \lambda_{D} \) is the Debye shielding length, given by

\[
\lambda_{D} = 7.4 \times 10^{3} (KT_{e} / e)^{1/2} / n^{1/2}
\]

Here \( n \) and \( T_{e} \) are the plasma electron density and temperature. From "Handbook of Vacuum Arc Science and Technology", \( T_{e} \approx 5 \text{ eV} \). Combining the two equations,

\[
J_{\text{ion}} \approx 3 \times 10^{-15} n \quad \text{MKS} \quad (1)
\]
Sheath Equation -

\[ \frac{e V_s}{kT_e} = \ln \left[ \frac{(\alpha J_i/J_e)/(2\pi m_e)^{1/2}}{(J_i/J_e)/(2\pi m_e)^{1/2}} \right] \]

\( \alpha \) = electron to ion mass ratio

\( = 8.6 \times 10^{-6} \) for Cu

For \( V_s \approx 10 \text{ eV}, \ kT_e \approx 5 \text{ eV}, \ J_i/J_e \approx 0.3 \) (spot currents of 20 - 100 A)

will need \( J_i/J_e = J_i/J_e \) later.
Because of the pronounced differences in mass and mobility between electrons and ions, the ionization region becomes positively charged and the electric potential in this region exhibits a local maximum, a potential hump, as indicated in Fig. 4.

As Mitchell and Harris\cite{11} point out, the hemispherical expanding plasma flow from the ionization region toward the anode provides an essentially neutral conducting medium that spans most of the interelectrode gap and permits the passage of electric current with only small voltage drop. It is as if the cathode were a tank of high pressure plasma, with tiny holes—the cathode spots—in its surface, through which the plasma shoots like jets into the vacuum gap. The plasma flow from the ionization zone toward the cathode provides both an intense energy flux and a high space charge field at the cathode surface, and consequently strong emission of both neutral atoms and electrons. The emitted atoms and electrons flow away from the surface, across the acceleration zone, to the ionization zone, where they mix by collisions to feed both energy and particles into the plasma.

![Diagram](image)

*Figure 4. Cathode cell geometry and potential distribution in the Harris model.*\cite{8}
From Fig. 5, we note that in the acceleration zone, a fraction of the current that we will designate \( s \) is carried by electrons. The ion current is therefore \((1 - s)I\). In the expanding plasma, there is also a component \((1 - s)I\) of ion current, but we note that this is flowing in opposition to the main current, thus the electron current in this region is \((2 - s)I\). Neutrality can be preserved by adjustment of ion and electron anode-directed velocities. The current density can be written

\[
\text{Eq. (1)} \quad J = nev
\]

where \( n \) and \( v \) are the concentration and velocity of the electrons or ions as the case may be. To preserve neutrality
PLASMA DENSITY AS LIMITED BY THE TWO-STREAM PLASMA INSTABILITY

The two-stream instability is a convective instability which amplifies disturbances in the electron flow moving through the plasma, much like a traveling -wave tube. For the system to be unstable, its dimension, \( L \), must be moderately larger than the longest wavelength of a disturbance.

Assume

\[
L \geq 10\lambda = 20\pi / k_{\text{Re}} = 20\pi v_o / \omega
\]

where \( k = 2\pi / \lambda \) and \( k_{\text{Re}} = \omega / v_o \) from the dispersion relation. The maximum gain, proportional to \( k_{\text{Im}} \), takes place near \( \omega / \omega_p \approx \alpha^{1/2} \). For instability therefore,

\[
\omega_p L \geq 20\pi v_o / \alpha^{1/2}.
\]

Using \( \omega_p = 56 \ n^{1/2} \) and \( v_o \approx 6 \times 10^4 \text{ m/s} \) (see paper),

\[
\begin{align*}
J_e &= e n u o \\
J_e &\propto 3 J_i^{15} \\
J_i &\propto 3 \times 10^{-15} n
\end{align*}
\]

\[
n^{1/2}L \geq 2.5 \times 10^7.
\]

Assuming \( L \) is of the order of the spot radius, using also Eq. (1), we have the order-of-magnitude estimate

\[
I_m = \pi^2 J_m = \pi \left( \frac{2.5 \times 10^7}{n^{1/2}} \right)^2 (3 \times 10^{-15} n) \approx 5A
\]
Dispersion relation for two-stream instability:

\[ \frac{\alpha}{x^2} + \frac{1}{(x-y)^2} = 1 \quad \left\{ \begin{array}{l}
\lambda = \frac{m e}{M_i} \\
x = \omega/\omega_p \\
y = \hbar \nu_0/\omega_p
\end{array} \right. \]

Solution (solve for \( y \) assuming \( x \) is real)

\[ k = (\omega/\omega_0) \left\{ 1+ j \left[ \lambda - (\omega/\omega_p)^2 \right]^{-1/2} \right\} \]

\[ \omega/\omega_p < \lambda^{1/2} \]

Thank you Norman Kroll & Roger Jones
Figure 53. Schematic section of a typical developed cathode arc spot on clean surfaces (spot type I-2), not true to scale. (1) Solid metal cathode below the spot (arrows indicate current and heat flow); (2) molten metal layer (thickness 0.2–0.5 μm, $2r = 5–10$ μm); (3) space charge layer (thickness 0.005–0.01 μm within the crater area, increasing $r/r_o$ outside); (4) ionization and thermalization layer of the spot plasma (thickness 0.1–0.5 μm); (5) dense central spot plasma; (6) plasma expansion region (plasma jet, arrows indicate plasma flow, current in opposite direction); (7) ejection of molten droplets.
CRATER FORMATION

Heat applied at a metal surface diffuses into the metal a distance $X_D$ in a time $t$, where

$$X_D = 2(Dt)^{1/2}$$

$D$ = thermal diffusivity

$$= 1.1 \times 10^{-4} \text{ m}^2/\text{s} \text{ (copper)}.$$

If the initial plasma spot size is on the order of 1 micron, and the power per unit area at the surface is

$$P_A = V_a I_o / \pi r_s^2 = (20V)(5A) / \pi(1\times10^{-6} \text{ m})^2 = 3 \times 10^{13} \text{ W/m}^2,$$

and if $r_s \gg X_D$ (semi-infinite surface limit), then the time to bring the surface to the melting temperature is

$$t_{melt} = \left(\frac{\pi}{D}\right)\left(\frac{K\Delta T}{2P_A}\right)^2 = 10^{-12} \text{ s}$$

$K$ = thermal conductivity = $380 \text{ W/m}\cdot\text{oC}$

$\Delta T = 1060^\circ$

$X_D = .02 \text{ microns} \ll r_s$

The initial melting time to form a 1 micron crater is very short indeed.

If plasma formation time $\approx 3\text{ ns}$, $X_D \approx 1\mu\text{m}$.
CRATER GROWTH

Assume the plasma has about the same radius as a melting hemispherical crater. Then

\[ 2\pi r^2 \rho (L_h + C_h \Delta T) \, dr = A r^2 \, dr = P(t) \, dt \]

\[ \rho = \text{density} = 9 \, \text{g/cm}^3 \]

\[ L_h = \text{latent heat of fusion} = 212 \, \text{J/g} \]

\[ C_h = \text{specific heat} = 0.42 \, \text{J/g - °C} \]

\[ A = 3.7 \times 10^4 \, \text{J/cm}^3 \]

Assume \( P = V_s I_{\text{ion}} = \text{constant} = (20V)(10A) \). Then

\[ r - r_0 = (3Pt/A)^{1/3} = 0.25 \, t^{1/3} \, \text{cm} \]

For \( t = 1 \, \text{ns} \) \quad \( r - r_0 = 2.5 \, \text{microns} \)

For \( t = 10 \, \text{ns} \) \quad \( r - r_0 = 5 \, \text{microns} \)

For \( t = 50 \, \text{ns} \) \quad \( r - r_0 = 10 \, \text{microns} \)

The latter is consistent with a typical spot residence time and crater diameter.
PLASMA EROSION CAN PRODUCE A MEASURABLE EFFECT ON STRUCTURE RF PARAMETERS

A typical copper erosion rate is 100 μg per Coulomb of electron current. Taking into account the relation between electron current and ion bombardment current, and assuming that about one-half of the eroded material falls back to the surface near the crater, we calculate that material is removed at the rate of about $1 \times 10^{-10}$ m$^3$/s (see paper for details). For the NLC structure, the phase shift per cell is related to the removal of material from the iris tip by (see paper)

$$\Delta \theta \approx 3 \times 10^7 \Delta V,$$

giving a phase shift per cell of

$$\Delta \theta \approx 3 \times 10^3 t_p,$$

where $t_p$ is the time a single plasma spot is present on the iris.

In an NLC structure, a phase change per cell of $6 \times 10^{-4}$ was measured after high power processing (C. Adolphsen). From the preceding relation, a plasma spot present for 2 seconds would produce the measured phase shift. Assume the spot is present some fraction of the time on each pulse during processing, or 150 fs for the NASTA. At a repetition rate of 60 Hz, the processing time needed to produce the observed phase shift is,

$$t_p \approx \frac{60}{f} \text{ hrs} \quad \left\{ \begin{array}{l} 400 \text{ hours measured} \nonumber \\ f \approx 0.15 \end{array} \right.$$
EROSION RATE PER SPOT

Assume ¼ of eroded material (ions, neutral Cu vapor) lands well away from plasma spot. Net erosion rate = 50µg/C

Assume ion current ≈ 20A (Ie x 60A)
Possibly from multiple spots

Erosion rate ≈ 1 x 10^{-3} g/sec
≈ 1 x 10^{-4} cm^3/sec
≈ 1 x 10^{-10} m^3/sec
CALCULATION OF $\Delta \phi / \Delta V$

The phase per cell $\phi$ for the $2\pi/3$ mode for a frequency error $\Delta \omega$ is

$$d\phi = \left(\frac{\pi}{3}\right) dh = \left(\frac{\pi}{3}\right) \frac{\Delta \omega}{\omega} \quad \text{and} \quad \omega \Delta h = \frac{\Delta \omega}{dh}$$

$$\Delta \phi = \frac{2\pi/3}{\nu g/c} \frac{\Delta \omega}{\omega}$$

From perturbation theory, for a volume of metal $\Delta V$ removed in an area of surface electric field $E_s$,

$$\frac{\Delta \omega}{\omega} = \frac{E_s E_a^2}{4U}$$

Here $U$ is the stored energy

$$U = \frac{\pi}{3} S = \frac{(\pi/3) E_a^2}{S} = \frac{(\pi/3) E_s^2}{F^2 S}$$

where $S$ is the surface area per unit length, $E_a$ is the average gradient and $F = E_s / E_a$.

Combining the preceding two expressions gives $\Delta \omega / \omega = 3E_s F^2 S \Delta V / \lambda$. Substituting in the expression for $\Delta \phi$,

$$\Delta \phi = \frac{(\pi/3) E_s S}{\lambda (\nu g/c)} F^2 \Delta V$$

Putting in $S \approx 900 \times 10^{12} \text{V/cm}$, $\nu g/c \approx 0.10$, $F = 2.5$ and $\lambda = 0.026 \text{m}$, we obtain

$$\Delta \phi \approx 3 \times 10^7 \frac{\Delta V}{(m^3)}.$$
Conclusion #1

Cathode spots can exist without rf breakdown (massive arc event with reflected power signal). They are probably present in nearly every pulse when processing at high power close to the breakdown limit. They are possibly a necessary precursor to breakdown. When present, they can erode material at a substantial rate.
The major premise of this paper is that the formation and subsequent dynamics of plasma spots on a copper surface is essentially the same phenomenon for both dc and rf. Once a plasma forms, the metal beneath it is shielded by the plasma and does not "know" whether the external field is dc or rf.

A simple calculation reinforces this conclusion. Rf field lines must terminate on a surface charge layer with density $\varepsilon_0 E$ per unit area. The rf field will peel off, at most, an electron and ion charge $Q = 2\pi r^2 \varepsilon_0 E$ per rf cycle, giving a current

$$I = \frac{dQ}{dt} = \omega r^2 \varepsilon_0 E$$

At 11 GHz, with $E = 2 \times 10^8$ V/m and $r = 10 \mu m$, this charge-removal current is about $10^{-2}$ A, which is negligible compared to the electron and ion current flow through the plasma of many Amperes.

Any scaling law of the breakdown field level as a function of frequency and pulse length must depend on the macroscopic parameters of the rf cavity or accelerating structure: physical dimensions (proportional to wavelength), stored energy and group velocity or filling time. Possible ingredients from the plasma model discussed here are the material erosion rate, spot energy dissipation and residence time. It remains to put all of these ingredients together, along with the macroscopic cavity parameters, into a coherent scaling theory for rf breakdown in accelerating structures.