Future Accelerators

The 2nd Advanced ICFA Beam Dynamics Workshop on Ground Motion in

SLAC, Stanford University, Stanford, CA 94309

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SSC

Theoretical Studies of Ground Motion Effects at the
### SSC parameters

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<th>Particles</th>
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<td>Energy</td>
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<td>Circumference</td>
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<tr>
<td>Revolution frequency</td>
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<tr>
<td>Betatron tune, $x$, $y$</td>
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<tr>
<td>$\epsilon_N$</td>
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<tr>
<td>$N_{quad}$</td>
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Underground Measurements of Seismic Vibration at the SSC Site

V. Shtil'stein, G. Stupakov Colliders

Slow Ground Motion and Operation of Large Experimental Simulation of Ground Motion Effects

N. Syphers, M. Ball et al., Newbeam Sextupole

Measurements of the Ground Motion Vibrations and Beam Instabilities Due to Dipole Ripple and Emittance Growth at the SSC

H. J. Shih, J. Ellison

For Suppression of the Emittance Growth Due to Noise in Large Hadron Colliders

Paper 194, 4th ILC Symposium

Computer Simulation of the Emittance Growth in Large Hadron Colliders

H. J. Shih, J. Ellison

Suppression with the Feedback System in Large Emittance Growth Due to Noise and its Growth Due to Ground Motion from Vibrations

R. R. Ackermann et al., V.A. Parmenovkh

Designing SSC Quadrupole Supports to Minimize Emittance

D. Ritsen

Author(s)
Summary:

- Slow ground motion.
- Beam-beam interaction and random motion at the IP.
- Feedback suppression of the emittance growth.
- Sextupoles.
- Resonant emittance growth due to vibrations of quadrupoles and dipolos.

Does slow ground motion cause beam emittance growth?

Issues
noise spectrum contributes to the emittance growth. This explanation hides the fact that only resonant frequencies in the tune spread translates the betatron oscillations into emittance dilution. Amplitude of the betatron oscillations, Beam dechannelling due to the can expect accumulation of the deflections and (dissipative) Growth of the For random offsets for each sequential pass through the quadrupole, one

$$\frac{b\Delta}{p} = \theta \nabla$$

When a quadrupole is offset by p, the particle is deflected by

Does low frequency motion cause emittance growth?
Resonant excitation of betatron oscillations due to ground motion

Assume that the field in a magnet fluctuates, \( \delta B(t) \).

Correlation function

\[
K(\tau) = \langle \delta B(t) \delta B(t - \tau) \rangle
\]

Spectrum

\[
S_B(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau K(\tau) e^{i\omega t}
\]

For linear betatron oscillations, the beam orbit on \( m \)th passage through the magnet

\[
\langle x_m^2 \rangle = \frac{m}{2} \Omega_0 \Sigma(\nu)
\]

where \( \Omega_0 = 2\pi f_0 \) is the revolution frequency,

\[
\Sigma(\nu) = \beta_0 \left( \frac{e \ell}{P_c} \right)^2 \sum_{n=-\infty}^{\infty} S_B [\Omega_0 (\nu - n)],
\]

(Michelotty and Mills, 1989; Lebedev et al., 1991; Stupakov, 1992)
Only the discrete set of frequencies is sampled, with the minimum frequency \( f_0\{\nu\} \) (if \( \{\nu\} < 0.5 \)). For the SSC, \( \{\nu\} = 0.28 \), \( f_0\{\nu\} = 960 \text{ Hz} \).

Due to the betatron frequency spread in the beam, the decoherence (phase mixing) of the betatron motion translates oscillations into the emittance growth:

\[
\frac{d\epsilon}{dt} = \frac{\beta_0 f_0^2}{4F_q^2} \sum_{n=-\infty}^{\infty} S_d[f_0(\nu - n)]
\]

where \( S_d \) is the spectrum of the quad offsets \( (\delta B = B'd) \), \( \int_0^\infty S_d(f)df = \langle d^2 \rangle \).

\[
\begin{array}{c|c|c|c|c}
 & 960 & 2480 & 4400 & 5920 \\
\hline
0 & f_0 & 2f_0 & & \\
\end{array}
\]

Characteristic time for the decoherence is of order of \( f_0\Delta \nu_{rms} \sim 1 \text{ s}^{-1} \).

Finite width of the resonances: \( \Delta f \sim f_0\Delta \nu_{rms} \).
Require emittance doubling be less then 20 hours, for 
$\epsilon = 10^{-6} \text{ m} \cdot \text{rad}$, $N_{quad} = 800$, $F_q = 60 \text{ m}$,

$$\sum_{n=-\infty}^{\infty} S_d [f_0(\nu - n)] < 5 \cdot 10^{-12} \mu m^2/\text{Hz}$$

For the white noise,

$$\sqrt{\langle d^2 \rangle} < 1 \text{ Å}$$

If $\delta B$ is due to the fluctuations of the dipole magnetic 
field (4200 magnets), assuming uncorrelated 
fluctuations

$$\frac{\sqrt{\langle \delta B^2 \rangle}}{B} < 7 \cdot 10^{-10}$$
Ground motion of the sextupole magnets
(fluctuation of the quadrupole field)

Random perturbation of the sextupole magnetic field results in parametric resonance and causes exponential growth of the emittance. Resonant frequency:

\[ \omega + n\Omega_0 = 2\nu\Omega \]
\[ f_{res} = (2\nu - n)f_0 \Rightarrow 1920 \text{ Hz} \]

\[ \frac{d\epsilon}{dt} = \epsilon \frac{f_0^2}{8} \left( \frac{B''l\beta_0 e}{Pc} \right)^2 \sum_{n=-\infty}^{\infty} S_d(f_0(2\nu - n)) \]

For the SSC, \( B''l = 2.4 \text{ T/m} \), \( N = 400 \), \( \beta_0 = 305 \text{ m} \) and requiring emittance doubling time of 20 h

\[ \sum_{n=-\infty}^{\infty} S_d(f_0(2\nu - n)) < 4 \cdot 10^{-4} \mu m^2/Hz \]
Feedback

To suppress the emittance growth one can use a feedback system. Assume phase advance \( \mu = \frac{\pi}{2} + 2\pi n \).

\[
\begin{align*}
\text{BPM} & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad
\end{align*}
\]

Deflection angle

\[ \alpha = g \frac{x_1}{\sqrt{\beta_1 \beta_2}} \]

Change in momentum

\[ \Delta p_2 = -gp_2 \]

In the limit \( g \ll 1 \), \( x \propto e^{-\lambda t} \) with the damping decrement is

\[ \lambda = \frac{1}{2} g f_0 \]

To overcome the decoherence process, one needs

\[ g \gg \pi \Delta \nu_{rms} \]

\[ \Delta \nu_{rms} \approx 3 \cdot 10^{-4} \]
The rate of emittance growth due to the ground motion with the feedback, in the limit \( g \gg \Delta \nu \)

\[
\frac{d\epsilon}{dt} = \frac{16\pi^2 \Delta \nu^2}{g^2} \left[ \left( \frac{d\epsilon}{dt} \right)_0 + \frac{f_0 g^2}{2\beta_1} X_{BPM}^2 \right]
\]

where \( X_{BPM} \) is the BPM resolution.

Emittance doubling time as a function of \( g \) and \( T_0 \): 1 - \( T_0 = 10 \) h, \( X_{BPM} = 3.5 \mu m \), 2 - \( T_0 = 10 \) h, \( X_{BPM} = 0.5 \mu m \), 3 - \( T_0 = 10 \) min, \( X_{BPM} = 3.5 \mu m \), 4 - \( T_0 = 10 \) min, \( X_{BPM} = 0.5 \mu m \).
Simulations by Shiltsev & Parkhomchuk (SSCL-639): measured noise drives the beam in a computer model.

\[ \Delta \epsilon = \epsilon_N(t) - \epsilon_{N0} \]

20-h On-Line Simulations of Emittance Growth During Noisy Weekday;
(a) Without Feedback System and with 20-h Damping Time;
(b) With Feedback System with \( g = 0.05 \).

In quiet conditions the rate of the emittance growth was found to be a factor of 3 below the acceptable level, 1 mm·mrad/20 h.
A more sophisticated computer model was developed by V. Lebedev (SSCL-Preprint-191).

- Bunch is modeled by macroparticles
- Two dimensions, $x$ and $y$
- Strong-weak beam-beam interaction
- The lattice is modeled by a linear map
The dependence of the emittance growth rate on the beam-Beam tune shift $\xi$ for different values of noise $\delta$.

FIGURE 3

The linear tune shift $\xi$ is equal to $0.0$. The model 1.2.3.4. nonlinear model 2.3.4. first model $A = 0.0$. Number of values of noise $\delta$. 

$\delta = 0.2$. The dashed lines are plotted using Eq. (8) with $\xi = 0.2$. For different values of the noise level $\delta$, the emittance growth rate $\gamma$ is shown.
The dependence of the emittance growth rate on the dimensionless gain g of the feedback system.
The dependence of the emittance growth rate on the tune $\nu$ is plotted using Eq. (8) with $\nu = 0.03 \pm 0.05$. The dashed line is the plot of $y = 0.24x + 0.03$. The fractional part of the tune $\nu_y = 0$ and $\nu_x = 0$. The plot shows the emittance growth rate ($\text{d}E/\text{d}n$) as a function of the tune $\nu$. The plot includes markers at specific tunes: $3/4$, $11/14$, $13/16$, $8/10$, and $5/6$.
Beam-beam interaction with random offset

Assume that with the use of a feedback system the betatron oscillations driven by magnet vibrations are stabilized at some level.

\[ \zeta(t) = x_1(t) - x_2(t) \] is a random function of time, \( S(\omega) \) is the spectrum of \( \zeta \).

Model (Stupakov, SSCL-560)

- One - dimension, \( x, p \)
- Strong - weak interaction
- Small displacements, \( \zeta \ll \sigma_x \)
- Gaussian distribution in \( x \) with \( \sigma_x \), rectangular in \( y \) with width \( d \).
The potential energy of the interaction (in dimensionless units)

\[ V(x, t) = 2T \xi h(x - \zeta(t)) \sum_{m=-\infty}^{\infty} \delta(t - mT) \]

where

\[ \xi = \frac{N_p r_p \beta_*}{\sqrt{2\pi d \sigma_x \gamma}} \quad h(x) = \int_0^x dy \int_0^y dz e^{-z^2} \]

\( J \) is the action variable

\( F(J) \) is the particle distribution function

\[ \frac{\partial \langle F \rangle}{\partial t} = \frac{1}{2} \frac{\partial}{\partial J} D(J) \frac{\partial \langle F \rangle}{\partial J} \]
with the diffusion coefficient

\[
D(J) = \frac{16\pi^3}{T} \xi^2 J e^{-J} \sum_{k=0}^{\infty} \left[ I_k \left( \frac{J}{2} \right) + I_{k+1} \left( \frac{J}{2} \right) \right]^2 \\
\sum_{n=0} \left[ S(\omega_{kn}^1) + S(\omega_{kn}^2) \right]
\]

where

\[
\omega_{kn}^1 = \Omega(n + \{\nu(2k + 1)\}) \\
\omega_{kn}^2 = \Omega(n + 1 - \{\nu(2k + 1)\})
\]

Luminosity degradation

\[
\frac{1}{\mathcal{L}} \frac{d\mathcal{L}}{dt} = -12.5 \frac{\xi^2}{T} \sum_n \left[ S(\omega_{kn}^1) + S(\omega_{kn}^2) \right]
\]
Simulations by J. Koga and T. Tajima (PRL, 72, 1994)

FIG. 1. $D(J)$ from (a) our $\delta f$ code and conventional tracking code, (b) tracking code particles with variations in $\langle x^2 \rangle$ input from the $\delta f$, and (c) tracking code particles with $\langle x^2 \rangle$ input using a band of frequencies around $2(\nu_0 - \Delta \nu)$ for $M = 10^5$ rotations. $D_1$ and $D_2$ have time scales of $\Delta N_1 = 1000$ and $\Delta N_2 = 10\,000$ rotations, respectively. (d) The diffusion $D(J)$ from $\delta f$ and our analytic theory.
Slow ground motion and closed orbits

Slow ground motion, with the frequencies $\ll$ then betatron resonances, causes closed orbit distortions. What is COD for a given correlated quad motion?

Discrete correlation function and spectrum:

$$K(n) = \langle d_i d_{i+n} \rangle$$

$$K(m) = \frac{1}{N} \sum_{m=0}^{N/2} k_m \cos \left( \frac{2\pi nm}{N} \right)$$

In the limit of large $N$, for FODO lattice

$$\langle \Delta x_{COD}^2 \rangle = \frac{\beta N}{64F^2 \sin^2 \pi \nu} \left( k_{[\nu]} \sqrt{\beta_F} + k_{N/2-[\nu]} \sqrt{\beta_D} \right)$$

(Parkhomchuk, Shiltsev & Stupakov, 1994).

For the ATL scaling

$$\langle \Delta x_{COD}^2 \rangle = \frac{\beta ATC(\beta_F + \beta_D)}{8F^2 \sin^2 \pi \nu}$$
### Conclusion

The ability of SSC correctors to correct the orbit will suffice for 1-2 years.

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<th>1 yr</th>
<th>0.5 yr</th>
<th>0.15 yr</th>
<th>28 days</th>
<th>5.7 days</th>
<th>0.74 day</th>
<th>0.74 min</th>
<th>23 min</th>
<th>( \langle \sigma_x \rangle^\wedge )</th>
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<tbody>
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<td>4.7 mm</td>
<td>0.86 mm</td>
<td>177 mm</td>
<td>23 min</td>
<td>( \langle \sigma_x \rangle^\wedge )</td>
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<tr>
<td>10 yr</td>
<td>yr</td>
<td>yr</td>
<td>yr</td>
<td>I day</td>
<td>I h</td>
<td>I min</td>
<td>I min</td>
<td></td>
<td>( \langle \sigma_x \rangle^\wedge )</td>
</tr>
</tbody>
</table>

\[
\frac{\sqrt{10^{-4} \text{mm}^2}}{\text{m} \cdot \text{s}} = A
\]

### Table I: RMS displacement for different time intervals

- **A**
Summary

The theoretical work done at the SSC was an important step in our understanding of the ground motion effects in large hadron colliders. The issues included:

- Emittance growth due to the resonant frequencies in the ground motion for both quadrupole and sextupole magnets
- Using feedback for suppression of the emittance dilution
- Beam - beam interaction with random motion at the IP
- Closed orbit distortion due to random misalignment of quadrupoles

The analytical models developed complement more sophisticated computer simulations. The results obtained can be used for other large machines such as LHC and VLHC.