Methods of ground motion modeling and applications

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Types of ground motion

- Ground motion can be divided into ‘fast’ and ‘slow’
- **Fast** motion \((f > \text{a few Hz})\) cannot be corrected by a pulse-to-pulse feedback operating at the \(F_{\text{rep}}\) of the collider \(\Rightarrow\) results primarily in **beam offsets at the IP**
  - primarily wave-like mechanism of motion
- The beam offset due to **slow motion** can be compensated by feedback and thus slow motion \((f << \text{a few Hz})\) results only in **beam emittance growth**
  - primarily non-wave-like mechanism
  - can be divided into **diffusive** and **systematic** components

The goal is to characterize ground motion and model it so that it can be used to predict a collider performance
Fast ground motion

- Fast ground motion usually presented as **power spectra** $p(\omega)$ of absolute motion. We also need **correlation** information to build a 2D spectrum of ground motion $P(\omega,k)$

- Fundamental behavior of $p(\omega)$ -- $1/\omega^4$ (corresponds to spectrum of a heavy pendulum influenced by random kicks) variations- due to specific distribution of sources on frequency.
Transverse waves, propagating in surrounding media, displace elements (e.g. quadrupoles) of the linear collider.

The relative displacement of elements is smaller, and correlation is better, if the wavelength is longer.

Wavelength is longer if the wave velocity is higher, since \( \lambda = \frac{v}{f} \)

Wave velocity is higher if surrounding media is more strong.

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Correlation measurements and interpretation

In a model of plane wave propagating on surface

correlation \( = \langle \cos(\omega \Delta L/v \cos(\theta)) \rangle \theta = \)

\( = J_0(\omega \Delta L/v) \) where \( v \) - phase velocity

SLAC 1995 measurements [ZDR]

BINP@CERN LEP 1993 measurements [V.J. et al.]

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Correlation measurements and interpretation

- **Phase velocity** found in correlation measurements characterize surrounding media.
- **Increase** of $v$ at lower frequency corresponds to **increase of rigidity** with depth.
- **Shallow** tunnels like HERA, SLAC, TT2A, show $v \sim 400-2000\text{m/s}$
- **Deep** tunnels like LEP show $v \sim 4-6\text{km/s}$

Phase velocity found in SLAC studies [ZDR]. Fit $V(f)=450+1900\exp(-f/2)$, m/s.

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Fast ground motion, summary

- Studies at UNK, CERN, SLAC, ...
- **Correlation** measurements =>
  - fast motion is **wave like**
  - correlation defined by phase velocity \( v(f) \)
  - \( v(f) \) is linked to geology:
    - larger \( v \) at lower \( f \) => more rigid ground at larger depth (e.g. SLAC: \( E \sim 10^9 \text{ Pa}@100\text{m}, 10^{10} \text{ Pa }@1000\text{m})
- **SLAC sector 10**: noise relatively **low**
- **Deep tunnels**: considerably more quiet
- **Concern**: cultural noise
- Low contribution of **cultural noise** at NLC must be ensured by a good site and proper engineering

Power spectra of absolute motion measured in different places, in UNK tunnel [B.B., et al.], at CERN (LEP tunnel, shutdown) [V.J. et al.], at DESY (HERA tunnel, operational condition) [V.S. et al.], in Finland (Hiidenvesi cave) [V.J. et al.] and at SLAC at night [ZDR].
Slow motion (minutes - years)

- **No beam offset** -- corrected by a pulse to pulse feedback
- **Emittance dilution** -- trajectory deviates from an ideal line
- **Diffusive or ATL motion**: $\Delta X^2 \sim \text{ATL}$ (minutes-month)
- **ZDR value**: $A = 5 \cdot 10^{-7} \, \mu m^2/(m \cdot s)$ => linac BB alignment every $\sim 30 \text{ min}$
- Observed ‘A’ varies by $\sim 5$ orders: $10^{-9}$ to $10^{-4} \, \mu m^2/(m \cdot s)$
  - in some cases due to inappropriate interpretation of year-to-year motion as diffusive rather than systematic
  - parameter ‘A’ should strongly depend on geology -- reason for the large range
  - ‘A’ reported to depend on tunnel construction method: blasting/TBM
- **Systematic motion** [R. Pitthan]: $\sim$linear in time (month-years)
- **ATTL law for systematic**:
  - SLAC 17 years motion suggests $\Delta X^2 = A_s T^2 L$ with $A_s \sim 4 \cdot 10^{-12} \, \mu m^2/(m \cdot s^2)$ for early SLAC
  - LEP 10 years of observation suggests $A_s \sim 0.6 \cdot 10^{-12} \, \mu m^2/(m \cdot s^2)$

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Very slow (year-to-year) motion

- **Year-to-year** motion observed in tunnels appears to be **systematic** (~linear in **time** or \(\Delta X^2 \sim T^2\)).
  - SLAC 17 year observation, LEP, etc.
- **Spatial** spectrum behave as \(1/k^2\) or \(\Delta X^2 \sim L\)

- RMS misalignment due to **systematic** motion: \(\Delta X^2 = A_S T^2 L\)  
  - "ATTL law"

- Early SLAC: \(A_S \sim 4 \cdot 10^{-12} \, \mu m^2/(m \cdot s^2)\)
- Diffusion: \(A \sim 5 \cdot 10^{-7} \, \mu m^2/(m \cdot s)\)
- ATL to systematic transition: \(T_{trans} \sim A/A_S \sim 10^5 \, s\)
Effect of fast motion on the beam

- **Produce beam offset at the IP**
  
  Rough scale of jitter tolerances:

- **Final quads**: tolerance ~1/3 IP
  beam size ~ 1 nm
  - rely on active methods

- **Some quads in beam delivery**: tolerate ~ a few beam size ~ 5nm
  - rely on natural quietness, maybe some active method

- **Linac quads** (many of them!):
  tolerate ~ 10nm
  - rely on natural quietness and good girders. Active(???)

- **Beam-based feedback acts** below ~F_{rep}/20 ~6Hz

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How ground motion influence on the beam

How to find trajectory offset or chromatic dilution?

Relative beam offset at exit and dispersion:

\[ x^*(t) = \sum_{i=1}^{N} c_i x_i(t) - x_{\text{fin}} \quad \eta(t) = \sum_{i=1}^{N} d_i x_i(t) \]

Linear model: \( c_i = \frac{dx^*}{dx_i} \approx -K_i t_{12}^i \quad d_i = \frac{d\eta}{dx_i} \approx K_i (t_{12}^i - t_{126}^i) \) } Approximate values are for thin lens, linear order

Then, for example, the \textbf{rms beam dispersion}:

\[ <\eta^2(t)> = \int_{-\infty}^{\infty} P(t,k) G_\eta(k) \frac{dk}{2\pi} \]

and \( G_\eta(k) = \left( \sum_{i=1}^{N} d_i (\cos(k s_i) - 1) \right)^2 + \left( \sum_{i=1}^{N} d_i \sin(k s_i) \right)^2 \)

- spectral response function

Sum rules. E.g. \( \sum_i d_i s_i = -T_{126} \) at small k then \( G_{\text{offset}}(k) = k^2 R_{12}^2 \quad G_\eta(k) \approx k^2 T_{126}^2 \) unless \( R_{12} \) or \( T_{126} = 0 \)

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“SLAC 2AM” model includes:
- Fast wave-like motion
- Diffusive ATL motion
- Systematic motion

Modeling $p(\omega)$ and $p(\omega,L)/2$ spectra for “SLAC 2AM” model

Can model ground motion:
- In frequency domain - spectra $P(\omega,k)$
- In time domain - harmonics composition

Problems:
- cultural noise
- unergodicity

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An example of the modeling power spectra $P(\omega, k)$. Waves + corrected ATL.

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Waves in $P(\omega, k)$

Fast motion is represented by waves in the modeling $P(\omega, k)$.

Elastic, transverse, propagating at the surface with uniform distribution over azimuthal angle.

Wave with frequency $\omega$ and phase velocity $v$ give $k_{\text{seen}} = \omega/v \sin(\theta)$ (seen by linac). Uniform distribution on $\theta$ gives distribution on $k$:

$$U_i(\omega, k) = \frac{2}{\sqrt{(\omega/v)^2 - k^2}} \text{ if } |k| \leq \omega/v \text{ and zero otherwise.}$$

Wave contribution to the 2-D spectrum $P(\omega, k) = \sum_i D_i(\omega) U_i(\omega, k)$

here $D(\omega)$ – contribution of waves to absolute spectrum $p(\omega)$,

e.g. $D_i(\omega) = a_i / (1 + [d_i(\omega - \omega_i)/\omega_i]^4)$. 

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Slow “ATL” motion needs to be corrected to be included into $P(\omega, k)$.

Pure ATL overestimates fast motion:
the spectrum of relative motion $\rho_{ATL}(\omega, L) = AL/\omega^2$ at some frequencies can be $>\;$ than spectrum of absolute motion $2p(\omega) = B/\omega^4$, that is impossible.

Let’s correct the ATL by taking the upper approximation:
$\rho_{corr.ATL}(\omega, L) = \min(AL/\omega^2, B/\omega^4)$

Corresponded contribution to 2-D spectrum is $P(\omega, k) = A^2/\omega^2 k^2 (1 - \cos(\frac{kB}{A\omega^2}))$

$\langle \Delta X^2 \rangle = ATL/(1 + T_0/T)$ for the corrected ATL, where $T_0 \approx \sqrt{AL/B}$.

It still rather overestimate the effect. Needs measured data on transition region.  

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Ground motion and Final Focus

- Final focus tolerances are most severe.

- **Fast ground motion** induces vibrations of optical elements resulting in the beam offset at the IP.
  - The inter- bunch-train feedback will keep the rms IP beam offset constant (and hopefully small).

- **Slow ground motion** induces misalignments of optical elements resulting in slow growth of the IP beam size.
Fast motion & Final Focus

- Can evaluate analytically:
  - rms IP offset, IP size, with/without feedback
- Simulations: other cases
- Example below:

Rms IP offset for New NLC FF for different ground motion models. Feedback with $f_0=6\text{Hz}$, $S_{FD}=8\text{m}$ (i.e. 16m between FD quad supports). Solid lines - no FD stabilization, dashed - relative motion of FD is excluded by active system. Only vertical, no tilt.

Acceptable without any active stabilization. Underestimate effect since detector adds noise and FD supports are not ideal.

(Not intended to evaluate any of these sites)
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Simulation of ground motion

ATL motion is too easy to simulate.

How to simulate ground motion given by an arbitrary $P(\omega, k)$?

Ground motion displacement $x(t, s)$ is modelized by summation of harmonics.

$[T_{\text{min}}, T_{\text{max}}], [L_{\text{min}}, L_{\text{max}}]$ are given. Analyze the modeling $P(\omega, k)$ to find the band of relevant $\omega$ and $k$. Split by cells ($\sim 50 \times 50$) equidistantly in logarithmic sense. Find rms amplitude $a_{ij}$. Seed two sets of random phases $\phi_{ij}$ and $\psi_{ij}$ and choose $\omega_i$ and $k_j$ randomly within cell.

The modeling displacement $x(t, s)$ for this seed $x(t, s) =$

$$\sum_i \sum_j a_{ij} \left[ \sin(\omega_i t) \sin(k_j s + \phi_{ij}) + (\cos(\omega_i t) - 1) \sin(k_j s + \psi_{ij}) \right] + \Lambda(t) \sum_i b_i \sin(k_i s + \gamma_i) \quad \text{Systematic motion} \quad \text{A.Seryi}$$
Simulation of ground motion

Simulation of $x(t,s)$ for a given $P(\omega,k)$ of a ground motion model

Modeling power spectra $P(t,k)$ for some given ground motion model. **Analytical results** (smooth lines) and spectra obtained in **simulations** using the harmonics model.

To obtain these simulated spectra we used many **different seeds** and **averaged** their spectra.

*Different seeds = different realizations of the reality*

Spectra obtained from **ONE** seed, even with averaged (over time) will have **sharp peaks** corresponded to **individual harmonics**.

$\Rightarrow$ **problems for simulations**

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Improving ground motion modeling

- **Cultural noise** can sharply depend on location, but $P(\omega,k)$ spectrum cannot. Use $P(\omega,k)$ and explicitly position dependent part of spectrum $p(\omega,s)$. For correlation information need mutual spectra $p_{12}(\omega,s_1,s_2)$.

- **Damping/amplification** (by girder or active system) can be taken into account:

  ![Diagram](image)

  Example, if girders amplify by $R_k(\omega) = r_k(\omega)e^{i\varphi_k(\omega)}$ then

  $$<\Delta x^2> = \iint P(\omega,k)2[1 - \cos(\omega T)]2[1 - \cos(kL)]\frac{d\omega}{2\pi}\frac{dk}{2\pi}$$

  $$[r_1^2 + r_2^2 - 2r_1r_2\cos(kL + \varphi_1 - \varphi_2)]$$

- **Time-domain** ground motion modeling using harmonic composition: unergodicity problem (average on seeds not equal average on time)

  Can be cured by wavelet composition of GM (in time and space)

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Improving ground motion modeling

- **Time-domain** ground motion modeling using harmonic composition: **unergodicity** problem: average on seeds (ensembles) not equal to average on time
- Can be cured by wavelet composition of GM (in time and space)

Instead of sin-like harmonics (that have forever defined frequency, phase and amplitude)

Use sequence of “wavelets” with slightly different frequency, phase, amplitude and shape

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