Experimental observations based on the Self-Modulated Laser Wakefield Acceleration (SMLWFA) have shown that a substantial number of electrons, nano Coulombs to be precise, are accelerated beyond relativistic energies (up into the neighborhood of 10 MeV) with a surprisingly small normalized transverse emittance of ~a few $10^{-7}$ m rad. The longitudinal emittance is also small and poses an experimental and theoretical problem and opportunity. This talk describes results of an extension of the (transverse) K-V formalism to include a coupled longitudinal phase space.
This talk is based on the recent work by

Alex Chao, Rainer Pitthan, Toshi Tajima and Dian Yeremian

with the title:

“Space Charge Dynamics of Bright Electron Beams”,

published in SLAC-PUB-9189,

and on the experimental observations by many groups, mentioned in the above abstract.

Our aim was to explore the suitability of plasma produced electrons for injection into an RF linac, while keeping the emittance below currently achieved values. To do so, we had to make a determination of the emittance. We realized that the longitudinal and the transverse phase space was to be coupled, enabling a better understanding of recent observation of low divergence electron beams emanating from plasma.
What Y’all Know (1)
(Not carrying coals to Newcastle)

- In recent years short and bright bunches of electrons have been accelerated to relativistic energies in the MeV range in plasma, and driven out of the plasma, by the effects of intense short laser pulses.

- General features of these laser driven beams are the very short bunch length (100’s of femto seconds) and the large energy spread of the electron beam.

- The short bunch length is due to the short pulse length of the laser and, therefore, within reason a variable parameter of the experiment.

- The capture of electrons from the plasma bulk through the plasma wave proceeds through an instability and makes the energy spread substantial.
• There are two remarkable properties of the ejected relativistic electrons:
  – the product of the bunch length and the energy spread, the longitudinal
    emittance, is comparable to conventional RF sources (in the range of MeV-ps)
  – the micron-size transverse spot size of the initial electron bunch corresponds to
    the laser spot size and may, therefore, lead to a small transverse emittance.

• Unfortunately, “beam” properties of the electrons are not well defined experimentally
  (but work is going on on this).

• In one experiment at $2 \times 10^{11}$ electrons were originally produced and space collimated
  to $5 \times 10^8$ $e^-$ in an 5 millirad cone. In a nuclear physics spectrometer the energy
  spectrum was measured and gave energy of $7\pm3$ MeV with an “apparent”
  normalized transverse emittance possibly as low as a few $10^{-7}$ m rad after $2\sim m$
  of drift.
Question is: is the core of the jet more energetic? Then radial collimation may help energy spread.

1 cm diameter collimator in 2 m distance from the gas jet (plasma) defines a cone of 5 mrad FWHM ($\sigma' = 2.5$ mrad). The momentum in this cone is $7\pm 3$ MeV (as shown on the next slide).

QQQDD Beamline

P. Gueye, C. Keppel, R. Ent, K. Assamagan, R. Green, J. Taylor and W. Buck

Hampton U/JLAB
Correlation Free Emittance Assumed

From the 5 μm radius plasma channel size and the divergence, one can calculate γε. This calculation is correct if there are no non-linear space charge effects.

Comparison between two UMichigan experiments give low apparent envelope emittances ε12 for both. Both used 4-5TW lasers with 400 fs (120 μm) pulse length on a gas jet. The main difference is the centroid collimation used in the second experiment.

<table>
<thead>
<tr>
<th>Meas./Coll. divergence /mrad</th>
<th>10</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nₑ</td>
<td>10¹⁰</td>
<td>5 10⁸</td>
</tr>
<tr>
<td>Avg Energy/MeV</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>Energy spread</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Laser/Plasma spot size/μm</td>
<td>9/5</td>
<td>14/8.5</td>
</tr>
<tr>
<td>γε₁₂/ 10⁻⁶ π m rad</td>
<td>0.06</td>
<td>0.3</td>
</tr>
</tbody>
</table>
The topic of the emittance is a subtle one, so we will define quantities which play a role. These quantities are a product of a spot size and a divergence, and sometimes loosely are called emittance, although maybe they shouldn’t, because they aren’t.

In the following we will denote the location of the plasma exit with subscript 0 (zero) and the place of measurement with subscript 1 (one).

So we define a quantity \( \varepsilon_{01} \) which is derived from the product of the initial spot size at the plasma channel exit (location 0) and the divergence of the beam measured after a certain drift (20~cm and more, location 1), as in

\[
\varepsilon_{01} = \sigma_0 \times \sigma_1'
\]

This quantity is often called emittance in the plasma literature (although is is not an emittance) and we call it “apparent” emittance (which maybe a conceptual mistake).
What Y’all Should Know 2

- But, this quantity $\varepsilon_{01}$ will be an upper limit on the original plasma exit emittance $\varepsilon_0$. Because, while we know the size $\sigma_0$ of the plasma channel quite precisely, the divergence $\sigma'_{1}$ at a distance has been subject to space charge forces during the drift.

- We define $\varepsilon_0$ to be the actual emittance at the exit point.

- The quantity

$$\varepsilon_{11} = \sigma_1 \times \sigma'_{1}$$

at the place of measurement 1 is an upper limit on the actual emittance at location 1 (and this upper limit might be huge, although the emittance itself at location 1 might be small), because correlation between particle displacement and divergence in phase space is ignored.
Now What? How to Process and Transport?

We put our money on RF Adiabatic Acceleration:

**Conventional:** built special X-band standing wave section with \( G = 200 \text{ MeV/m} \). In lieu of continuous focusing.
\[ \Delta E/E (@2\text{MeV}) = 50\% \Rightarrow \Delta E/E(@200\text{MeV}) = 0.5\%. \]

**Non-Conventional:** use Plasma Acceleration with \( G = 150 \text{ GeV/m} \). Only need a few mm (as shown by RAL). Can be integrated into the plasma gun, so no drift space. Further advantages: the charge is neutralized in the plasma channel during acceleration.

The question then is: what happens with the emittance, before you can RF-capture it?
A Puzzle (in Emittance?)

The observation of tightly collimated electron beams coming from micron size spots has been a puzzle as to their very existence (and their possible emittance).

Classical transverse K-V theory fails to explain it, because in K-V the beams should not be that collimated.

Moreover, since the beams have a very large energy spread, classical methods of determining the emittance with measurements are not possible.

Fortunately, the coupled theory presented here shows a dependence of the asymptotic divergence on the intrinsic emittance at the plasma exit, giving an upper limit on the possible emittance.
What Was Observed in Plasma Physics?

Michigan:
Phys. Plasmas 6 (1999) 4739

Fact:

High Power Lasers, on interacting with matter, produce narrow beams of relativistic electrons, but with large energy spread (~100%)
Most general: emittance is a measure of the parallelism of a beam. And: not necessarily and not everywhere the product of spot size and divergence, in particular if these two are measured at different locations.

More practical: phase space area at a waist or after a pinhole: $\varepsilon \approx \sigma_x \sigma'_x$

Normalized emittance $\varepsilon_N = \beta \gamma \varepsilon$ is a useful quantity:

- Conserved in a linear system
- Adiabatic invariant under acceleration
- Gives geometric beam size and divergence:
  $$\sigma_x = \sqrt{\varepsilon_N \beta_x / \gamma}; \quad \sigma'_x = \sqrt{\varepsilon_N / \gamma \beta_x}$$
Why Do We Care: Needs of Future Accelerators

Linac based Coherent Light Sources need **small emittances** (but not small spot sizes) for SASE to produce high brilliance:

- Future light sources in the sub-Angstrom regime need $\sim 10^{-7}$ m rad
- LCLS/TESSA for $\sim 1-2$ Angstrom wavelength need $\sim 10^{-6}$ m rad

Linear Colliders need small emittances, high polarization (>80%), and small spot sizes, for high luminosity:

- SLC was 400 nm vertical
- FFTB was $\sim 60$ nm vertical
- 1 TeV linear colliders need $\sim 5$ nm
- 3 TeV linear colliders need $\sim 1$ nm

Additional requirement for colliders: polarization (80% or more), flat beams.
## Emittances Already Achieved, and Required

**Emittances from existing “Injectors”, with currents of typically 1 nCoul, are:**

<table>
<thead>
<tr>
<th>Injectors</th>
<th>Horizontal $\varepsilon_N (\pi \text{ m rad})$</th>
<th>Vertical $\varepsilon_N (\pi \text{ m rad})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF Photocathode Gun</td>
<td>$\sim 1 \times 10^{-6}$</td>
<td>$\sim 1 \times 10^{-6}$</td>
</tr>
<tr>
<td>SLC Damping Ring</td>
<td>$3 \times 10^{-5}$</td>
<td>$3 \times 10^{-6}$</td>
</tr>
<tr>
<td>ATF (KEK) Damping Ring</td>
<td>$5 \times 10^{-6}$</td>
<td>$5 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

**Future linear colliders need to achieve the requirements of the ATF damping rings, $3 \times 10^{-6}$ and $3 \times 10^{-8}$ in the horizontal and vertical, respectively.**

**Linear collider damping ring emittance is not good enough for Coherent Light Sources. Bunches are too long (several mm) and horizontal emittance is much too large.**
Enemies of Small Emittance

• Large Phase Space \((x \times x')\) at Origin
• Space Charge Effects
• Transport
• Intra Beam Scattering
• Quantum Excitation
• Transverse Wake Fields
• Oide Effect

The first two impact mostly at the beginning of the emittance chain for Plasma Based Beam Sources:
• Small phase space of a plasma injector determined by the small transverse size at origin: good
• Large space charge because of the small transverse size: bad

Balancing act: small emittances are difficult to make and keep!
General Transverse Emittance definition:

\[ \varepsilon_{\perp}^2 = \langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2 \]

\( X \) is determined from the plasma channel size \( x_1 \) at Position 1. Nothing is known about \( x'_1 \), it could be as small as indicated in the small ellipse.

\( X' \) was taken from position 2, from the projection onto the \( x' \)-axis, \( x'_2 \).

If ellipsoid area is not enlarged through space charge effects, or only through linear forces, then there are no correlations \( \langle x x' \rangle^2 \).

In this case the emittance calculation as \( x x' \) would be acceptable, but is really only an “apparent” emittance, composed of quantities measured at different locations \( \varepsilon_{12} = x_1 x'_2 \).
Envelope Equation Describes Transverse Emittance Only

\[ r'' + k^2 r + \gamma \frac{r''}{\gamma^2} - 2 \frac{I}{(\gamma^3 l_A r)} - \frac{\varepsilon^2}{(\gamma r^3)} = 0 \] (if uncoupled)

- Solen. field
- Adiabatic acceleration
- Charge spreading term
- Emittance term

\( \varepsilon \) is sometimes called the edge (or envelope) emittance. Pay attention to the different powers of \( \gamma \) and \( r \). They lead to the subtle dance of emittance preservation in RF guns.

The emittance term dominates in classical electron transport near a sharp focus, or after the beam goes through a pinhole. But the whole equation probably does not apply to the non-classical situation here.

In the following we use \( \sigma \) instead of \( r \) to denote the transverse coordinate.
Another important equation (adapted from Fubiani, Esarey and Leemans, AAC2000, p423) describes the bunch lengthening (and, therefore, line charge or current reduction) \( \Delta l \) over a drift distance \( D \), if the bunch has a momentum spread \( \Delta p/p \):

\[
\Delta l = \frac{D}{\gamma^2} \cdot \frac{\Delta p}{p}
\]

Example: after 50 mm drift a bunch with a 50% momentum spread will have a lengthening (and a corresponding line charge reduction) of

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( \Delta l/\text{mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6.25</td>
</tr>
<tr>
<td>4</td>
<td>1.56</td>
</tr>
<tr>
<td>8</td>
<td>0.39</td>
</tr>
<tr>
<td>16</td>
<td>0.10</td>
</tr>
</tbody>
</table>
In the longitudinal case derivation with a parabolic Ansatz (analog the K-V distribution) leads to the following differential equations. The solutions are self-consistent.

\[ \langle \delta^2 \rangle' = 2\kappa^2 <z\delta> \]

with \( \Delta P = mc\delta \) the momentum deviation, or \( \delta = \Delta(\beta\gamma) \).

\[ <z\delta>' = <\delta^2> / \beta^2\gamma^3 + \kappa^2 <z^2> \]

\[ <z^2>' = 2 <z\delta> / \beta^2\gamma^3 \]

With \( \kappa^2 = 2N r_0 / [5^{3/2} \beta^2 \gamma^2 <z^2>^{3/2}][\ln \left\{ (\gamma^2 <z^2> + 4\sigma^2)^{1/2}/2\sigma \right\} + 1/2] \)

Coupling of the longitudinal dynamics to the transverse dynamics is described by the (relatively) weak dependence of \( \sigma \) in the logarithm of \( \kappa^2 \).

The time derivative of the longitudinal emittance, \( [\langle \delta^2 \rangle <z^2> - <z\delta>^2]' \), is zero, which means it is a constant of the motion and determined by the initial condition of the beam.
In the transverse case, derivation with the parabolic Ansatz of the K-V (Kapchinskij-Vladimirskij) distribution also leads to a differential equation. The solutions are also self-consistent.

In the absence of acceleration and focusing fields we get

\[ \sigma'' - \varepsilon_{N,\text{RMS}}^2/\beta^2 \gamma^2 \sigma^3 = \zeta/8\sigma \]

where the coupling to the longitudinal is contained in the parameter \( \zeta \)

\[ \zeta = 3 N r_0 / \sqrt{5} \beta^2 \gamma^3 <z^2>^{1/2} \]

with \( \zeta \) a modified line charge density evaluated at the bunch center.

Longitudinal and transverse emittances are constant of the motion, even with coupling. There are no intrinsic emittance growths in the model. These would have to come from the motion kinematics or the non-linearities in the space charge force.

The 4 differential equations derived need to be solved simultaneously.
Example: Coupled and Uncoupled Case

- **Coupled case**: thick line
- **Uncoupled case**: thin line

- **20 cm** drift, as in a typical Laser-Plasma experiment

- **Charge** (assumed) $10^{10}$ e⁻, original laser pulse length (FWHM) 400 fs, equal to length of 54 μm rms

- Energy spread and bunch length are substantially different between coupled and uncoupled case
Problems in Emittance Preservation

Good news: PARMELA simulations are in good agreement with the experimentally derived beam sizes and divergences,

Main cause of emittance increase for plasma based electron injector is the energy spread, even if everything else is perfect.

Bad news: in PARMELA simulations the emittance increases even after the divergence saturates. Transverse dynamics depends on $1/\gamma^3$, so low energy particles rotate faster in phase space $x'-x$, leading to the so-called bow-tie effect.

Consequence: need to capture and accelerate beam early.
Look at coupled calculations only for the experiments just discussed. Take the \((7\pm3)\) MeV case.

Initial values:

- Plasma spot size \(\sigma_0 = 8.5\, \mu m\)
- Emittance \(\varepsilon_0 = 0.01\, mm\, mrad\)
- Divergence \(\sigma'_0 = 0\, mrad\)
Similar Parameters With Acceleration

**Graphs:*

1. *Energy Spread* $\sqrt{\langle \Delta \gamma \rangle^2}$
2. *Cross Term* $\langle z \Delta \gamma \rangle / \mu m$
3. *Bunch Length* $\langle z^2 \rangle / \mu m$
4. *Size* $\sigma / \mu m$
5. *Divergence* $\sigma' / \text{radian}$
6. *Divergence* $\sigma' / \text{radian}$

---

Rainer Pitthan

10th Advanced Accelerator Concepts Workshop, 2002
The flat dependence gives an upper limit on the emittance $\varepsilon_0$, $<\approx 10^{-8} \pi$ m rad. That is to say, the emittance could be smaller, but we can not determine it from the data.

Since the $5 \times 10^8$ case is due to collimation, we conclude that the original beam of $2.6 \times 10^{11}$ did undergo more rapid expansion. A bunch length of $250 \mu$m would yield a $\sigma'$ consistent with the collimator parameters.
Increase control of plasma (and electron) parameters by using 3 lasers: a high intensity pump laser with amplitude \( a_0 \), and two counter propagating lower intensity lasers \( a_1 \) and \( a_2 \) which create a beat wave.

The beat wave corresponds to a physical RF structure designed to create ultra short bunch.

Timescale of 1 fs determined by pre-bunching of electrons in the plasma wave buckets. Number of electrons can be augmented by having many buckets.

Similar parameters for LILAC proposal.

Simulations from LBL give typically (there is wide variation in the parameter space):

- Power (pump) \( \approx 15 \text{TW} \)
- \( E \approx 40 \text{MeV} \)
- \( \Delta E/E \approx 0.2\% \) (=80KeV!)
- \( \tau \approx 1 \text{fs} \)
- \( N > 1.5 \times 10^7 \)
-\( \gamma E_\perp < 1 \text{ mm mrad} \)
-\( \Sigma_\| \approx \sim 0.1 \times 10^{-9} \text{ eV sec} \)

C. Schroeder et al., PRE 59(2000)6037
Plasma spot size: $\sigma_0 = 5 \, \mu m$ (not self-consistent with Berkeley simulation, larger spot size needed with higher charge)

- Energy 40 MeV
- Energy spread 80 keV
- Emittance: $\varepsilon_0 = 0.01 \, mm \, mrad$
- Bunch length 1 fsec
- Divergence $\sigma'_0 = 0 \, mrad$ (waist at exit)

- Problem: Invariant emittance is large (0.8 mm mrad) because of the high energy, even so the energy spread is small.
Plasma spot size: $\sigma_0 = 2, 5, 15 \, \mu m$
(now self-consistent with Berkeley simulation, larger spot size needed with higher charge)

Varying the laser spot size has little effect on the longitudinal, but reverses the order in growth of the transverse dimension.

There might be surprises in exploring this because even $\varepsilon_{01}$ after 5 mm is smaller than $10^6 \, \pi \, \mu m \, rad$. 
2 Laser Experiment: Pre-formed Plasma

- 100 TW 500 fs Laser light reflected from a pre-formed plasma produces jet of electrons in the direction of the reflected light.

- Electrons self-generate magnetic field (2-3 $10^6$ Tesla!). Explains collimation of the electrons (there is also some self-self-focusing).

- Electron motion from the Laser E-field in the magnetic field produces 10-30 keV X-rays.

- X-ray pinhole camera maps the jet: divergence is constant. No space charge effects noticeable.

**Constant divergence:**

**What does it mean?**

Important to remember: the emittance of an RF gun is spot size dominated, the emittance of a Plasma Gun is divergence dominated.

The divergence allowed for the beam from the Plasma Gun is 10 mrad but only 10-100 µrad for a RF gun. The plasma gun is, for equal emittance, a factor of 100-1000 more robust against Space Charge kicks.

The neutralized electron bunch does not experience a space charge force ($F_c$) in the plasma channel.

The experimental evidence from the 2-Laser experiment does not show any obvious space charge effect (linear curve). However, K-V based calculation show the bunch radius should grow rapidly – but it doesn’t.
Do Coupled Calculations Do Better Than K-V?

- Well, yes, some, but....
- The 2 curves are for $4 \times 10^{10}$ and $8 \times 10^{10}$ e-, respectively. They are higher than the experiment, lower than K-V.
- Phase space coupled simulations show the importance of initial energy, which is only 600keV (average) here. If it would be 1.5 MeV, the simulation would describe the experiment quite well. But it isn’t 1.5 MeV!!!
- There must be other physics be buried in it.
- Simulation by the authors of the experiment shows plasma acceleration and focusing outside of the primary plasma as a possibility.
Summary

- Due to the availability of high intensity lasers with short pulse length, plasma acceleration has made great progress. Using such an “accelerator” as an injector for a coherent light source Linac seems attractive. The electrons come in short bunches, making the use of less charge than ~1 nC feasible for some applications.

- Progress has been made in understanding the dynamics of short bunches emanating from μ-size “cathodes” and in overcoming the gut reaction: this can’t work!

- FEL: It will take much work to produce an “industrial” grade reliable injector, but the potential pay-off, both in cost savings for an X-FEL Wiggler, and in enhancement of the physics, are large. On a sociological plane, shorter wigglers would allow to construct more beam lines, thus creating a true “Synchrotron User Facility”.

- LC: Low emittance beams by a Plasma Gun do reduce damping ring requirements for electrons. Short beams (femto seconds and below) for e⁻e⁻ collisions reduce the beamstrahlung and coherent pair production background at high energy (Derbenev). If these beams can be polarized, they could make e⁻e⁻ competitive with e⁺e⁻.
Acknowledgements

Many people have contributed to the ideas, insights and the directions to go expressed here.

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