Table 1. Parameters of the structure

**Solenoid**
Max field                  1.1384 Tesla  
Field length              15 cm  

**Bending magnets**
Field, B                                    0.15 Tesla  
Bending radius, R                   12.5 cm  
Pole angle, α                   0  
Gap                                          5 cm  
Fringe field coefficients           K$_1$ = 0.7  
                                      K$_2$ = 4.4  
Vertical aperture                       5 cm  
Horizontal aperture                  10 cm  

**Aperture** at the exit of transport system      5 x 5 cm$^2$  
**Drift** between bending magnets, L     20 cm  

![Fig.1 . Layout of spectrometer (A.Michailichenko, 01/05/2004)
1. Equation of motion

\[
\begin{align*}
\frac{dx}{dt} &= \frac{p_x}{m \gamma}, \\
\frac{dy}{dt} &= \frac{p_y}{m \gamma}, \\
\frac{dz}{dt} &= \frac{p_z}{m \gamma (1 + \frac{x}{R})}, \\
\gamma &= \sqrt{1 + \frac{p_x^2 + p_y^2 + p_z^2}{m^2 c^2}}, \\
\frac{dp_x}{dt} &= \frac{p_x^2}{m \gamma (R + x)} + qE_x + \frac{q}{m \gamma} (p_y B_z - p_z B_y), \\
\frac{dp_y}{dt} &= qE_y + \frac{q}{m \gamma} (p_z B_x - p_x B_z), \\
\frac{dp_z}{dt} &= - \frac{p_x p_z}{m \gamma (R + x)} + qE_z + \frac{q}{m \gamma} (p_x B_y - p_y B_x). 
\end{align*}
\]

(1.1)

2. Calculation of magnetic field

**Bending magnets**

The magnetic field inside a bending magnet is described by the Taylor expansion up to the terms of second order:

\[
\begin{align*}
B_x(x, y, z) &= B_y (- n \frac{y}{R} + 2 \xi \frac{xy}{R^2}), \\
B_y(x, y, z) &= B_y [1 - n \frac{x}{R} + \frac{n}{2} \frac{y^2}{R^2} + \xi \frac{(x^2 - y^2)}{R^2}], 
\end{align*}
\]

(2.1)

(2.2)

where \(B_y\) is the vertical component of magnetic field along the reference trajectory with radius of curvature \(R\), \(n\) is the field index and \(\xi\) is a nonlinear coefficient in the magnetic field expansion:

\[
\begin{align*}
n &= - \left[ \frac{R}{B_y} \frac{\partial B_y}{\partial x} \right]_{x=0, y=0}, \\
\xi &= \left[ \frac{R^2}{2! B_y} \frac{\partial^2 B_y}{\partial x^2} \right]_{x=0, y=0}.
\end{align*}
\]

(2.3)

At the entrance and at the exit of the magnet, the slope of the particle trajectory is changed because of the pole angle \(\alpha\) according to the linear matrix transformation (TRANSPORT code):
\[
\begin{pmatrix}
  x \\
  x' \\
  y \\
  y'
\end{pmatrix} = \begin{pmatrix}
  1 & 0 & 0 & 0 \\
  \frac{\tan \alpha}{R} & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & -\frac{\tan(\alpha - \psi)}{R} & 1
\end{pmatrix} \begin{pmatrix}
  x_o \\
  x'_o \\
  y_o \\
  y'_o
\end{pmatrix}.
\]

(2.4)

The correction angle \( \psi \) is given by the expression

\[
\psi = K_1 \left( \frac{g}{R} \right) \left( \frac{1+\sin^2 \alpha}{\cos \alpha} \right) \left[ 1 - K_1 \frac{g}{R} \tan \alpha \right],
\]

(2.5)

where \( g \) is the gap of the magnet and coefficients \( K_1, K_2 \) are defined by pole geometry.

*Calculation of solenoid field*

Magnetic field of solenoids and axial-symmetric permanent magnets is calculated as

\[
B_z = B(z) - \frac{r^2}{4} \frac{d^2B}{dz^2}, \quad B_r = -\frac{r}{2} \frac{dB}{dz} \frac{d^3B}{dz^3} \frac{r^2}{8},
\]

(2.6)

where field at the axis \( B(z) \) is given at fixed points.

3. Acceptance of 2 x 90° bending magnets

![Fig.2. Particle trajectories in bending magnets: Pz = 5.6 MeV/c, dx/dz= ±0.3.](image1)

![Fig.3. Particle trajectories in bending magnets: Pz = 5.6 MeV/c, Δp/p = ± 0.1.](image2)
Maximum horizontal particle deviation (after first bend and drift):

\[ x_{\text{max}} = - \frac{L}{R} x_0 + Rx_0' + (R + L) \frac{\Delta p}{p_0} \]  

(3.1)

Maximum deviation from axis is \( x_{\text{max}} = a = 5 \text{ cm} \), therefore, horizontal acceptance of the channel is:

\[ x_{0, \text{max}} = a \frac{R}{L} = 5 \frac{12.5}{20} = 3.125 \text{ cm} \]  

(3.2)

\[ x_{0, \text{max}}' = a \frac{R}{L} = 5 \frac{12.5}{12.5} = 0.4 \]  

(3.3)

\[ \varepsilon_x = 1.25 \pi \text{ cm rad} \]  

(3.4)

Maximum energy spread is defined from Eq. (3.1) as:

\[ \left( \frac{\Delta p}{p_0} \right)_{\text{max}} = a \frac{R}{R + L} = 5 \frac{12.5}{12.5 + 20} = 0.15 \]  

(3.5)

Maximum vertical slope of particle trajectory:

\[ y_{0, \text{max}}' = 0.027 \]  

(3.6)

therefore, vertical acceptance is

\[ \varepsilon_y = 0.027 \times 2.5 = 0.0675 \pi \text{ cm rad} \]  

(3.7)

From initial distribution, the fraction of positrons within phase space - 3 cm < \( x < 3 \) cm, - 2.5 cm < \( y < 2.5 \) cm, - 0.4 < \( x' < 0.4 \), 0.027 < \( y' < 0.027 \), -0.15 < \( \Delta p/p < 0.15 \) is

\[ \frac{\Delta N}{N} = 10^{-2} \]  

(3.8)
Fig. 5. (Left) initial positron distribution distribution, (right) positrons within acceptance of bending magnets:
- -3 cm < x < 3 cm, -2.5 cm < y < 2.5 cm, -0.4 < x' < 0.4, 0.027 < y' < 0.027, -0.15 < Δp/p < 0.15.
4. Particle trajectories in solenoid

Fig. 6. Solenoid field profile.

Fig. 7. Single particle trajectories in solenoid, $z_0 = -15 \text{ cm}$, $x' = 0.1$, $y' = 0.1$, $p_z = 5.6 \text{ MeV/c}$.

Fig. 8. Single particle trajectories in solenoid, $z_0 = -10.8 \text{ cm}$, $x' = 0.1$, $y' = 0.1$, $p_z = 5.6 \text{ MeV/c}$.
5. Results of beam transport simulation

![Graphs showing beam transport simulation results](image_url)

Fig. 9. Output beam after solenoid and two 90° bends, \( z_0 = -10.8 \) cm.

Table 2. Results of simulation.

<table>
<thead>
<tr>
<th>Transport system</th>
<th>Average energy, MeV</th>
<th>Energy spread, ( \sigma_E/E )</th>
<th>Positron transmission</th>
<th>Longitudinal positron polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 x 90° bends, ( z_0 = 0 )</td>
<td>5.4</td>
<td>0.09</td>
<td>0.88 ( \times 10^{-2} )</td>
<td>0.67</td>
</tr>
<tr>
<td>2 x 90° bends, ( z_0 = -15 ) cm</td>
<td>5.2</td>
<td>0.08</td>
<td>0.23 ( \times 10^{-2} )</td>
<td>0.60</td>
</tr>
<tr>
<td>Solenoid + 2 x 90° bends, ( z_0 = -10.8 ) cm</td>
<td>5.3</td>
<td>0.08</td>
<td>4.6 ( \times 10^{-3} )</td>
<td>0.63</td>
</tr>
<tr>
<td>Solenoid + 2 x 90° bends, ( z_0 = -15 ) cm</td>
<td>5.3</td>
<td>0.08</td>
<td>0.76 ( \times 10^{-2} )</td>
<td>0.63</td>
</tr>
</tbody>
</table>