On Depolarization of Fast Positrons by Bremsstrahlung (*)

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The recent experimental measurement of the polarization of positrons from $\mu^+$ decay performed at CERN by Dick et al. (1) yields a value of $(28 \pm 10)$%, a result which disagrees with the $(V-A)$ theory and other measurements (2). This number has been corrected for depolarization due to multiple scattering and bremsstrahlung. The contribution due to multiple scattering is the more significant and arises largely by collisions which occur while the positrons slow down from 3 MeV to rest, causing a depolarization of the order of 50% (3). This result is supported by data on the decay products of $^9$B (4). On the other hand, the bremsstrahlung mechanism is most effective at the highest attainable energies. This note is an attempt at obtaining an estimate of this effect.


(*) This work was supported in part by the U. S. Atomic Energy Commission.


(3) C. Bouchiat and J. M. Levy-Leblond: (to be published).

Olsen and Maximow (4) have calculated this contribution; however, a simple result due to Dyson and McVoy (5) permits a more transparent treatment of this effect. It should be noted that whereas the latter authors work with helicity states, Olsen and Maximow do not. They consider a spin-flip transition to be one in which the helicity changes sign. Since at extremely high energies both the scattered positron and emitted photon are sharply peaked in the forward direction, one would expect both methods to yield the same result in the high-energy limit in which we are working.

We define the polarization $P$,

$$P = \frac{(N_+ - N_-)}{(N_+ + N_-)},$$

where $N_+$ and $N_-$ are the relative number of particles with positive and negative helicity respectively ($N_+ + N_- = 1$). The differential form of (1) is

$$dP = 3N_+ - 3N_- = 2dN_+.$$

If $T$ is the transition probability for


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(2) \[ \delta N_+ = TN_+ - TN_+ = -TP, \]

or

(3) \[ \delta P = -2TP. \]

If \( \sigma \) is the energy-dependent cross-section for spin flip, then the probability for spin flip in traveling a distance \( \delta x \) is \( N \sigma \delta x; N \) is the nuclear density. Therefore, one obtains

(4) \[ \frac{\delta P}{P} = -2N \sigma \delta x \]

which can be integrated,

(5) \[ P = P_0 \exp \left[ -2 \int_{x_0}^{x} N \sigma \frac{dE}{dx} \right]. \]

\( E_0 \) and \( E_f \) are the initial and final particle energies, and the energy loss is approximately given by

(6) \[ \frac{dE}{dx} = \left( \frac{dE}{dx} \right)_{\text{ionization}} + \left( \frac{dE}{dx} \right)_{\text{radiation}} \approx -I - \frac{E}{x_0}; \]

\( I \) and \( x_0 \) (radiation length) will be taken as constants although they are weakly energy-dependent in the range of interest. Both depend on the slowing down medium.

Equations (5) and (6) define the probability for spin flip assuming the cross-section \( \sigma \) is known. For this quantity we shall rely on the result of McVoy and Dyson (4) for the differential cross-section (lowest order in \( aZ \)) for helicity change

(7) \[ \frac{d\sigma}{dk} = \frac{k}{3\left(2E^2 \right)}, \]

where \( k \) is the photon energy, \( E \) the initial energy of the positron, and \( r = \text{radiation length} \times N \). It must be noted that this is an extreme relativistic approximation where only the leading terms in \( m/E \) and \( m/k \) have been retained. The cross-section is calculated by integrating the photon momentum to its maximum. The result is

(8) \[ \sigma = (E - m)^2/(6E^2), \]

\( \approx 1/6E \) for high energies.

Substituting (6) and (8) into (5), we obtain

(9) \[ P = P_0 \left( \frac{E_f + I \delta \alpha}{E_0 + I \delta \alpha} \right)^4. \]

The accompanying table shows the depolarization \( 1 - (P/P_0) \) due to energy changes of 50 to 10 MeV and 50 to 5 MeV for various media. It is of interest here to note that the absorber used in the CERN experiment (5) was Be. In this case, (9) gives a depolarization of approximately 10% for the most energetic positrons. Values of \( I \) and \( x_0 \) are taken from the table of Barkas and Rosenfeld (4).

**Table 1. Depolarization of positrons in various media from eq. (9).**

<table>
<thead>
<tr>
<th>Material</th>
<th>% Depol. (50–10) MeV</th>
<th>% Depol. (50–5) MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>C</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>Al</td>
<td>18</td>
<td>21</td>
</tr>
<tr>
<td>Pb</td>
<td>34</td>
<td>41</td>
</tr>
<tr>
<td>CH scintilator</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

For our energy range, both \( I \) and \( x_0 \) should be somewhat larger than those given in ref. (4). It is of interest to note that while \( I \) is an increasing function of energy, \( x_0 \) decreases with energy.

(4) W. Barkas and A. Rosenfeld: Data for Elementary-Particle Physics, UCRL-5030 Rev. unpublished, 1948.

(5) W. Barkas and A. Rosenfeld: Data for Elementary-Particle Physics, UCRL-5030 Rev. unpublished, 1948.)
Thus the product $I_2$, is relatively constant over our energy range. A numerical evaluation of eq. (5) using values of $dE/dx$ obtained from equations given by Heitler (7) yields a depolarization of 7% for positrons slowed down from 50 to 10 MeV in Be. This number should be compared with that obtained from (9), approximately 9%. This amount of depolarization is much too small to explain the experimental result (7).

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Multiple Coulomb Scattering for Very-High-Energy Particles.

P. K. Aditya

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(Nuovo Cimento, 31, 473 (1964))

An investigation on large cell scattering of $\mu$-mesons, carried out in continuation of the above work, communicated later for publication and agreed upon for inclusion as an Appendix to the above work, has appeared as Appendix III. Prior to the knowledge of its having been accepted as an appendix, these results were included briefly in the form of Appendix II added in proofs.

The duplication of subject matter, on account of both the appendices having been published is regretted.

The caption of Fig. 2 should read "Integral Distribution ..." instead of "Internal ...".