7.4 Emittance Dynamics

Figure 7-37. Tolerance on the structure alignment for a 50% emittance growth in the NLC-IIb linacs as a function of alignment length, when considering only single-bunch effects, only multi-bunch effects, and both effects together. $L_0$ is the 1.8-m structure length.

From ZDR
N = number of st. fronds
W = transverse section
N = punch population
L = length of structure
x = structure offsets
d = punch spacing
z = position in machine
x' = offset of punch
\[ \text{with } e = E_0 \]

\[ \frac{E}{d} \left( e \frac{dx}{dz} \right) + \frac{e}{2} \frac{w_{	ext{air}}}{2w_{	ext{air}}} \]
if $b$-tron term is small compared to alignment term:

at the final position:

$$X_{mf} = -e^2 N L_a \sum B_i \frac{N_a}{E_i} \sum_{i=1}^{N_a} X_a \sin \mu_i \sqrt{B_i}$$

with 

$$\sum B_i = \sum_{m=m_0}^{m_{max}} W[\text{m.m.} \text{os}] - \text{sum wake}$$

$\mu_{if}$ - phase advance from $i$ to $f$

Suppose $\beta \approx E^{\frac{1}{2}}$ (i.e. $\beta_e \approx \beta_0, E_e \gg E_0$); errors uncorrelated:

\[ \text{ran ensemble of lines:} \]

\[ (X_{mf})_{\text{rms}} = e^2 N L_a \sum (X_a)_{\text{rms}} \sqrt{N_a} \frac{B_0}{\beta_0} \left[ 1 - \frac{(E_0/E_f)^{\frac{1}{2}}}{E_0 E_f} \right]^{\frac{1}{2}} \]

Emittance growth follows a $X^2$ distribution of degree 2; i.e. $\sim \exp \left[-\frac{(x-E_f)^2}{\sigma_e} \right] / \sigma_e$

\[ \sqrt{\frac{\Gamma_e}{\Gamma_0}} = e^2 N \beta_0^{\frac{1}{2}} N_a^{\frac{1}{2}} \left[ (X_a)_{\text{rms}} (S_a)_{\text{rms}} \right] \left[ \frac{1 - (E_0/E_f)^{\frac{1}{2}}}{E_0 E_f^{\frac{1}{2}}} \right]^{\frac{1}{2}} \]

Notes: if $E_0 = 10$ keV
oscillations, particularly for the bunches near the end of the train. Therefore the approximation of dropping the betatron term in Eq. (1) will be more valid than before. As for the emittance growth due to the alignment term it can be shown that its expectation value is still given by Eq. (5), provided that now the sum wake is understood to be averaged over the 4 structure types in quadrature the emittance distribution, however, will no longer follow a simple exponential distribution.

To benefit from the use of 4 structure types in the misalignment term effects (as we did in the betatron term effects) we need to align the structures particularly well within each group of 4. Let us suppose each group is on its individual girder, to facilitate the alignment. To include the effect of girder misalignments on emittance we need to add a second term to Eq. (5), one that differs only in that the combination of parameters $N_g L_g^2 x_0^2 \sigma_t^2 (S_0)^2 \gamma_m^2$ is replaced by the one corresponding to the girder scale. If we take subscript g to represent girder quantities, we have $N_g = N_0 / 4$, $L_g = 4 L_0$, and $S_m$ is found by simply averaging the sum wake over the 4 types. Fig. 2 displays $S_m$ rms bunch number m: here $(S_m)_{rms}$ = 0.38 V/pC/mm/m.

**Fig. 2.** The sum wake for the combination of the 4 structure types $S_m$ v.s. bunch number m.

**Sensitivity to Slight Frequency Changes**

The distribution of dipole modes is approximately gaussian with a central frequency of 15 GHz and an rms of 2.3%, and a total width of 10%, and the bunch spacing is 42 cm. Therefore a kind of resonance can develop between a mode frequency and the 20th, 21st, or 22nd harmonic of the bunch frequency. For one structure type the relative mode spacing at the center of the distribution is $4.3 \times 10^{-3}$ [1]; therefore, a small shift in the mode frequencies relative to the bunch frequency can result in a large change in effect. To show this effect, in Fig. 3 we plot $(S_0)_{rms}$ for the 4 structure types as function of small changes in relative bunch spacing (the dashed curves). We see more than a factor of 5 variation when the relative bunch spacing is changed by only $2.5 \times 10^{-4}$. (However, in a real accelerator, where each structure has different, random manufacturing errors, the effect of the fluctuations will be reduced.) The solid line in Fig. 3 gives the sum of the contributions of the 4 structure types added in quadrature. This gives the effect on emittance growth when the errors are on the structure scale and 4 structure types are used. We note that the fluctuations are much smaller. Fig. 4 gives $(S_0)_{rms}$ for 4 structure types are used. We note that the average value of $(S_0)_{rms}$ = 1/20$(S_0)_{rms}$, therefore the expected gain in tolerance on the girder scale (remember the factor $\sqrt{N_g N_0 L_g / L_0}$) is about 10.

**Fig. 3.** $(S_0)_{rms}$ for the 4 different structure types.

**Comparison with Numerical Results**

For the numerical comparisons we use the NLC parameters: eN = 1 nC, $E_0 = 10$ GeV, $E_I = 250$ GeV, $r_b = 8$ m, $L_0 = 1.8$ m, $N_g = 3600$, and normalized emittance $\epsilon_0 = 3 \times 10^{-9}$m. The lattice is a piecewise 90 degree-per-cell FODO type; the number of structures between quadrants is given by the integer part of $\sqrt{4E_0 / E_I}$. Fig 5 displays the results of numerical tracking when there are 4 independently misaligned structure types, and when $(x_0)_{rms} = 5 \mu$m. The dashes give an exponential approximation, with $\sigma_t$ given by Eq. (5). The average growth obtained numerically, 0.09, agrees with that obtained analytically, 0.085, but the distributions differ slightly.

In Table 1 we compare the results of three methods of finding the tolerance for 25% emittance growth: (i) using Eq. (5); (ii) numerically performing the sum Eq (2)
Fig. 5. Tracking results for 4 independently misaligned structure types. with \((x_0)_{\text{rms}} = 5 \, \text{µm}\).

(and its counterpart for \(z_0\)), but with \(x_0\) replaced by \((x_0)_{\text{rms}}\) to find the expected emittance for the real lattice: and (iii) tracking. The error factors give the rms variation due to bunch spacing. We give results for the case of only 1 structure type and for 4 structure types on both the structure and lattice scales. In the first case the analytical tolerances are much larger than the tracking results, indicating that dropping the betatron term of Eq. (1) is not a good approximation. Also, the variation terms are large, showing a great sensitivity to frequency changes. (With random fabrication errors along the linac in this sensitivity should decrease.) In case 2 the analytic approximation agrees with the tracking results. Finally, in case 3, 4 structure types on the girder scale. we note that the accurate inclusion of the amplitude and phase of the betatron sum is important for a good estimate.

<table>
<thead>
<tr>
<th>Scale</th>
<th>N-type</th>
<th>Eq. (5) Num</th>
<th>sum Tracking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure (a)</td>
<td>10.9 ± 6.6</td>
<td>11.5 ± 5.4</td>
<td>4.3 ± 3.2</td>
</tr>
<tr>
<td>Structure (a)</td>
<td>8.3 ± 0.6</td>
<td>8.9 ± 0.6</td>
<td>8.7 ± 0.6</td>
</tr>
<tr>
<td>Girder (g)</td>
<td>120 ± 60</td>
<td>32.0 ± 5.4</td>
<td>32.0 ± 4.9</td>
</tr>
</tbody>
</table>

Table 1. Alignment tolerances in microns for 25% emittance growth.

THE HIGHER DIPOLE BANDS

The analysis of the detuned structure has focused almost entirely on the modes of the first dipole band (including the effects of the second band modes on the first) since the kick factors of these modes are at least an order of magnitude larger than those of the other bands. However, when we perform the uncoupled calculation including also the effect of the modes of bands 3-5 we find that the wakefield amplitude is now an unacceptable 10% at \(s = 42 \, \text{cm}\), the position of the second bunch, and it decreases only slowly as we move further back in the bunch train.

Fig. 6a illustrates the cause of the problem. In this figure we plot the dispersion curves representing a cell near the beginning, middle, and end of the detuned structure for dipole bands 3 to 6. Note that in the vicinity of the speed of light line the 3rd, 6th, and 7th bands for the three cells are closely spaced, resulting in little detuning. The 3rd band, which contributes about 12% to the total wake.

This tolerance is a factor of 2 tighter than that presented in Ref. 2. a discrepancy accounted for by differences in parameters and the tolerance definition used.
PARAMETERIZATION OF STRAIGHTNESSES

SUS Juv STRUCTURES HAVE AN A.L LIKE STRAIGHTNESSES i.e. \( <\Delta y^2> \sim A \cdot \Delta L \)

\[ \text{STRUCTURE CENTER} \]

\[ \text{Beam} \]

\[ \text{Girder} \]

\[ y_b \quad y_s \]

\[ L_y \]

\[ \Delta y \sim L_y \quad <y_s - y_b>^2 \cdot N_{girder} \]

\[ <y_s - y_b>^2 \sim <\Delta y^2> / N_{bpm} \]

\[ = A \cdot L_y / L_y \sim A \]

\[ \text{and} \quad N_{girder} \sim 1 / L_y \]

\[ \Delta y \sim A \cdot L_y \]

\[ \text{or} \quad A \sim \frac{1}{\# \text{ of structures/girder}} \quad \text{for constant} \]

\[ y_s \quad y_b \]