Post-Target Beamline Design for Proposed FFTB Experiment with Polarized Positrons

Y. K. Batygin and J. C. Sheppard

Stanford Linear Accelerator Center
Stanford University
Menlo Park, CA 04025

Abstract: The beamline after positron production target for the proposed experiment E-166 is discussed. The beamline includes bending magnets and solenoid to deliver polarized positron beam from the target to polarimeter. Results of simulation indicate that transmission efficiency of 1...3 % with beam polarization of 60...80 % can be obtained if beam energy resolution is required while the transmission of 40...77 % and polarization of 40% can be obtained without beam energy resolution.
POST-TARGET BEAMLINE DESIGN FOR PROPOSED FFTB EXPERIMENT WITH POLARIZED POSITRONS

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SLAC, Stanford University, Stanford, CA 94309

Abstract

The beamline after positron production target for the proposed experiment E-166 is discussed. Beamline includes bending magnets and solenoid to deliver polarized positron beam from the target to polarimeter. Results of simulation indicate that transmission efficiency of 1...3 % with beam polarization of 60...80 % can be obtained if beam energy resolution is required while the transmission of 40...77 % and polarization of 40% can be obtained without beam energy resolution.

INTRODUCTION

The proposed experiment E-166 is designed to demonstrate the possibility of producing longitudinally polarized positrons from circularly polarized photons. Layout and general description of experiment are given in Ref. [1]. It utilizes a low emittance 50 GeV electron beam passing through a helical undulator in the FFTB. Circularly polarized photons generated by the electron beam in undulator hit a target and produce electron-positron pairs. Outcoming positron and electron beams have to be separated from each other and from outcoming photon beam. The purpose of post-target optics is to select the positron beam and to deliver it to a polarimeter keeping positron beam polarization as high as possible. An additional solenoid might be required to transport the beam through the wall to increase signal-to-background ratio in polarimetry. We consider several design schemes (see Figs. 1 - 10). Parameters of the beamlines are summarized in Table 1.

POSITRON TRACKING

Particle tracking is performed using BEAMPATH code [2]. Initial positron distribution after interaction of photon beam with 0.5 RL Ti target was calculated using the EGS4 program [3] adapted for polarized beam [4, 5]. It was assumed that positrons are produced by 11.7 MeV γ-flux. Outcoming positrons have a large energy spectrum (from 0 to 10 MeV) and transverse emittance of 0.016 π·m·rad (see Fig. 11).

Particle tracking was accompanied with integration of the Thomas-BMT equation [6], describing the precession of the spin vector $\vec{S}$:
\[
\frac{d\hat{S}}{dt} = \frac{e}{m \gamma} \left[ (1+G\gamma)\vec{B}_\perp + (1+G)(1+G)\vec{B}_\parallel + \left( \gamma - \frac{\gamma}{1+\gamma} \right) \frac{\vec{E} \vec{B}}{c} \right],
\]

where \( G \) is the anomalous magnetic moment of the positron, \( \vec{E} \) is the electrical field, and \( \vec{B}_\perp \) and \( \vec{B}_\parallel \) are components of the magnetic field perpendicular and parallel to particle velocity. The spin advance at a small distance \( dz \) is described as a matrix [7]:

\[
\begin{bmatrix}
S_x \\
S_y \\
S_z \\
\end{bmatrix} =
\begin{bmatrix}
1 - a(B^2 + C^2) & ABa + Cb & ACa - Bb \\
ABa - Cb & 1 - a(A^2 + C^2) & BCa + Ab \\
ACa + Bb & BCa - Ab & 1 - a(A^2 + B^2)
\end{bmatrix}
\begin{bmatrix}
S_{x,o} \\
S_{y,o} \\
S_{z,o}
\end{bmatrix},
\]

\[
A = \frac{D_x}{D_0}, \quad B = \frac{D_y}{D_0}, \quad C = \frac{D_z}{D_0}, \quad D_0 = \sqrt{D_x^2 + D_y^2 + D_z^2}.
\]

\[
a = 1 - \cos(D_0 \, dz), \quad b = \sin(D_0 \, dz),
\]

where components \( D_x, D_y, D_z \) are defined by the equations:

\[
D_x = \frac{e}{m \gamma v} \left[ \left( 1+G\gamma \right)(B_x - x'B_z) + (1+G)(B_z - x'B_z) + \frac{v}{c^2} \left( \frac{\gamma}{1+\gamma} + G\gamma \right)(E_y - y'E_z) \right],
\]

\[
D_y = \frac{e}{m \gamma v} \left[ \left( 1+G\gamma \right)(B_y - y'B_z) + (1+G)(B_z - x'B_z) + \frac{v}{c^2} \left( \frac{\gamma}{1+\gamma} + G\gamma \right)(E_y - y'E_z - E_x) \right],
\]

\[
D_z = \frac{e}{m \gamma v} \left[ \left( 1+G\gamma \right)(-x'B_x - y'B_y) + (1+G)(x'B_x + B_z + y'B_z) + \frac{v}{c^2} \left( \frac{\gamma}{1+\gamma} + G\gamma \right)(y'E_x - E_y x') \right],
\]

and prime means derivative over longitudinal coordinate, \( ' = d/dz \). Matrix (2) describes spin precession in Cartesian coordinates. In a bending magnet with a design orbit radius \( R \), spin is corrected according to the matrix

\[
\begin{bmatrix}
S_x \\
S_y \\
S_z \\
\end{bmatrix} =
\begin{bmatrix}
\cos\theta & 0 & -\sin\theta \\
0 & 1 & 0 \\
\sin\theta & 0 & \cos\theta
\end{bmatrix}
\begin{bmatrix}
S_{x,o} \\
S_{y,o} \\
S_{z,o}
\end{bmatrix},
\]

which describes rotation of a system of coordinates to the angle of \( \theta = -dz/R \) at every integration step.
Initially, the spin vector of each positron is pointed along momentum vector. During beam transport, the spin vector precesses, resulting in the depolarization of the beam. We define the longitudinal polarization as an average of the product of the longitudinal component $S_z$ and the value of polarization, $P$, summed over all positrons:

$$<P_z> = \frac{1}{N} \sum_{i=1}^{N} S_z^{(i)} P^{(i)}.$$  \hspace{1cm} (9)

The initial value of longitudinal polarization is $<P_z> = 0.41$. Depending on the beamline configuration, the polarization of the final beam will be within the range of 0.41...0.88, depending on energy of selected particles. Several possible configurations of post-target beam optics are considered below.

**BEAMLINE WITH 90° MAGNET**

*Beam dynamics in bending magnet*

Fig. 12 and Table 2 illustrate the beam dynamics in Beamline # 1 which contains only a 90° bending magnet with a radius of curvature $R = 17$ cm and an aperture of 1.75 cm. Depending on the value of the magnetic field, one can obtain different energy resolutions. In case of $B = 0.13$ Tesla, the positron transmission efficiency is 1.5% and the polarization of the beam is 76%. The value of transmission efficiency might be increased if an additional flux concentrator is used after the target. The SLAC flux concentrator [8] is a solenoid with a sharp increase in magnetic field up to peak value of 5.8 Tesla at a distance of 5 mm from the target and an adiabatic decrease of the field over a distance of 15 cm (see Fig. 9). An additional magnetic field of 1.2 Tesla at the target is required to confine the emitted positron beam with its large momentum spread. In the Cornell flux concentrator [9], the field is distributed over a distance of 8 cm and has a peak value of 0.8 Tesla (see Fig. 10).

In Table 3, the results of beam dynamics in a Beamline # 2 with a SLAC flux concentrator and 90° bending magnet are presented. Simulations indicate that in presence of flux concentrator, the value of transmission efficiency is increased by a factor of 3.

*Beamline with an additional solenoid*

In order to improve a signal-to-background ratio in positron polarimetry of the positron beam, an additional shielded alcove of the length of a few meters might be required [10]. To deliver beam through the shield, a solenoidal field can be used along the beam transport (Beamline
The required magnetic field to confine a circular beam with normalized emittance $\varepsilon$ and radius $a$ is

$$B = 2 \frac{mc}{e} \frac{\varepsilon}{a^2}.$$  \hspace{1cm} (10)

After the bending magnet the beam emittances are $\varepsilon_x = 0.04 \pi \text{ m-rad}$, $\varepsilon_y = 0.0027 \pi \text{ m-rad}$. Taking the larger value of beam emittance, $\varepsilon = 0.04 \pi \text{ m-rad}$, the required magnetic field to confine the beam with radius $a = 1.75 \text{ cm}$ is $B = 0.46$ Tesla. Due to unequal emittances, $\varepsilon_x \neq \varepsilon_y$, the beam is not in equilibrium with magnetic field and a solenoid with an aperture larger than $1.75 \text{ cm}$ is required. Table 4 and Fig. 13 show the results of the beam dynamics simulation in Beamline #3 containing a $90^\circ$ bending magnet followed by a $3 \text{ m}$ long solenoid with an aperture radius of $D_x = 2.5 \text{ cm}$. Simulations show that in a solenoid with a magnet field of $B \geq 0.5$ Tesla, the beam is successfully transmitted through the transport system without significant losses.

**BEAMLINE WITH TWO 90° MAGNETS**

Numerical integration of Eq. (1) in the considered beamlines indicates that spin depolarization is small. Spin precession in a bending magnet is

$$\varphi = \alpha \gamma G,$$  \hspace{1cm} (11)

where $\alpha$ is a bending angle. For $\alpha = \pi/2$ and $\gamma = 20$ the precession angle is $\varphi = 3.6 \cdot 10^{-2}$ or 2°. In order to avoid any depolarization, the S - bend scheme containing two $90^\circ$ magnets can be used (see Figs. 4, 5, 6). In the S - bend beamline, particles pass through two $90^\circ$ bends of opposite polarity and depolarization in the first bending magnet is compensated by the second bending magnet. Additionally, a flux concentrator can be used just after the target to confine more particles. In Table 5 and in Figs. 14, 15 results of the positron beam dynamics are presented for different combinations of S-bend magnets. Results indicate that the final beam transmission is $0.4\% - 3.5\%$ while the positron polarization is $0.54...0.68$.

**BEAMLINE WITH 20° MAGNET**

In the considered beamlines, the positron beam transmission efficiency is low (around several percent). This is a result of the energy selection which imposes limitations on the transverse acceptance of the beam. To increase the value of beam transmission, a beamline with a small bending angle of $20^\circ$ and large aperture of $d_x = d_y = 5 \text{ cm}$ followed by the $3 \text{ m}$ solenoid with the same aperture has been modeled (see Figs. 7, 8). Parameters of the structure are selected to assure
a high value of transmission. Table 6 and Fig. 16 illustrate the results of the beam dynamics simulation in the structures. Transmission efficiency as high as 42% without a flux concentrator and 77% with a flux concentrator can be obtained. In case of Beamline # 8 with a flux concentrator, it was assumed that the target is placed in the area with strong magnetic field of B = 6.4 Tesla to provide higher positron capture. Polarization of the final beam is 0.42...0.47, which is close to that of original beam because of absence of energy resolution in that structures.

**SUMMARY**

Proposed beamlines provide various design opportunities for polarized beam transport in the experiment E-166. In a final design, a compromise has to be made between the value of positron beam transmission efficiency, energy resolution, and beam polarization.

**REFERENCES**

Fig. 1. Beamline #1.

Fig. 2. Beamline #2.
Fig. 3. Beamline # 3.

Fig. 4. Beamline # 4.
Fig. 5. Beamline # 5.

Fig. 6. Beamline # 6.
Fig. 7. Beamline # 7.

Fig. 8. Beamline # 8.
Fig. 9. Magnetic field of SLAC flux concentrator.  
Fig. 10. Magnetic field of Cornell flux concentrator.

Fig. 11. Initial distribution of polarized positrons.
Fig. 12. Positrons polarization after Beamline # 1.
Fig. 13. Polarized positron beam after Beamline # 3 with solenoid field $B = 0.5$ Tesla.

Fig. 14. Polarized positrons after Beamline # 4.

Fig. 15. Polarized positron beam after Beamline # 6 with SLAC flux concentrator.
Fig. 16. Polarized positron distribution after Beamline # 7.
Table 1. Parameters of the structure

**Target**
- Material: Ti, 0.5 r.l.
- Initial positron distribution: generated by EGS program
- Drift space after target (Beamline # 5), h: 18 cm

**SLAC flux concentrator**
- Magnetic field at the target: 1.2...6.4 Tesla
- Max field: 6.4 Tesla
- Aperture: 0.45......2.5 cm
- Field length: 16 cm

**Cornell flux concentrator**
- Magnetic field at the target: 0
- Max field: 0.8 Tesla
- Aperture: 0.6....1.6 cm
- Field length: 8 cm

**Bending magnets**
- Bending angle: 20°......90°
- Bending radius, R: 17 cm
- Aperture, d_x, d_y: 1.75 ....2.5 cm
- Magnetic field: 0.05....0.17 Tesla

**Solenoid**
- Magnetic field: 0.5.....1 Tesla
- Length: 3 m
- Aperture: 2.5....5 cm

Table 2. Results of positron tracking in Beamline # 1.

<table>
<thead>
<tr>
<th>Bending field, Tesla</th>
<th>Mean Energy (MeV)</th>
<th>Energy spread, $\sigma_E/E$</th>
<th>Positron transmission (%)</th>
<th>Longitudinal positron polarization, $P_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>2.8</td>
<td>0.21</td>
<td>0.38</td>
<td>0.28</td>
</tr>
<tr>
<td>0.09</td>
<td>4.8</td>
<td>0.17</td>
<td>1.1</td>
<td>0.53</td>
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<tr>
<td>0.11</td>
<td>5.3</td>
<td>0.17</td>
<td>1.5</td>
<td>0.67</td>
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<tr>
<td>0.13</td>
<td>6.5</td>
<td>0.16</td>
<td>1.5</td>
<td>0.76</td>
</tr>
<tr>
<td>0.15</td>
<td>7.2</td>
<td>0.13</td>
<td>1.3</td>
<td>0.84</td>
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<tr>
<td>0.17</td>
<td>7.8</td>
<td>0.12</td>
<td>0.97</td>
<td>0.87</td>
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Table 3. Results of positron tracking in Beamline # 2.

<table>
<thead>
<tr>
<th>Bending field, Tesla</th>
<th>Mean Energy (MeV)</th>
<th>Energy spread, $\sigma_E/E$</th>
<th>Positron transmission (%)</th>
<th>Longitudinal positron polarization, $P_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>2.3</td>
<td>0.15</td>
<td>0.8</td>
<td>0.20</td>
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<tr>
<td>0.09</td>
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<td>0.1</td>
<td>2.3</td>
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<td>0.15</td>
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<td>0.08</td>
<td>2.4</td>
<td>0.85</td>
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<tr>
<td>0.17</td>
<td>8.1</td>
<td>0.07</td>
<td>2.8</td>
<td>0.88</td>
</tr>
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</table>

Table 4. Results of positron tracking in Beamline # 3.

<table>
<thead>
<tr>
<th>Bending field, Tesla</th>
<th>Solenoid field, Tesla</th>
<th>Mean Energy (MeV)</th>
<th>Energy spread, $\sigma_E/E$</th>
<th>Positron transmission (%)</th>
<th>Longitudinal positron polarization, $P_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.11</td>
<td>0.5</td>
<td>5.3</td>
<td>0.12</td>
<td>1.2</td>
<td>0.67</td>
</tr>
<tr>
<td>0.11</td>
<td>0.7</td>
<td>5.5</td>
<td>0.15</td>
<td>1.3</td>
<td>0.68</td>
</tr>
<tr>
<td>0.11</td>
<td>1.0</td>
<td>5.5</td>
<td>0.16</td>
<td>1.4</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Table 5. Results of positron tracking in Beamlines # 4, 5, 6.

<table>
<thead>
<tr>
<th>Beamline</th>
<th>Bending field, Tesla</th>
<th>Mean Energy (MeV)</th>
<th>Energy spread, $\sigma_E/E$</th>
<th>Positron transmission (%)</th>
<th>Longitudinal positron polarization, $P_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beamline # 4</td>
<td>0.11</td>
<td>5.5</td>
<td>0.12</td>
<td>1.0</td>
<td>0.68</td>
</tr>
<tr>
<td>Beamline # 5</td>
<td>0.11</td>
<td>5.3</td>
<td>0.07</td>
<td>0.4</td>
<td>0.61</td>
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<tr>
<td>Beamline # 6 with Cornell flux concentrator</td>
<td>0.11</td>
<td>5.1</td>
<td>0.07</td>
<td>1.4</td>
<td>0.54</td>
</tr>
<tr>
<td>Beamline # 6 with SLAC flux concentrator</td>
<td>0.11</td>
<td>5.3</td>
<td>0.07</td>
<td>3.5</td>
<td>0.61</td>
</tr>
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Table 6. Results of positron tracking in Beamlines # 7, 8.

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<tr>
<th>Beamline</th>
<th>Bending field, Tesla</th>
<th>Mean Energy (MeV)</th>
<th>Energy spread, $\sigma_E/E$</th>
<th>Positron transmission (%)</th>
<th>Longitudinal positron polarization, $P_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beamline # 7</td>
<td>0.11</td>
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<td>0.41</td>
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<td>0.42</td>
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<tr>
<td>Beamline # 8</td>
<td>0.11</td>
<td>4.7</td>
<td>0.42</td>
<td>77</td>
<td>0.47</td>
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</tbody>
</table>