Abstract: The yield of positrons as a function of drive beam energy for a K=1, planar undulator-based positron source is evaluated using the EGS4 simulation code. Raw yield, yield into a fixed phase space acceptance, and rms emittance of emitted positrons is calculated. For a fixed geometry, the yield varies as the square of the drive beam energy. A faster fall off at the low energy end is seen and is due to reduced emission of positrons produced with initially low energies (in the range of a few MeV).
Positron Yield as a Function of Drive Beam Energy
for a K=1, Planar Undulator-Based Source

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References

Abstract

The yield of positrons as a function of drive beam energy for a K=1, planar undulator-based positron source is evaluated using the EGS4 simulation code. Raw yield, yield into a fixed phase space acceptance, and rms emittance of emitted positrons is calculated. For a fixed geometry, the yield varies as the square of the drive beam energy. A faster fall off at the low energy end is seen and is due to reduced emission of positrons produced with initially low energies (in the range of a few MeV).

Question

How does the yield of positrons from an undulator-based source vary as a function of drive beam energy?

Answer

In reference (1) the yield of positrons was estimated for two values of electron drive beam energy; 250 GeV and 160 GeV. This estimate is extended over a range of drive beam energies from 100 GeV up to 250 GeV. The positron production simulations are made using EGS4 and analyzed with a Matlab script file (see Appendix A for details of the routines used). For the simulations, a K = 1, $\lambda_p = 1.42$ cm period planar undulator photon spectrum is used in conjunction with a 0.4 r.l. thick Ti converter target. A gaussian, transverse spatial profile for the incident photon beam is modeled; a 40 MeV upper limit cut on the emitted positrons is made. Yield into a 4-D phase space acceptance of 0.036 m-rad per plane is tabulated. For each energy point, the total number of incident photons has been set to $N_{in} = 100,000$; the same random seed was used for each case and only one EGS4 run was made for each energy run. Table 1 lists the inputs and results of the simulations.

In Table 1: $E_e =$ electron drive beam energy, $E_{c10} =$ cutoff energy of the undulator first harmonic, $\sigma_x = \sigma_y =$ incident rms beam size, $N_{out} =$ the raw number of outgoing positrons of all energies, $\gamma e_i =$ normalized rms emittance of the out going positrons with energies less than the cut off energy (of 40 MeV); $N_{EC} =$ fraction of $N_{out}$ with energies less than the cut off energy; $N_{PS} =$ fraction of $N_{out}$ with energies less than the cut off energy and within a specified 4-dimensional phase space acceptance (of 0.036 m-rad per $x-p_x$, $y-p_y$ plane).
The acceptance ellipses are geometrically similar to the positron emittance distributions and therefore result in the maximum acceptance into a fixed aperture for the simulated distributions.

Table 1

<table>
<thead>
<tr>
<th>$E_e$ (GeV)</th>
<th>$E_{c10}$ (MeV)</th>
<th>$\sigma_x=\sigma_y$ (cm)</th>
<th>$N_{out}$</th>
<th>$\gamma_{E_x}$ (m-rad)</th>
<th>$N_{EC}$</th>
<th>$N_{PS}$</th>
<th>$N_{EC}N_{PS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>4.48</td>
<td>0.175</td>
<td>247</td>
<td>0.0067</td>
<td>1.000</td>
<td>0.874</td>
<td>0.874</td>
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<tr>
<td>130</td>
<td>7.57</td>
<td>0.135</td>
<td>853</td>
<td>0.0089</td>
<td>1.000</td>
<td>0.791</td>
<td>0.791</td>
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<tr>
<td>160</td>
<td>11.50</td>
<td>0.109</td>
<td>1730</td>
<td>0.0095</td>
<td>1.000</td>
<td>0.758</td>
<td>0.758</td>
</tr>
<tr>
<td>190</td>
<td>16.17</td>
<td>0.092</td>
<td>2892</td>
<td>0.0100</td>
<td>0.994</td>
<td>0.754</td>
<td>0.749</td>
</tr>
<tr>
<td>220</td>
<td>21.68</td>
<td>0.079</td>
<td>4117</td>
<td>0.0101</td>
<td>0.972</td>
<td>0.743</td>
<td>0.722</td>
</tr>
<tr>
<td>250</td>
<td>28.00</td>
<td>0.070</td>
<td>5395</td>
<td>0.0113</td>
<td>0.939</td>
<td>0.723</td>
<td>0.679</td>
</tr>
</tbody>
</table>

The cutoff energy of the undulator first harmonic is given as $E_{c10}$:

$$E_{c10} = \frac{h}{\gamma} \frac{4\pi \gamma^2 c/\lambda_p}{1 + K^2/2}$$  \hspace{1cm} (1)

wherein $\gamma$ is given by the electron drive beam energy. The incident photon rms beam size is scaled as $1/\gamma$ from the 0.70 mm size taken for the 250 GeV point.

The results given in Table 1 are shown graphically in Figures 1-4. In Figures 1 and 2, a yield curve $Y = aE_x^2$ is drawn for comparison. These yield curves are normalized to the $E_e = 250$ GeV point in each figure.
Figure 1.: Raw positron yield as a function of electron drive beam energy for an incident \( K = 1 \), planar undulator photon spectrum. A line \( Y \propto E^2 \) is drawn for comparison.
Figure 2.: Positron yield into 5D phase space cut, normalized to the 250 GeV point, as a function of electron drive beam energy for an incident $K = 1$, planar undulator photon spectrum. A line $Y = aE_e^2$ is drawn for comparison.
Figure 3.: Raw positron yield and yield within 5D phase space cut as a function of electron drive beam energy for an incident $K = 1$, planar undulator photon spectrum. The phase space cut includes all positrons with energies less than 40 MeV and with transverse coordinates within a 0.036 m-rad per plane phase acceptance ellipses.
Discussion

In Figure 5, the probability of pair production and the probability of positron emission are plotted as a function of photon energy for monoenergetic photons incident on 0.4 r.l. of Ti. The production probability is taken from the average pair production in air and in Pb given by Rossi\textsuperscript{2}. ESG4 has been used to calculated the emission probabilities (raw yield). In the photon energy range of 10-40 MeV the pair production cross section is essentially linear with respect to energy but rolls off at lower energy. Positron emission from the converter target, which includes energy loss and absorption during drift to the surface, is linear in photon energy over a range of 5-50 MeV. Below about 5 MeV incident photon energy, very few positrons emerge from the target.
Since the photon energy from a fixed geometry undulator is proportional to the square of the drive beam energy, the resultant positron production is expected to vary quadratically with drive beam energy. However, at the low end of the spectrum the emission probability of produced positrons falls off due to absorption. Hence the yield falls slightly faster than the square of the drive beam energy. The smaller phase space of positrons produced with lower energy photons results in somewhat higher capture efficiency; this enhanced capture does not compensate for the drop due to decreased emission.

As noted above, the acceptance phase space used for counting has been drawn geometrically similar to the emittance distribution of the positrons. This is an optimistic picture of the capture process. Design studies need to be done to maximize the overlap of the downstream acceptance channel with the anticipated positron distribution. Preliminary studies indicate that the capture of positrons for an undulator based source is in the range of 25% rather than the value of about 70% shown in Table 1.
Appendix A: Simulation Package Routines

An EGS4 user code has been written to read in an arbitrary incident photon energy
distribution and rms gaussian beam size. These files (ucRTZspc.mortran, ucRTZ.data,
pscum_k1_120.out, ww10c_k1_120.out, and Xegs4run*) are stored in:
/afs/slac.stanford.edu/u/ad/jcs/egsruns/yieldcalc/phtnspetrm in Unix land. EGS4 output
files are FTP'ed to the NT system and analyzed using partcounter3.m. The relevant NT
files are stored on: jcsZ:\Positrons\EGS4Runs\ti_spectrum\partcounter3.m.