CP VIOLATION
&
B Physics

Angular Analysis

Lecture #13

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Problem

Modes $B \to VV$ or even higher spins

$\ell = 0,1,2$ higher ℓ's

CP of final state

$= (\text{intrinsic CP of particles})\,(-1)^{\ell}$

from P

ℓ even and ℓ odd $\Rightarrow$ mixture of even + odd CP

asymmetry (coefficient of $\sin \Delta m t$ term)

$$= f_e \sin 2\phi - f_o \sin 2\phi \qquad f_e + f_o = 1.$$

where $\sin 2\phi = \operatorname{Im} \lambda_{VV}$

$$= (1 - 2f_o) \sin 2\phi$$

"Dilution" factor

- reduces asymmetry magnitude
- confuses relationship between a and CKM parameters

Possible Ways Out

(i) Theorists' challenge — calculate f_0

— Very difficult (hopeless?) for any exclusive channel, depends on meson form factors
$\Leftrightarrow$ model dependent.

— for inclusive rates e.g. $B \to$ charmless hadrons
see Beneke Buchalla & Dunietz
Phys. Lett. B 393 132 (1997)

BVT
- uses "local quark-hadron duality"
i.e. quark diagram kinematics to determine final hadron kinematics
- f_0 is probably ~~cut~~ dependant & dependence is sensitive to assumptions

(ii) Experiments' challenge — measure f_0

$\Rightarrow$ angular analysis of decays of pseudo-two-body channels

Example: $B \to \psi K^*$

— presents general ideas

see Dunietz Quinn Snyder Toki and Lipkin
Phys. Rev D 43 2193 (1991)
for tables of many channels for which such approaches could be used.

$$B \to \psi \quad K^*$$
$$K^* \hookrightarrow K\pi$$
$$\psi \hookrightarrow \mu^+\mu^-$$

vector particle decays via vector current to two "massless" fermions

up to m_μ / m_ψ corrections

angular analysis of decay pattern determines helicity of ψ

vector particle $\Rightarrow$ two spin zero particles

$\Rightarrow$ angular pattern of decays fully reconstructs helicity of K^*

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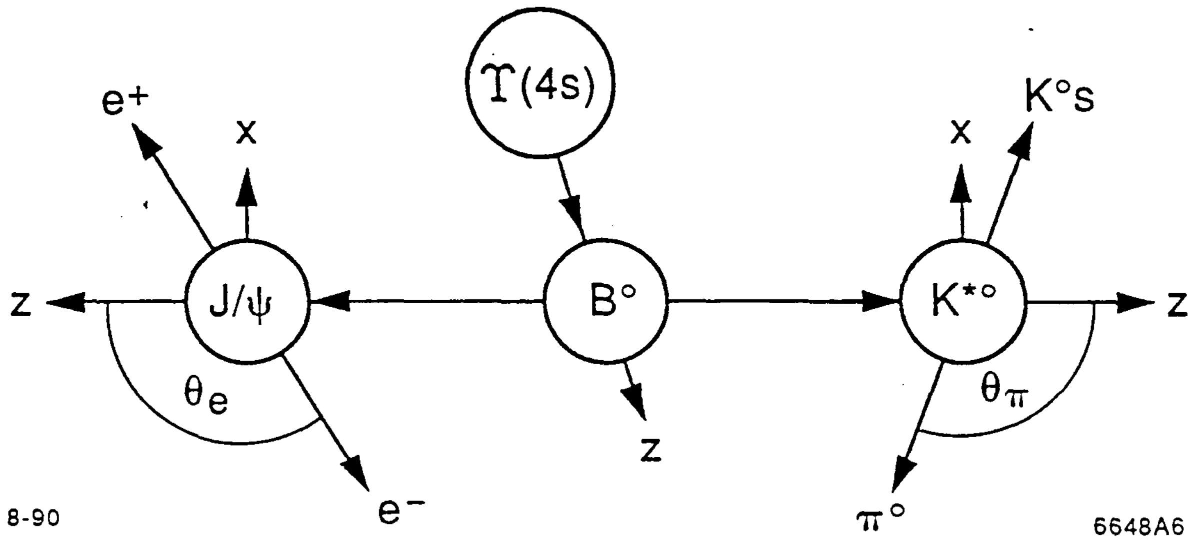
General Warning for angular treatments

— care with conventions is essential



—picture is deceptive, each decay is analysed in rest-frame of parent

ψ $K\pi$ define a plane

$\mu^+\mu^-$ (or e^+e^-) are not in the same plane (in general).

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12...

Several approaches

Theorists prefer

- Transversity — projection of helicities on direction transverse to $\psi K\pi$ plane
- Moments of angular distributions
  $\Rightarrow$ pick out particular helicity contributions

i.e. $\int d\Omega \, (\text{Rate}) \, Y_{lm}(\theta,\phi)$ projection

(These quantities are also useful for displaying results and checking stability of fits.)

Experiment will eventually use

- maximum likelihood fit to all data to extract helicity amplitudes
  i.e. multiparameter fitting

Note — isospin related charged channels may help constrain the relevant parameters, should be used!

e.g. $B^+ \to \psi K^{*+}$

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13.5

Now some grubby details

Jackson Convention
$R = (\phi, \theta, 0)$ rotations

Consider $B^0(t) \rightarrow \psi\,(K\pi)$ ← K^*

R_ψ θ, ϕ of ψ decay

$J_\psi = 1 \qquad J_{K^*} = 1$

R_{K^*} θ, ϕ of K^* decay

$$A_\alpha = \sum_{\lambda = 0, \pm 1} \sqrt{\frac{2J_\psi + 1}{4\pi}} \sqrt{\frac{2J_{K^*}+1}{4\pi}} \underbrace{D^{J_\psi *}_{\lambda,\alpha}(R_\psi)\, D^{J_{K^*}}_{\lambda, 0}(R_K)}_{\text{angular dependence}} A_\lambda(t)$$

$\alpha = \pm 1$ total lepton helicity in ψ decay

helicity of ψ and K^*

time dependence

$$\boxed{\text{Rate} = \sum_{\alpha = \pm 1} |A_\alpha|^2}$$

N.B. Factorization of angular dependence and time dependence

$\Rightarrow$ allows angular analysis for each time bin

(with sufficient data)

$$\text{Rate} = \left(\frac{3}{4\pi}\right)^2 \sum_{\alpha=\pm 1} \sum_{\lambda, \lambda' = \pm 1} A^{(t)}_{\lambda} A^*_{\lambda'}(t)\; D^{1*}_{\lambda\alpha}(R_\psi) D^{1*}_{\lambda'\alpha}(R_\psi) D^{1}_{\lambda 0}(R_{K}) D^{1}_{\lambda' 0}(R$$

Now we use a standard trick or two

$$D^{J\,*}_{m\,m'}(R) = (-1)^{m-m'} D^{J}_{-m\,-m'}(R)$$

and

$$\sum_{J_3=|J_1-J_2|}^{J_1+J_2}{}' (J_1 m_1\ J_2 m_2 | J_3 m_3)(J_1 m_1'\ J_2 m_2' | J_3 m_3') D^{J_3}_{m_3' m_3}(R)$$

J_3 for ψ = ~~J_2~~ side $= J_L$
J_3 for K side $= J_R$

$$= D^{J_1}_{m_1' m_1}(R)\, D^{J_2}_{m_2' m_2}(R)$$

this may look like a step backwards

but it lets us do the sum over α explicitly

$$|M|^2 = \left(\frac{3}{4\pi}\right)^2 \sum_{\lambda\lambda'} A_\lambda(t) A^*_{\lambda'}(t) \sum_{\alpha=\pm 1} (-1)^\alpha \sum_{J_L, J_R = 0,1,2}$$

$$(1\,\alpha\ 1\,{-\alpha} | J_L 0)(1\,\lambda\ 1\,{-\lambda'} | J_L M_L') D^{J_L}_{-M_L' 0}(R_\psi)$$

$$(1\,0\ 1\,0 | J_R 0)(1\,\lambda\ 1\,{-\lambda'} | J_R M_R') D^{J_R}_{-M_R' 0}(R_K)$$

Note $(1\,0\,1\,0|1\,0) = 0 \Rightarrow J_R = 1$ does not contribute

$$\sum_\alpha (-1)^\alpha (1\,\alpha\ 1\,{-\alpha} | J_L, 0) = 0 \text{ for } J_L = 0$$

13.7 so $J_L = 1$ does not contribute either.

So finally we have (after α sum)

$$\text{Rate} = -2\left(\frac{3}{4\pi}\right)^2 \sum_{\lambda\lambda'=0,\pm1} A_\lambda A^*_{\lambda'} \sum_{J_L,J_R=0,2}$$

$$(1\,1\,1\,{-1}\,|\,J_L\,0)(1\,\lambda\,1\,{-\lambda'}\,|\,J_L\,\lambda-\lambda')\;D^{J_L}_{\lambda-\lambda',0}(R_\psi)$$

$$(1\,0\,1\,0\,|\,J_R\,0)(1\,\lambda\,1\,{-\lambda'}\,|\,J_R\,\lambda-\lambda')\;D^{J_R}_{\lambda-\lambda',0}(R_{K^*})$$

Further we have $\boxed{D^L_{m,0}(R) = \sqrt{\frac{4\pi}{2L+1}}\,Y^*_{Lm}(\theta,\phi)}$

$\Uparrow$

explicit angular functions
(Jackson convention used)

From here on you can proceed in 3 ways

(i) transversity analysis

(ii) moments $\int d\Omega_\psi \int d\Omega_K (\text{Rate})\, Y_{JM}(\theta\varphi)_{K^*}\, Y_{J'm}(\theta\varphi)_L$

$\Uparrow$ $= T_{JJ'm}$

angular projections

(iii) fit for $A_\lambda(t) A_{\lambda'}(t) \sum_{J,J'} D^J_{\lambda-\lambda',0}(\theta,\varphi)_\psi\, D^{J'}_{\lambda\lambda',0}(\theta,\varphi)$

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in each t-bin

Transversity $\Longleftrightarrow$ simple angular moments

with transparent physical interpretation

τ = projection of helicities of

$\psi + K + \pi$ on an axis

$\perp$ (transverse) to plane of $\psi\ K\ \pi$

here clearly $\tau_K = \tau_\pi = 0$

(for cases where they are nonzero further decays may be used to fix them)

Note: Reflection about plane

$$R = P e^{i\pi J_T}$$

but $J_T = 0 = \tau + \ell$

and $P = P_{intrinsic} (-1)^\ell$

so $R = P_{intrinsic} (-1)^\ell (-1)^{\tau+\ell} = P_{intr} (-1)^\tau$

Also $CR = CP e^{i\pi J_T} = CP$

$= (CP)_{intrinsic} (-1)^\tau$

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Transversity projections $B \to \psi K^*$
(simpler moments)

- let θ = angle between e^- (μ^-)

and transverse axis to $\psi K \pi$ plane

$$\Gamma(\theta, t) = \text{Rate} = \Gamma_+(t)\, r_+(\theta) + \Gamma_-(t)\, r_-(\theta)$$

$\Gamma_+(t) \Rightarrow$ time dependent CP even rate

$\Gamma_-(t) \Rightarrow$ time dependent CP odd rate

$$r_+(\theta) = 3/8 \left(1 + \cos^2\theta\right)$$

$$r_-(\theta) = 3/4 \sin^2\theta$$

Thus integrating $\int_{-1}^{1} d\cos\theta \, \Gamma(\theta, t) = M_0$

$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$ $\int_{-1}^{1} d\cos\theta \, P_2(\cos\theta)\, \Gamma(\theta, t) = M_2$

$$\Rightarrow \quad M_0 = \Gamma_+(t) + \Gamma_-(t)$$

$$M_2 = \frac{\Gamma_+(t)}{10} - \frac{2}{5}\Gamma_-(t)$$

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13.10