Impedance and Beam Stability in the SLC

Damping Rings

See:

L. Risten, et al - 1988 EPAC

K. Bane - 1988 EPAC


R. Holzapple - Thesis - Bunch length measurements
B. Podobedov - Thesis - Saw tooth measurements

Impedance Calculations
Instability Calculations

Theory of Weak Instability in Resistive Machines
Longitudinal Wakefield Effects

- Potential well distortion → bunch lengthening
- Microwave instability
  - Increased energy spread
  - Bunch lengthening
- Transient phenomena - e.g. "saw-tooth"
- Heating of components

Effects on Downstream Linear Collider

- Longer bunch will have stronger wakefield effects in linac
- More initial energy spread will lead to more chromatic emittance growth
- Transient behavior will tend to amplify in linac
Want:
(1) a "green function" wakefield
(2) to analyze which objects are important

For most objects use time domain, Maxwell Eq solver, like MAFIA

![Diagram showing a green function bunch]

Typical bunch, length $L_z$

$3$ instability

to model instability need an oscillation over the bunch

$\Rightarrow \Delta z \leq \frac{L_z}{3}$

Note:
- for accuracy MAFIA requires mesh size $\Delta \leq \frac{\Delta z}{3}$

$\therefore$ for $30$ objects (eg septum) may be impossible to get an accurate green function

$\Rightarrow$ may need to use very simple models, or ignore

- for a few simple, small objects (eg. a small hole in beam tube

well) analytic formulas exist

see eg reports by S. Karenney

For a careful ring impedance calculation (which doesn't completely satisfy all these problems) see work on Daphne ring

Types of Impedances

And Inductive Example: $V_{\text{ind}} \sim -L \frac{dI}{dt}$

Note: $V_{\text{ind}} = eNW$

A Capacitive Example: $V_{\text{ind}} \sim \frac{1}{\varepsilon} \int I dt$

Model cavity
A Resistive Example: $V_{ind} \sim -RI$

One cell of SSL of cavity.
**SLC Damping Ring**

<table>
<thead>
<tr>
<th>History</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Old, old ring</td>
<td>original</td>
</tr>
<tr>
<td>Old ring</td>
<td>bellows shielded</td>
</tr>
<tr>
<td>Current ring</td>
<td>new vacuum chamber</td>
</tr>
</tbody>
</table>

**Old Ring**

**Layout**

![Diagram of SLC damping ring with notations](image)

*Circumference = 3.5 m*

**Fig. 6.** The girders of the SLC north damping ring.

**Fig. 7.** The cross-section of the bend chamber. The dashed circle shows the size of a quad chamber. **Bend chamber cross-section**
Fig. 8. The vertical profile of a QD segment (top) and a QF segment (bottom). The noncylindrically symmetric portions are drawn with dashes.
Table 1. The inductive vacuum chamber elements.

<table>
<thead>
<tr>
<th>Single Element Inductance</th>
<th>Contribution in Ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>L/(nH)</td>
</tr>
<tr>
<td>QD bellows</td>
<td>0.62</td>
</tr>
<tr>
<td>QD &amp; QF masks</td>
<td>0.47</td>
</tr>
<tr>
<td>QD &amp; QF trans.</td>
<td>0.52</td>
</tr>
<tr>
<td>Ion pump slots</td>
<td>1.32</td>
</tr>
<tr>
<td>Kicker bellows</td>
<td>2.03</td>
</tr>
<tr>
<td>Flex joint</td>
<td>0.18</td>
</tr>
<tr>
<td>1&quot; BPM trans.</td>
<td>0.10</td>
</tr>
<tr>
<td>Other</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 9. The geometries used to calculate $\ell$ for: (a) the QD bellows, (b) the QD mask, (c) the QD transition, and (d) the pump slots.

Resistive Objects

RF cavities - $R = 411 \Omega$
BPM cavities - $R = 227 \Omega$
Fig. 10. The longitudinal wakefield of a 1 mm Gaussian bunch in the SLC damping ring.

Fig. 13. The impedance $|Z/n|$ of the damping ring. The dots give what remains when the QD bellows (with their antechambers) are perfectly shielded. The power spectrum of a 6 mm Gaussian bunch is also shown.
Fig. 11. The longitudinal wakefield of a 6 mm Gaussian bunch in the SLC damping ring. The current distribution is also shown.

Fig. 12. The loss factor $k$ and the effective inductance $l$ of the damping ring as function of bunch length. The dotted curve gives the loss contribution of the rf cavities alone.
Potential Well Distortion

Below threshold the steady-state distribution is given by

the Haissinski Equation:

\[ I(t) = K \exp \left( \frac{-t^2}{2\sigma_0^2} + \frac{1}{V_+ \sigma_0^2} \int_0^\infty S(t') I(t-t') dt' \right) \]

with \( S(t) = \int_0^t W(c(t')) dt' \);

- \( \sigma_0 \) - nominal bunch length
- \( V_+ \) - slope of rf voltage
- \( K \) - normalizing constant

Note: \( V_+ \langle t \rangle = eNk_B T_0 \)

Boussard Criterion of threshold to strong instability

\[ \frac{\hat{I}^2 \sqrt{2m}}{2\pi \kappa E_0^2} < 1 \]

- \( \hat{I} \) peak current
- \( \kappa \) momentum compaction
- \( E_0 \) energy
- \( \sigma_e \) energy spread
- \( n \) w/wo, w/ revolution frequency

Note - independent of radiation damping time
Fig. 20. An inductive impedance: (a) the bunch shape for several values of bunch population and (b) the bunch length variation as a function of current.

Fig. 21. (a) The tune distribution and (b) the dependence of tune on $\tilde{x}$ and $h$ when $\Gamma = 7.5$ for an inductive impedance.
Fig. 26. (a) The bunch shape for various currents and (b) bunch shortening as a function of current, for a capacitive impedance.
Resistive Model

Fig. 23. A resistive impedance: (a) the bunch shape for several values of total charge and (b) the change of bunch length and centroid position (dots) with current. For analytic solution see, A. Ruggiero, et al., IEEE Trans. Nucl. Sci. NS-24, 1977.

Fig. 24. (a) The tune distribution and its integral (dashes) and (b) the dependence of tune on $\hat{x}$ and $h$ when $\Gamma = 3.4$ for a resistive impedance.
Fig. 28. The calculated damping ring bunch shapes for several current values, when $V_{rf} = 0.8$ MV. Superimposed on the curves are the measurement results.

Fig. 27. (a) Bunch lengthening and (b) the centroid shift calculated for the SLC damping rings at $V_{rf} = 0.8$ MV. The symbols indicate the measurement results.
Tracking

follow \((\epsilon_i, z_i)\) for 100,000's of macro-particles

\[
\begin{align*}
\Delta \epsilon_i &= -\frac{2 T_0}{\tau_d} \epsilon_i + 2 T_0 \sqrt{\frac{T_0}{\tau_d}} \upsilon_i + V_{r_i} z_i + V_{\text{ind}}(z_i) \\
\Delta z_i &= \frac{\alpha c T_0}{E_0} (\epsilon_i + \Delta \epsilon_i)
\end{align*}
\]

with \(V_{\text{ind}}(z) = -e N \int_{-\infty}^{\infty} W(z-z') \lambda_0(z') \, dz'\)

- \(T_0\): revolution period
- \(\tau_d\): damping time
- \(\alpha\): momentum compaction
- \(E_0\): energy
- \(r_i\): random number \(<r_i> = 0\), \(<r_i^2> = 1\)
- \(\epsilon_0\): nominal energy spread

Vlasov Eqn Solution

Computer program that solves perturbatively the time

independent Vlasov Eqn., including the effects of potential

well distortion, looking for unstable modes

Fig. 5. A snapshot of the beam, at two phases 180° apart, when \( N = 3.5 \times 10^{10} \).

Fig. 6. The shape of the unstable mode from two views at \( N = 3.5 \times 10^{10} \).
Fig. 3. The turn-by-turn skew when $N = 3.5 \times 10^{10}$ (a), and the rms when $N = 5.0 \times 10^{10}$ (b).

Fig. 4. The absolute value of the Fourier transform of the skew signal for two currents.

Fig. 7. The positions of the major peaks in the Fourier transform of the skew signal vs $N$. 
Frequency and Amplitude Dependence of Excited Modes as a Function of Beam Current with $V_e = 960$ kV

Near 6 $\nu$ values

<table>
<thead>
<tr>
<th>Frequency [kHz]</th>
<th>Amplitude [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>540</td>
<td>-100</td>
</tr>
<tr>
<td>550</td>
<td>-98</td>
</tr>
<tr>
<td>560</td>
<td>-96</td>
</tr>
<tr>
<td>570</td>
<td>-94</td>
</tr>
</tbody>
</table>

Near 3 $\nu$ values

<table>
<thead>
<tr>
<th>Frequency [kHz]</th>
<th>Amplitude [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>540</td>
<td>-88</td>
</tr>
<tr>
<td>550</td>
<td>-86</td>
</tr>
<tr>
<td>560</td>
<td>-84</td>
</tr>
<tr>
<td>570</td>
<td>-82</td>
</tr>
</tbody>
</table>

Measured

M. H. H. M. y
### Summary for Old Machine

\[ [\text{old, old} \rightarrow \text{old}] \] shielding bellows: \( N_{\text{tw}} = 1.5 \times 10^{10} \rightarrow 3.0 \times 10^{10} \)

**Old:**
- \( N_{\text{tw}}, \sigma \) vs \( N \): good agreement between calculation & measurement

<table>
<thead>
<tr>
<th>( \frac{N_{\text{tw}}}{N} )</th>
<th>Meas.</th>
<th>Calc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (V_{\text{th}}/V_{\text{so}}) )</td>
<td>3 ( \times 10^{10} )</td>
<td>1.5 ( \times 10^{10} )</td>
</tr>
<tr>
<td>2.6</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>( \frac{N_{\text{tw}}}{V_{\text{so}}}(10^{10}/N) )</td>
<td>+0.25</td>
<td>+0.27</td>
</tr>
</tbody>
</table>

- Saw tooth
  - Not reliably seen in simulations
  - Could not run SLC above threshold
New Vacuum Chamber - to shorten bunch length, increase threshold

Measurement: Bunch Lengthening in Old vs New Vacuum Chamber

![Graph showing Bunch Length vs Current](image)

Figure 2: Bunch length dependence on current. Bunch lengths are FWHM/2.35. \( V_{RF} = 800 \text{ kV} \).

- Shielding bellows: \( N_m = 1.5 \times 10^{10} \) to \( 3.0 \times 10^{10} \)
- In 1994 new, low impedance vacuum chambers were installed

- bunch lengthening reduced
- threshold went down from \( 3 \times 10^{10} \) to \( 1.5 - 2.0 \times 10^{10} \)
- coherent frequency just below \( 2V_0 \)
- less severe: ran routinely above threshold \( \approx 4.5 \times 10^{10} \);
  old machine could not run above threshold
The Bend-to-Quad Transitions

Fig. 3. The new bend-to-quad transition.
K. Oide has shown that a purely resistive machine is unstable, that it has a (weak) growth rate varying as \( \omega = \left| A^{\text{st}} \right| \), and that it can be stabilized by a small amount of inductance (Landau damping).

Whereas the normal (strong) instability is often characterized by two azimuthal modes coupling, this instability can be characterized by two radial modes, with the same azimuthal mode number, coupling.

Note: Boussard criterion does not apply to the weak instability.

- See Chao et al. double waterbag model. Asymmetry of mode is important.
- That such a mode can exist was never appreciated before.
Fig. 1. The wakefield used for the simulations.

Fig. 2. A potential well example.

Fig. 5. Average bunch properties vs $N$. Shown are tracking results (plotting symbols) and the Vlasov solution (curves).
$I = 4.5 \times 10^{10}$

![Graph showing distribution of $dN/dz/\left(10^{10}/\text{mm}\right)$ with 'Old Ring' label and axis labels: (tail), z/mm, (head).]
Fig. 3. The turn-by-turn rms energy spread just above threshold (a) and at a higher current. (b)

Fig. 4. $N_{th}$ vs. $\tau_d$ obtained by tracking.
mode of instability
$N = 4.5 \times 10^{10}$, New Ring, Simulation

Tracking

(b) View

Direction of wave motion
Fig. 7. Modes obtained by the Vlasov method.

Fig. 6. Mode shape at $N = 2 \times 10^{10}$.
Measurements of Sawtooth Behavior in New Chamber

- B. Podobedov
- B. Podobedov, R. Siemann,
  1997 Pac.

Figure 4. Oscilloscope traces of the instability signal for different values of stored charge.

Figure 3. Spectrum analyzer data vs. stored charge.
Summary for Current Machine

- Simulation: threshold (results) very sensitive to small amount of pure inductance (Landau damping)

\[ \Delta (2/n) = 0.1 \Rightarrow 0 \leq N_{th} = 1 \times 10^{10} \]

- if 0.1 \( \mu \) added, good agreement with measurement \( T_3, T_6 \), though large fluctuations in simulation for \( N > N_{th} \)

<table>
<thead>
<tr>
<th>Meas.</th>
<th>Calc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{th} )</td>
<td>1.5-20 \times 10^{10}</td>
</tr>
<tr>
<td>( \frac{1}{2} \lambda_{th} / \lambda_{so} )</td>
<td>1.77</td>
</tr>
<tr>
<td>( \frac{\Delta \xi}{\xi_{so}} (10^{10} / N) )</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

- Saw tooth 1 few percent of beam performs transient behavior
  - Not as serious as before: routinely operated above threshold (4.5 \( \times 10^{10} \))
  - Not reliably simulated
How can we understand that when impedance was reduced $N_k$ dropped?

**Old machine:**

inductive $\Rightarrow$ large tune spread $\Rightarrow$ weak instabilities are Landau damped

strong instability at $3 \times 10^{10}$

**New machine:**

resistive $\Rightarrow$ little tune spread $\Rightarrow$ we see a weak instability at $1.5 - 2.0 \times 10^{10}$

strong instability has not been seen

$I > 5 \times 10^{10}$ according to calculation

Why was weak instability not predicted?
New Machine

- average of Pe, but over bunch spectrum accurate
- unstable mode characteristics, eg $\frac{dv}{dt}$ agree pretty well
- Threshold?
- Sawtooth?

To avoid weak instability in NLC

- higher harmonic cavity - possible
- adjustable inductance (?) - preferably slots/holes in cavity walls

only a little fine spread if needed