“Emittance Preservation” in Beam Delivery Systems

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DESY

Snowmass 2001
Introduction

• Sources of Emittance Growth in BDS
• Time Scales
• Effects of Magnet Motion (Ground Motion)
  – Uncorrelated motion
  – Frequency Domain & feedback
  – “slow” drifts: ATL motion

Goal: to look at response of lattice independent of beam parameters
Sources of Emittance Growth in the BDS

- Wakefields from
  - collimator apertures
  - bellows
  - beam pipe (resistive wall)
- Synchrotron Radiation in magnets
- Optical aberrations due to magnet misalignments
  - time dependent
  - ground motion effects
Two effects wrt luminosity:
- beam-beam separation at IP
- beam size increase due to optical aberrations
  (emittance growth)

Two time scales:
- fast: those effects which occur too fast to correct
- slow: those effects which can be actively compensated.
Notes on Beam-Beam Effects

• Not considered in this talk
• Especially important for TESLA since
  – high disruption regime ($D_y = 25$)
  – lumi. very sensitive to beam-beam offsets
    (2% tolerance at $\Delta y \sim \sigma_y/10 = 0.5 \text{ nm}$)
  – so-called “banana” effects should be considered!
• Hence results shown for TESLA are probably optimistic!
Notes on Ground Motion

Long history of R&D and now relatively well understood

From A. Seryi, SLAC
## Important Parameters

<table>
<thead>
<tr>
<th></th>
<th>TESLA 2000</th>
<th>NLC stage 1</th>
<th>NLC stage 2</th>
<th>CLIC 2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{beam}}$ [GeV]</td>
<td>250</td>
<td>250</td>
<td>500</td>
<td>1500</td>
</tr>
<tr>
<td>$N_{\text{bunch}}$</td>
<td>2820</td>
<td>190</td>
<td>190</td>
<td>154</td>
</tr>
<tr>
<td>$T_{\text{train}}$ [$\mu$s]</td>
<td>950</td>
<td>0.27</td>
<td>0.27</td>
<td>0.1</td>
</tr>
<tr>
<td>$\Delta T_{\text{bunch}}$ [ns]</td>
<td>337</td>
<td>1.4</td>
<td>1.4</td>
<td>0.67</td>
</tr>
<tr>
<td>$f_{\text{rep}}$ [Hz]</td>
<td>5</td>
<td>120</td>
<td>120</td>
<td>100</td>
</tr>
<tr>
<td>$\delta_{\text{rms}}$ [%]</td>
<td>0.16 (0.04)</td>
<td>0.23 (±0.4)</td>
<td>0.23 (±0.4)</td>
<td>0.28 (±0.5)</td>
</tr>
<tr>
<td>$\sigma_y^*$ [nm]</td>
<td>5</td>
<td>2.7</td>
<td>2.1</td>
<td>1</td>
</tr>
</tbody>
</table>
Beam Delivery Systems

TESLA 500 GeV

\[ \sigma_{x,y} = 550/5 \text{ nm} \]
\[ \delta_{\text{rms}} = 0.16\% \text{ (electrons)} \]
\[ \delta_{\text{rms}} = 0.04\% \text{ (positrons)} \]

CLIC 3TeV

\[ \sigma_{x,y} = 43/1 \text{ nm} \]
\[ \delta_{\text{rms}} = 0.28\% \pm 0.5\% \]
Analysis of Magnet Motion

\[ y_j = \left( - \sum_{i=1}^{j} g_{ij} K_i y_j \right) - Y_i \]
Analysis of Magnet Motion

Original Equation

\[ y_j = \left( -\sum_{i=1}^{j} g_{ij} K_i Y_j \right) - Y_i \]

Defining *Response Matrix* \( Q \):

\[ Q = G \cdot \text{diag}(K) + I \]

Hence beam offset becomes

\[ y = -Q \cdot Y \]

*Covariance* of beam motion

\[ \langle y_i y_j \rangle = V_y = Q \cdot V_Y \cdot Q^T \]

\( V_Y \) contains all the information of the magnet motion (ground motion)
Uncorrelated Motion

- $V_{ij} = 0; i \neq j$
- $V_{ii} = \sigma^2$ (for all $i$)
- $V_Y = \sigma^2 I$

$Q \cdot Q^T$ is a matrix of amplification factors

Most interested in IP motion factor

→ last row vector of $Q$
IP Vertical Motion due to Uncorrelated Magnet Motion

<table>
<thead>
<tr>
<th></th>
<th>Entire Lattice</th>
<th>No Final Doublet</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>factor</td>
<td>$1\times \sigma$ motion</td>
</tr>
<tr>
<td>CLIC 3TeV</td>
<td>1.77</td>
<td>0.56 nm</td>
</tr>
<tr>
<td>TESLA 500GeV</td>
<td>2.17</td>
<td>2.3 nm</td>
</tr>
</tbody>
</table>

Need:
- Beam-based Feedback
- Active (mechanical) Stabilisation of Magnets
CLIC Quadrupole Tolerances

Motion required to give $\sigma/3$ at IP

Final doublet

Sensitivity ($\mu$m$^{-1}$)

Quadrupole #
Mechanical Stabilisation of Final Doublet (and other magnets)

- Mechanical stabilisation using piezoelectric mover
  - achieved $70 \rightarrow \sim 25$ nm $f >$ few Hz
    [Montag, DESY]
  - *move the field and not the magnet?*

- Use of vibration insulation stands
  - Factor of 20 achieved at PEP ($40 \rightarrow 2$ nm $f > 2$Hz)
    [Bowden, Mazaheri, SLAC]

- Optical Anchoring techniques
  - demonstrated resolution of 0.2nm [Woods, Mattison, SLAC]
  - as yet not used as part of active feedback

- Other areas interested too
  - e.g. gravitation wave antennas (LIGO, $10^{-14}$ m)

On-going R&D (cf CLIC stability study www.cern.ch/clic-stability)
## Emittance Dilution

<table>
<thead>
<tr>
<th>Magnet</th>
<th>Generator</th>
<th>Type</th>
<th>IP generator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrupole</td>
<td>$+ K \Delta y y\delta$</td>
<td>Linear Dispersion</td>
<td>$- R_{34} K \Delta y y'\delta$</td>
</tr>
<tr>
<td>Sextupole</td>
<td>$- S \Delta y x y$</td>
<td>x-y coupling</td>
<td>$- R_{12} R_{34} S \Delta y x' y'$</td>
</tr>
<tr>
<td>Skew-sextupole</td>
<td>$- S D_x \Delta y y\delta$</td>
<td>Linear dispersion</td>
<td>$+ R_{34} S D_x \Delta y y'\delta$</td>
</tr>
</tbody>
</table>

$x, y$ = phase space coordinates  
$\Delta y$ = beam offset wrt magnetic centre  
$K, S, H$ = integrated strength (quad, sextupole, octupole resp.)
RMS Linear Dispersion

• define the following vectors:
  \( K \)  quad strengths
  \( S \)  sextupole strengths
  \( D_x \)  horizontal dispersion function
  \( R_{34} \)  Green’s functions from magnets to IP

• define \( D = (S \otimes D_x - K) \otimes R_{34} \) such that \( D_y = D \cdot \Delta y \)

• Hence

\[
  D_y = -D \cdot Q \cdot Y \\
  <D_y^2> = (D \cdot Q) \cdot V_Y \cdot (D \cdot Q)^T
\]

• Uncorrelated:

\[
  <D_y^2> = \sigma^2 \ D \cdot Q \cdot Q^T \cdot D^T
\]

  similar expressions for coupling, waist shift etc.
RMS Aberrations for Uncorrelated Magnet Motion

\[
\begin{align*}
\Delta y_{rms} &= D_y \delta_{rms} \\
\Delta y_{rms} &= C_{xy} \theta_x^* \\
\Delta y_{rms} &= \eta D_{2,y} \delta_{rms}^2
\end{align*}
\]

\( D_y, C_{xy}, D_{2,y} \propto \sigma_{Y, rms} \)

<table>
<thead>
<tr>
<th>Lattice Factor</th>
<th>RMS motion for 2% tolerance (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>dispersion</td>
<td>25</td>
</tr>
<tr>
<td>TESLA 500GeV x-y coupling</td>
<td>585</td>
</tr>
<tr>
<td>2\textsuperscript{nd}-order dispersion</td>
<td>7754</td>
</tr>
<tr>
<td>dispersion</td>
<td>3.6</td>
</tr>
<tr>
<td>CLIC 3TeV x-y coupling</td>
<td>5161</td>
</tr>
<tr>
<td>2\textsuperscript{nd}-order dispersion</td>
<td>1264</td>
</tr>
</tbody>
</table>
Motion required to give 2% increase in $\sigma_y^*$
CLIC Comments

• RMS magnet motion quoted is the required uncorrelated motion

• Values should be achieved by application of
  – mechanical stabilisation or
  – beam based feedback correction or
  – both (most likely)

• ground motion: $p(f) \propto f^{-4}$, CLIC $f_{\text{rep}} = 100$ Hz
Beam-Based Feedback
(orbit correction)

\[ \theta_{n+p} = \theta_n - g \frac{y_n}{R_{34}} \]

\( p = 1 \) 1 pulse delay
\( p = 2 \) 2 pulse delay etc.

\( y_n = y_{n-p} (1 - g) + w_n - w_{n-p} + b_n - b_{n-p} \)

beam “jitter” BPM noise

gain factor
\( 0 < g \leq 1 \)
$p = 1$ Frequency Response

$g = 1.0$

$g = 0.5$

$g = 0.1$

$g = 0.01$

$f/f_{rep}$

2.5 Hz (TESLA)
60 Hz (NLC)
50 Hz (CLIC)
$p = 2$ Frequency Response

- 2.5 Hz (TESLA)
- 60 Hz (NLC)
- 50 Hz (CLIC)
Frequency Spectrum

Original Covariance
\[ \langle y_i y_j \rangle = [Q \cdot V_y \cdot Q^T]_{ij} \]

\[ \langle y_i y_j \rangle = \int_{-\infty}^{\infty} F\{y_i(t)\} F^*\{y_i(t)\} df \]

mutual (or cross) power spectrum

For Ground Motion:
\[ \langle Y_i Y_j \rangle = \int_{-\infty}^{\infty} F\{Y_i(t)\} F^*\{Y_i(t)\} df \]
\[ = \int_{-\infty}^{\infty} p(f) C_{ij}(f) df \]

absolute power spectrum

correlation function
Frequency Spectrum

Original Covariance
\[ \langle y_i, y_j \rangle = \mathbf{Q} \cdot \mathbf{v}_Y \cdot \mathbf{Q}^T \]_{ij}

\[ F\{y_i(t)\} F^* \{y_j(t)\} = p(f) \left[ \mathbf{Q} \cdot \mathbf{C}(f) \cdot \mathbf{Q}^T \right]_{ij} \]

Lattice Response ("filter")

SLAC Ground Motion Model:

Wave-like motion with phase velocity \( v(f) = 450 + 1900 \ e^{-f/2} \)

\[ C_{ij}(f) = J_0 \left( \frac{2\pi f \Delta L_{ij}}{v(f)} \right) \]
IP Motion Lattice Response

CLIC 3TeV
TESLA 500GeV

$G(f) \propto f^2$

uncorr. limit
Feedback and Rep. Rate
Beam-Beam Feedback

- Use beam-beam kick as offset signal
- Two types:
  - feedback $\leq f_{rep}$ (cf SLC)
  - intra-bunch-train (fast) feedback

TESLA:
2820 bunches in 950$\mu$s bunch train ($\tau_b = 337$ ns)
TESLA IP Feedback

- Fast (>300KHz) IP FDBK
- Forget tolerances based on $\Delta y^*$
- Tolerances set only by $\Delta \sigma_y$ considerations

(a) Separation Response

- offset feedback OFF
- offset feedback ON

Vertical Offset ($\Delta y/\sigma_y$) vs Bunch #
Status of NLC fast (intra-train) feedback
(reported by Philip Burrows and Glen White,
10/5/01, CERN, see also note LCC – 0056 03/01, Steve Smith)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLC bunch spacing:</td>
<td>1.4 ns</td>
</tr>
<tr>
<td>Vertical spot size:</td>
<td>2.7 nm</td>
</tr>
<tr>
<td>Number of bunches:</td>
<td>190</td>
</tr>
<tr>
<td>Bunch train length:</td>
<td>266 ns</td>
</tr>
<tr>
<td>(100 ns for CLIC)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Offset</th>
<th>Initial offset</th>
<th>Start of steering</th>
<th>Full luminosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 nm (3 $\sigma_y$)</td>
<td>after 36 ns</td>
<td>after 42 ns</td>
<td>(16 % of bunches)</td>
</tr>
<tr>
<td>100 nm (37 $\sigma_y$)</td>
<td>after 36 ns</td>
<td>after 120 ns</td>
<td>(45 % of bunches)</td>
</tr>
</tbody>
</table>

Beneficial for NLC and CLIC as well but not sufficient.
Slow diffusive ground motion: \( \sigma^2 = A \cdot T \cdot L \)

\[
\langle y_i y_j \rangle = [Q \cdot V_Y \cdot Q^T]_{ij}
\]

\[
V_Y = A \cdot T \cdot \begin{pmatrix}
z_1 & z_1 & z_1 & \cdots \\
z_1 & z_2 & z_2 & \cdots \\
z_1 & z_2 & z_3 & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix} \equiv A \cdot T \cdot Z_{ATL} \quad \text{assuming } z_i > 0
\]

\[
\langle y_n^2 \rangle = A \cdot T \cdot [Q_n \cdot Z_{ATL} \cdot Q_n^T]
\]

Effective ATL length

- TESLA 500 GeV: 15 m
- CLIC 3TeV: 28 m
# ATL Lattice Factors

Define: $\langle \Delta y^2 \rangle = \Delta Y \cdot A \cdot T$

Where:

- $\Delta Y$ beam-beam offset (effective length)
- D Dispersion
- C x-y Coupling ($\Delta y \propto \theta_x$)
- $D_2$ 2nd-order dispersion ($\Delta y \propto \delta^2$)

**Two “Extremes”**

<table>
<thead>
<tr>
<th></th>
<th>$\Delta Y$</th>
<th>D</th>
<th>C</th>
<th>$D_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TESLA 500 GeV</td>
<td>15</td>
<td>4477</td>
<td>$3 \times 10^6$</td>
<td>$6 \times 10^7$</td>
</tr>
<tr>
<td>CLIC 3 TeV</td>
<td>28</td>
<td>878</td>
<td>$5 \times 10^8$</td>
<td>$2 \times 10^6$</td>
</tr>
<tr>
<td>$A^*T = 10^{-4} \mu m^2/m$</td>
<td>nm</td>
<td>% increase in $\sigma_y$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TESLA 500 GeV</td>
<td>36</td>
<td>2</td>
<td>0.82</td>
<td>0.16</td>
</tr>
<tr>
<td>CLIC 3 TeV</td>
<td>53</td>
<td>34</td>
<td>0.00</td>
<td>0.49</td>
</tr>
</tbody>
</table>
ATL Simulation Results for TESLA

Effect of ATL Ground Motion on Luminosity

- No Feedback
- IP Feedback Only
- IP+Orbit Correction

A = 4 \times 10^{-6}
ATL Simulation Results for TESLA

\[ A = 4 \times 10^{-6} \]
ATL Simulation Results for CLIC

Effect of ATL Ground Motion on Luminosity

A = 5\times10^{-7}
ATL Simulation Results for CLIC

A = 5\times10^{-7} \, \mu \text{m}^2/\text{m/s}

1 \mu \text{m BPM noise}
taking dispersion dominates:

\[ \frac{\Delta L}{L} \propto C_{lattice} AT \left( \frac{\delta_{RMS}}{\sigma_y^*} \right)^2 \]

suggests:

\[ C_{NLC} \approx 2 \times C_{TESLA} \]

\( A = 5 \times 10^{-7} \)
Summary

• Response of Lattices of all machines are similar
  – “Raimondi” style FF seems better wrt dispersion, but worse wrt X-Y coupling (sextupoles)
• Performance (tolerance to magnet motion) dominated by beam parameters ($\varepsilon_y$, $\delta$)
• Low rep. rate (5Hz) of TESLA is disadvantage
  – fast IP feedback solves beam-beam offset problem, BUT
  – optical aberrations are still an issue
• Improved simulations with more ‘realistic’ models
  – better ground motion simulation (at least for TESLA/CLIC)
  – effects of supports etc
  – Cultural noise sources etc
• Continued R&D on stabilisation techniques
  (fc CLIC study)