Sources of Emittance Dilution

- **Conservative and non-conservative dilutions**
- **Non-conservative**
  - Synchrotron radiation – relevant in damping rings and BDS
  - Beam-gas scattering – not an issue for emittance
- **Conservative**
  - Dispersion
  - Wakefields; CSR
  - Betatron coupling; beta-matching
  - Multi-bunch couplings \((W_{\perp}, \text{photo e-}, \text{ions, etc.})\)
  - Nonlinearities; space charge; photo e-; ions
  - Etc.
- **Conservative dilutions just couple degrees of freedom and increase the projected emittance; the 6-D emittance is not changed and the couplings can be removed (in theory)!**
Filamentation

- Injection trajectory and betatron mismatches will filament due to chromaticity or nonlinear fields.
- Betatron mismatches can be characterized in terms of $B_{mag}$:

$$\Delta \gamma \epsilon \approx \gamma \epsilon (B_{mag} - 1) \quad B_{mag} = \frac{1}{2} \left[ \left( \frac{\beta}{\beta^*} + \frac{\beta^*}{\beta} \right) + \left( \alpha^* \sqrt{\frac{\beta}{\beta^*}} - \alpha \sqrt{\frac{\beta^*}{\beta}} \right)^2 \right]$$

* parameters are for the beam
Damping Rings

- Two primary terms:
  - Local increase of the beam spot (and divergence) at extraction
  - Increase of the beam emittance due to synchrotron radiation or IBS which is coupled to the vertical plane by some aberration
  
  \[
  \frac{\sigma_y^2(s)}{\beta_y(s)} = \epsilon_y + \frac{\sigma_{y\text{local}}^2(s)}{\beta_y(s)}
  \]

  - In the weak coupling limit, \( \epsilon_y \) is invariant around the ring while the local increase depends on the local betatron coupling and dispersion
  - The local increase can be corrected after extraction provided this is done before the correlations are lost

  - There is also a limiting contribution from the radiation opening angle but this small:
    
    \[
    \gamma \epsilon_y \approx 0.24 J_\epsilon \overline{\beta_y} \frac{\sigma^2}{\gamma}
    \]
Vertical Dispersion

- Vertical dispersion generated by sextupole misalignments and quadrupole rotations in regions of $\eta_x$ and orbit errors
  - Usually tightest tolerance on sextupoles
  - Chromaticity correction reduces effect of orbit – do not over-correct
  - Simple estimates for random errors
    \[
    \frac{\langle \eta_y^2 \rangle_{\text{sext misalign}}}{\beta_y} = \frac{1}{8 \sin^2 \pi \nu_y} \sum_{\text{sext}} (K_2 L)^2 y_m^2 \beta_y \eta^2_x
    \]
    - Resonant denominator arises from periodicity – stay away from the integer?

- Expected effect on emittance is larger than that on local beam size
  \[
  \frac{\langle \sigma_y^2(s) \rangle_{\text{local}}}{\beta_y(s)} = \frac{\langle \eta_y^2(s) \rangle}{\beta_y(s)} \sigma^2_{\epsilon} \quad \langle \epsilon_y \rangle = 2 \frac{C_g \gamma^2}{J_y} \frac{\langle \eta_y^2 \rangle}{\beta_y} \int |G|^2 ds = 2J_\epsilon \frac{\langle \eta_y^2 \rangle}{\beta_y} \sigma^2_{\epsilon}
  \]
Betatron Coupling

- Betatron coupling generated by sextupole misalignments, quadrupole rotations, and orbit errors in sextupoles
  - Again, usually tightest tolerance on sextupoles
  - Two terms: sum and difference contributions

\[
\frac{\langle \sigma_y^2 \rangle_{\text{local}}}{\beta_y} = \frac{\epsilon_x}{4} \frac{1 - \cos 2\pi \nu_x \cos 2\pi \nu_y}{\left( \cos 2\pi \nu_x - \cos 2\pi \nu_y \right)^2} \sum_{\text{sext}} (K_2 L)^2 y_m^2 \beta_x \beta_y
\]

\[
\langle \epsilon_y \rangle = \frac{\epsilon_x \alpha_x}{4 \alpha_y} \frac{1 - \cos 2\pi \nu_x \cos 2\pi \nu_y}{\left( \cos 2\pi \nu_x - \cos 2\pi \nu_y \right)^2} \sum_{\text{sext}} (K_2 L)^2 y_m^2 \beta_x \beta_y
\]

- Cosine expression equal sum over sum and difference resonances:

\[
\frac{1}{\sin^2 \pi \Delta \nu_-} + \frac{1}{\sin^2 \pi \Delta \nu_+}
\]
Emittance Correction (1)

- Lots of different techniques to measure and correct residual dispersion and coupling
  - Present generation diagnostics allow for very accurate measurements and construction of beam transport matrices
  - Tolerance on rms $\eta_y$ is 1~2mm in NLC ring
    - Variation of beam energy by 0.3% with BPMs having 1µm resolution gives plenty of resolution for global correction techniques
  - Correction of betatron coupling when far from coupling resonance is probably most effectively performed by measuring 4-D transport matrix
    - Minimization of the tune separation is done at a different set-point and does not treat contribution from the sum resonance
Emittance Correction (2)

- Must correct dilutions locally at all sources of radiation/scattering to fully correct emittance
- Simple correction effectively removes resonant denominator
  - Using one pair of correctors separated by $90^\circ$ to correct the average of $\hat{H}_y$, one can expect to reduce the emittance by:
    \[ \langle \epsilon_y \rangle = \frac{2}{3} \sin^2 \pi \nu_y \langle \epsilon_y \rangle_0 \]
  - Be careful – correction to zero $\hat{H}_y$ at one point, for example extraction, can increase average error and thus emittance
  - Additional correctors decreases residual error slowly although can be very effective for special areas such as wigglers
  - Similar performance with 4 skew quadrupoles to correct betatron coupling
Distribution of Machines

- Tolerances are set to ‘ensure’ that emittance will be attained
- Random errors in one degree-of-freedom, ie. dispersion ($y-\delta$) or betatron coupling ($y-x$), generate an approximately exponential distribution in the emittance
- The 95% c.l. occurs at $3\sigma$ in an exponential distribution
Other Issues (1)

• Transients!
  – Linacs are very sensitive small changes in injected beam parameters
  – Non-uniform fill causes transient beam loading and variations in trajectory and tunes along bunch train
  – Tune variation can make emittance correction and dynamic aperture difficult
  – Instabilities are not a good thing!

• Stability
  – Corrections are difficult and time consuming to implement
  – Must maximize stability of trajectory
  – After establishing a ‘gold’ trajectory, stability of BPMs and sextupoles are the most important
  – Place BPMs in sextupoles??
Other Issues (2)

- **Circumference control**
  - Extraction energy must be maintained at fraction of $\sigma_{\Delta E/E}$
  - Small momentum compaction means large sensitivity of energy to circumference: $\frac{\Delta E}{E} = \frac{1}{\alpha} \frac{\Delta C}{C}$
  - Tolerance on circumference is roughly 10$\mu$m in NLC ring – need feedback and chicane for control!

- **Intrabeam Scattering**
  - Expected to increase emittance by 10~20% in NLC ring and similar(?) in TESLA ring – discrepancy between theory and KEK ATF measurements may be resolved but need additional data
Other Issues (2)

- **Electron cloud and ions**
  - These can drive single bunch dilutions as well as multi-bunch dilutions and tune variations

- **Space charge**
  - Can decrease dynamic aperture due to tune spread, increase vertical emittance due to coupling, and provides a 4\textsuperscript{th} order coupling term which can increase the vertical emittance

- **Wiggler systems**
  - Large wigglers generate nonlinearities and massive radiation – are there any collective effects not seen in normal rings?
Linac Dilutions

- **Trajectory and energy jitter**
  - Sets quad vibration and rf stability (esp. in bunch compressors)

- **Dispersion errors**
  - Sets quad-to-bpm alignment for $\Delta \varepsilon$
  - Three E sources: incoming $\Delta E$, E jitter, and BNS energy spread

- **Short-range wakefields**
  - BBU $\Rightarrow$ BNS - requires rf energy overhead or rf quadrupoles
  - Sets tolerances on structure-to-beam alignment for $\Delta \varepsilon$

- **Long-range wakefields**
  - BBU $\Rightarrow$ wake reduction - sets dipole frequency errors
  - Sets internal structure alignment tolerances for $\Delta \varepsilon$
  - Sets charge tolerances

- **Other (coupling, bookshelf, slow component motion)**

- **Dynamics in linacs are ‘simple’ compared to rings**
BBU and BNS Damping (1)

- Short-range wakefield will amplify incoming trajectory jitter
- Single bunch BBU well studied theoretically and experimentally – analytic models are quite good
- BNS technique prevents BBU due to short-range wakefields
- Add correlated energy spread to compensate dipole wakefield deflection with quadrupole chromaticity or use rf quadrupoles
  - Works locally in smooth focusing and on average in strong focusing

\[
\frac{1}{\gamma} \frac{d}{ds} \gamma \frac{d}{ds} y(s, z) - (1 - \delta)K(y - y_q) = (1 - \delta)G - 4\pi \epsilon_0 N r_e \frac{1 - \delta}{\gamma_0} \\
\times \int_{z}^{\infty} W_{\perp}(s, z' - z) (y(z) - y_a) \rho(z') dz'
\]
BNS Damping (2)

- Different regimes: autophasing; BNS damping; filamentation

Rigid oscillation: \[ \delta_{\text{auto}}(z) = \frac{4\pi e_0 N r_e}{K \gamma_0} \int_z^\infty W(z' - z) \rho(z') \, dz' \]

Gaussian bunch with \( \sigma_z = 100 \mu m \)

- BNS energy spread also reduces growth of other correlations: y-z tilts etc.
BNS Damping (3)

- BNS damping reduces sensitivity to disturbances along the linac – cannot operate linac with significant BBU
  - single bunch is handled with BNS presently induced by running off the rf crest - requires energy overhead
  - rf quadrupoles allow for more uniform compensation and looser quadrupole tolerances but tight tolerances on rf quad alignment
Multiple Bunch BBU

- Long-range wakefield drives coupled bunch BBU
- First observed at SLAC in 60’s (sometime) – well studied!
- Primary solution is direct reduction of (averaged) wake between bunches, ie. detuning mode freq. or damping

- Calculation of MB-BBU is difficult: nearest neighbor or daisy chain model, single mode models, etc.

\[
\left| \frac{2\pi \epsilon_0 N r_e W_\perp (\Delta_b) \beta_i}{\alpha G} \right| \left( \frac{\gamma_f}{\gamma_i} \right)^\alpha < 1
\]

For NLC parameters,

\[ W_\perp (\Delta_b) \leq 1\% \quad W_\perp \]

- Difficult to get accurate representation of wakefield
- Computer simulation is straight-forward
- Three DDS-style structures have been built and tested
- Wakefield model agrees well with measurements

- Lowest band is around 15 GHz
- Next band (25 GHz) is visible after 1.4 ns
‘BNS’ for Long-Range Wakefield

• LR wake tends to be much larger than SR: \( \frac{\lambda}{6} \gg \sigma_z \)

• If magnitude can be made comparable to that of the short-range across a bunch

\[ \Rightarrow \text{Use BNS damping-like technique, i.e. add bunch-to-bunch energy deviation to bunch train - actually more detuning than BNS damping but ...} \]

• Long-range longitudinal wake and rf compensation provides route: need 0.5% rms for ‘worst-case’ DDS3 wakefield
Jitter and Static Errors

- Linear colliders cannot operate in BBU regime!
- If BBU is small and operating close to autophasing condition, transport matrices are close to nominal optics
- First-order perturbations are all that are important!
- Beam jitter arises from incoming errors and quadrupole motion (faster than that correctable)
  - Typical jitter tolerances are 10nm in NLC and TESLA
- Correct ‘static’ alignment errors to reduce emittance dilution
- Alignment scales are roughly ~1000 times larger than beam jitter scales: $1/\sigma_{\Delta E/E}$
Example ‘Bare’ Machine Tolerances

- ‘Bare’ machine tolerances are the component alignment tolerances without any non-local tuning or correction
  - NLC is designed with emittance correction like storage rings are designed with orbit correction (many rings use $\eta$-correction)

Bare Tolerances for 500 GeV cms

<table>
<thead>
<tr>
<th></th>
<th>Injector</th>
<th>Main Linac</th>
<th>BDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quad jitter</td>
<td>10 nm</td>
<td>10 nm</td>
<td>1-10 nm</td>
</tr>
<tr>
<td>Quad alignment</td>
<td>10 $\mu$m</td>
<td>2 $\mu$m</td>
<td>2-5 $\mu$m</td>
</tr>
<tr>
<td>Structure</td>
<td>40 $\mu$m</td>
<td>10 $\mu$m</td>
<td>-</td>
</tr>
<tr>
<td>alignment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quad roll</td>
<td>100 $\mu$r</td>
<td>300 $\mu$r</td>
<td>300 $\mu$r</td>
</tr>
<tr>
<td>$\Delta\varepsilon/\varepsilon$</td>
<td>10%</td>
<td>40%</td>
<td>25%</td>
</tr>
</tbody>
</table>
Local and Non-Local Alignment

• To preserve emittance must correct net effect of individual dilution sources

• ‘Local’ correction - directly correct dilution sources
  – Beam-based alignment
  – Most robust solution / least sensitive to energy or strength errors

• ‘Quasi-Local’ correction - correct dilution effects over short distance, i.e. betatron wavelength
  – Dispersion-Free steering
  – Based on ‘measurements’ of dilution / sensitive to systematics

• ‘Global’ correction - tune emittance using direct ε diagnostics
  – Directly corrects desired quantity / sensitive to phase advance
Emittance Correction (Global Correction)

• Most sources of emittance dilution come from conservative processes: transverse wakefields, dispersive errors, etc.

⇒ 6-D phase space *does not* increase
(only the projected phase space increases)

• Any conservative dilution can be removed but this is difficult after phase space mixing from betatron frequency variations

• Filamentation is significant in NLC
Examples of Global Correction

• Create knobs for orthogonal control of dilution:

• Dispersion:
  – Quadrupole and skew quadrupoles in regions of dispersion
  – Sextupoles for 2nd-order dispersion
  – Betatron oscillations with acc. structures aligned to beam

• Wakefields and RF deflections
  – Movable accelerator structures
  – RF deflecting cavities

• Multi-bunch wakefields / alignment
  – Fast kickers (350 MHz)

• Betatron coupling
  – Skew quadrupoles or rotational control on quadrupole magnets
Dispersion errors arise from misaligned quadrupoles and/or BPMs, rf and wakefield deflections, and stray fields and beam energy spread.

In a 90° FODO lattice, correction to a single offset BPMs generates a dispersion with amplitude ~4x the BPM offset.
Dispersive Errors (2)

- Component of errors that is important is that at the betatron oscillation frequency – insensitive to long wave lengths
Dispersive Errors (3)

• **Different approaches to problem:**
  - Local correction – measure offset of individual quadrupole magnetic center to trajectory (and local BPM) by shunting or similar technique
  - Quasi-local – measure offsets of quadrupoles by varying beam energy or the strength of strings of quadrupole
    - Can be used to correct for errors other than just quadrupoles
    - Sensitive to systematics
  - Global correction – measure beam emittance and minimize by introducing beam bumps, skew quadrupoles in regions of $\eta_x$, etc.
    - Sensitive to variation of phase advance along linac and filamentation

• **Limitations due to:**
  - ‘Alignment’ resolution scales with BPM resolution
  - Variation of quadrupole centers with excitation
  - Knowledge and stability of beam optics
Wakefield Dilutions (1)

- Transverse wakefields are very strong functions of rf frequency: \( dW_\perp/ds \sim 1/a^4 \quad W_\perp \sim 1/a^3 \)

- Short-range wakefield depends on average offset of beam in structure irises
  - For an offset structure: \( \Delta \varepsilon \approx 0.9 \frac{\beta}{8\pi} y^2 \left( \frac{Nre}{\gamma} W_\perp (2\sigma_z) \right)^2 L_a \)
  - Length scales and alignment procedures are important: random offsets of single cells is less important that offsets of all structures between quadrupoles
  - Ability to measure structure offsets and move structures to beam trajectory can remove this sensitivity
Wakefield Dilutions (2)

- Long-range wakefield has similar dependence although some details differ
  - Replace $W_\perp(2\sigma_z)$ with the rms ‘sum’ wakefield $4W_{\text{rms}}$:
    \[
    W_{\text{rms}}^2 = \frac{1}{n_b} \sum_{j=0}^{n_b-1} W_{\text{sum}}^2(j\Delta s) - \left(\frac{1}{n_b} \sum_{j=0}^{n_b-1} W_{\text{sum}}(j\Delta s)\right)^2
    \]
    \[
    W_{\text{sum}}(n\Delta s) = \sum_{j=0}^{n} W_\perp(j\Delta s)
    \]
  - However offset now depends on ‘location’ of dipole mode along the structure
  - Internal deformation of structures can reduce effectiveness of detuning techniques
  - Frequency errors in fabrication can help or hurt
In NLC, length scales are different for short-range and long-range wakefields because of detuning techniques.
• Dipole frequency errors will cause a breakdown of the intentional detuning but can also help

• Different categories of errors with different effects:
  – random/random -- random cell-to-cell and random from structure-to-structure (very likely)
  – systematic/random -- systematic error in each cell but random from structure-to-structure (depends on manufacturing model)
  – systematic/systematic and random/systematic -- arise from modeling errors (these are most damaging -- important to verify model and perhaps design in errors in manufacturing!)

• TESLA is relying on random frequency errors to generate detuning
Alignment Schemes (1)

- NLC first aligns quadrupoles and then aligns structures to the beam trajectory
  - Have to iterate a few times to reduce kicks from structure moves
• For NLC – further details from Peter Tenenbaum
• Conventional alignment at installation and after long down periods
  – 100 µm rms variation of magnets and girders over 100 m lengths
• Establish ‘gold’ trajectory using local and/or quasi-local correction techniques
  – Use quadrupole shunting techniques align BPMs to quadrupoles at the level of 1 to 20 µm depending on variation of quadrupole centers
  – Dispersion-Free steering to establish a ‘gold’ trajectory which minimizes the dispersion - need to correct $\eta$ to 100 µm level
• Steer to ‘gold’ trajectory as magnets move (few hr timescale)
  – Use feedbacks to maintain trajectory between steering
• Beam-bumps to tune emittance based on emittance meas.
  – Ideally create ‘dither’ feedbacks to tune emittance bumps
NLC - The Next Linear Collider Project

Alignment Schemes (3)

- TESLA fixes structures and steers between quadrupoles
  - Same technique as used in SLC
  - Results are sensitive to details of alignment and steering approach
    - If reasonable alignment – steer to BPMs at focusing quadrupoles only
    - Alignment of structure girders more important than alignment of cavities
Diagnostic Requirements

• Resolution of alignment procedures depends on diagnostics!

• Lesson from SLC: diagnostics and control
  – Need 300 nm Beam Position Monitor resolution
    • FFTB/SLC FF striplines have 1 µm resolution
    • FFTB RF cavity BPM had 25 nm resolution
  – Need 10 µm accelerator structure alignment
    • DDS3 and RDDS1 S-BPM has 1-2 µm absolute precision
  – Need beam size resolution of 300 nm
    • SLC laser wire had between 500 and 230 nm resolution
    • FFTB ‘Shintake’ BSM had 40 nm resolution
  – Need magnet movers with 50 nm step size (for $L$ during steering)
    • FFTB magnet movers have 300 nm step sizes
• Microns are small numbers!
• Alignment/correction procedures are only as good as diagnostics
• Damping ring req. are close to tolerances attained elsewhere
  – Stability is very important!
• Linac issues are well understood
• Dispersion in NLC linac must be limited to $\sim 100\mu m$
  – With a 10% energy change, this implies a 10$\mu m$ trajectory change
  – More difficult because cannot correct entire linac at once
    • 25 sections $\rightarrow$ 2$\mu m$ trajectory changes
• Global emittance correction is sensitive to optics changes
  – Maintain energy profile along linac