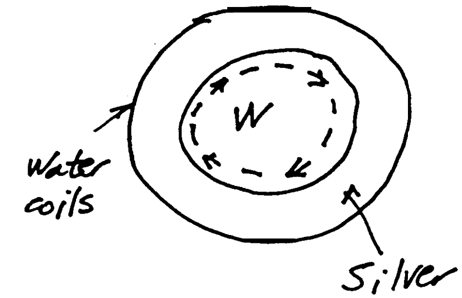
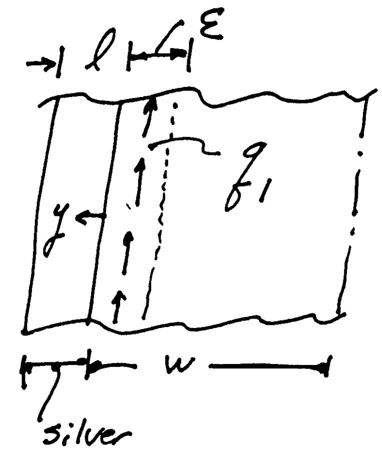
FORMULATION

TWO SOLUTIONS SUPERIMPOSE (LINEAR EQUATIONS):



T_1 = FIRST IS FOR A CONTINUOUS ANNULUS OF HEATING ACROSS PATH WIDTH ε (ABOUT 4.4 KW).

T_2 = SECOND IS FOR AN INDIVIDUAL PULSE MOVING AT VELOCITY u AND HAVING PULSE STRENGTH Q_p

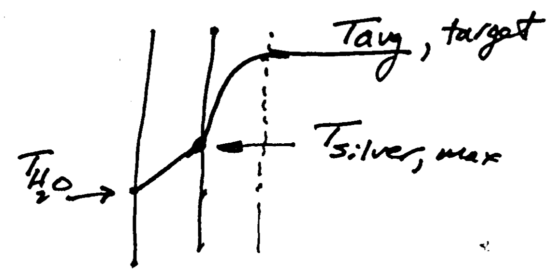


$T_1 = T_1(y)$

$$\frac{d^2 T_1}{dy^2} = \begin{cases} 0, & y < -\varepsilon \\ q_1/k_w, & -\varepsilon < y < 0 \\ 0, & 0 < y < l \end{cases}$$

obtain: For $-\varepsilon < y < 0$:

$$T_1 = T_{avg,target} + \frac{q_1}{2k_w}(\varepsilon^2 - y^2) - \frac{q_1}{k_w}\varepsilon(y+\varepsilon)$$



$0 < y < l$:

$$T_1 = T_{avg,target} \quad \frac{q_1 \varepsilon y}{k_{silver}} - \frac{q_1 \varepsilon^2}{2k_w}$$

Set $T_1 \cong T_{H_2O}$ to obtain $T_{avg,target}$ for $y = l$

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6/28/99

(2)

$$q_i = \frac{Q_{TOTAL}}{\pi D_o \cdot 2\varepsilon \cdot Z} = \frac{4.4\ KW \times 3{,}413\ \frac{B/h}{KW}}{\pi\left(\frac{2.5}{12}\right)\left(\frac{0.12}{30.48}\right)\frac{0.4}{12}\ ft^3} = 1.7 \times 10^{+8}\ \frac{B/h}{ft^3}$$

$$\varepsilon \cong \frac{0.12}{30.48} = 0.004\ ft \quad ; \quad \ell = 0.10\ mil = 0.0083\ ft$$

$$k_w = 95\ \frac{B}{h\ ft\ {}^\circ F} \quad ; \quad k_{silver} \cong 200\ \frac{B}{h\ ft\ {}^\circ F} \quad ; \quad T_{H_2O} \cong 60^\circ F$$

$$T_{avg,\ target} = T_{H_2O} + \frac{q_i \varepsilon^2}{2k_w} + \frac{q_i \varepsilon \ell}{k_{silver}}$$

$$= 60 + \frac{(1.7 \times 10^8)(0.004)^2}{2(95)} + \frac{(1.7 \times 10^8)(0.004)(0.0083)}{200}$$

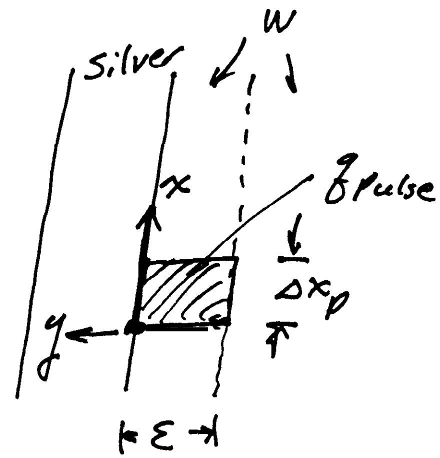
$$= 60 + 14 + 28 = 102^\circ F = 39^\circ C$$

too low

$T_2 = T_2(y, x)$ for an individual, moving pulse

$-\varepsilon < y < 0$:

$$(1) \quad \frac{\partial^2 T_2}{\partial x^2} + \frac{u}{\alpha_w}\frac{\partial T_2}{\partial x} + \frac{\partial^2 T_2}{\partial y^2} = \begin{cases} 0, & -\infty < x < 0 \\ -\dfrac{q_{pulse}}{k_w}, & 0 < x < \Delta x_p \\ 0, & x > \Delta x_p \end{cases}$$



$0 < y < \ell$:

$$(2) \quad \frac{\partial^2 T_2}{\partial x^2} + \frac{u}{\alpha_{silver}}\frac{\partial T_2}{\partial x} + \frac{\partial^2 T_2}{\partial y^2} = 0$$

Coordinates are fixed on heating spot, target travels under at u

Subject to: $\frac{\partial T_2}{\partial y} = 0$ on $y = -\varepsilon$

$T_2 = 0$ on $y = \ell$

because boundary conditions for T_1 and T_2 must also superpose.

The complete solution is then $T_1 + T_2$, so cooling coil fluxes are additive

- Note that the solution for T_1 need not be performed by numerical means since a closed form solution is available.
- Solution for T_2 to be obtained numerically. Another solution strategy for T_2 is by Fourier transforms.

Fourier transform:

$$\forall(\omega, y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\omega x}\, T_2(x,y)\, dx$$

Inversion $\Rightarrow$ $$T_2(x,y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega x}\, \forall(\omega,y)\, d\omega$$

$$\mathcal{F}\left(\frac{\partial T_2}{\partial x}\right) = -i\omega \forall \quad ; \quad \mathcal{F}\left(\frac{\partial^2 T_2}{\partial x^2}\right) = -\omega^2 \forall$$

(1) transforms to an ODE:

$$\frac{d^2 \forall_a}{dy^2} - \omega\left(\omega + i\frac{u}{\alpha_w}\right)\forall_a + \frac{i\, q_{pulse}}{\sqrt{2\pi}\,\omega}\left(1 - e^{i\omega x_p}\right) = 0$$

(2) transforms to another:

$$\frac{d^2 \forall_b}{dy^2} - \omega\left(\omega + i\frac{u}{\alpha_{silver}}\right)\forall_b = 0$$

At $y=0$: $k_w \dfrac{d\forall_a}{dy} = k_{silver}\dfrac{d\forall_b}{dy}$ and $\forall_a = \forall_b$

Do inversion numerically (by integration)

$\boxed{\text{FEEDBACK ON } \Delta T_{\text{cooling water}}}$