

Appendix 2

THE HEAT CONDUCTION CALCULATIONS

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To define the temperature distribution in conical part of concentrator, at first, the problem of heat conduction between two planes (See Fig. 1) was solved in case of point heat source.

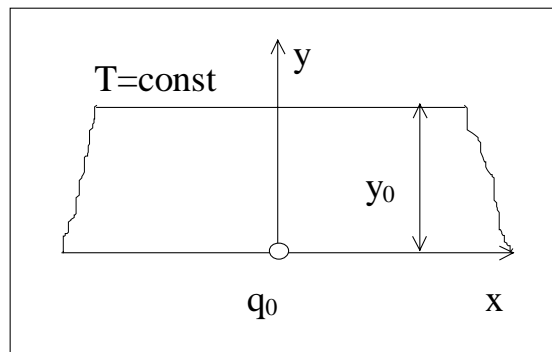


Fig. 1. Plane geometry

One plane is held under constant temperature, the heat removal is realized from this plane. The point heat source is situated on the another plane. Really the heat deposition has some distribution on the surface of concentrator, but if we solve problem for point heat source we will solve problem for any distribution of heat deposition. The process of heat conduction is described by the following equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -\delta(x)\delta(y) \cdot \frac{q_0}{\kappa} \quad (1)$$

Where T – is the temperature, q_0 – is the total power of the heat source ($q_0 \cong 500 \text{ W/cm}^3$ at $H_{max} = 10T$), κ – is the thermal conductivity of material ($\kappa_{Cu} = 3.9 \frac{\text{W}}{\text{cm} \cdot \text{deg}}$). We will find the solution as a sum of a row:

$$T(x, y) = \sum_n T_n(x) \cos \lambda y \quad (2)$$

$$\text{where } \lambda = \frac{2n+1}{2} \cdot \frac{\pi}{y_0} \text{ and } T_n = \frac{2}{y_0} \cdot \int_0^{y_0} T(x, \xi) \cos \lambda \xi \cdot d\xi .$$

After the substitution of (2) into (1) we get the new one-dimensional equation instead of two-dimensional one at the beginning.

$$T_n'' - \lambda^2 T_n = -\frac{2}{y_0} \cdot \delta(x) \cdot \frac{q_0}{\kappa} \quad (3)$$

The solution of this equation should fade in infinity, so we can find the solution as:

$$T_n = A_n \cdot e^{-\lambda|x|}, \quad A_n = \text{const.} \quad (4)$$

The result is: $A_n = \frac{q_0}{\kappa y_0 \lambda}$, so the temperature distribution is:

$$T = \sum_{n=0}^{\infty} \frac{2}{\pi} \cdot \frac{q_0}{\kappa(2n+1)} \cdot \cos\left(\frac{2n+1}{2} \cdot \frac{\pi}{y_0} \cdot y\right) \cdot \exp\left(-\frac{2n+1}{2} \cdot \frac{\pi}{y_0} |x|\right) \quad (5)$$

Our real geometry is not plane, but we can use the special coordinate system similar to the real geometry which can be converted into the plane geometry. One of possible coordinates are:

$$u + iv = (x + iy)^2; \quad u = x^2 - y^2; \quad v = 2xy. \quad (6)$$

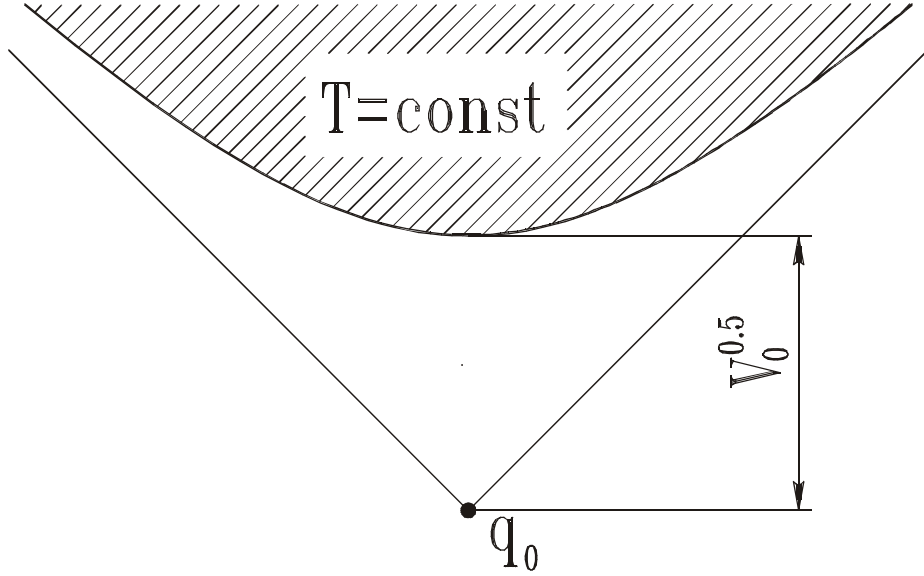


Fig. 2. Hyperbolic geometry

The axis of this system are hyperbolas in ordinary Cartesian system. The point source is situated on the top of one hyperbola $v = 0$, the constant temperature is held on another hyperbola $v = v_0$. In coordinates u, v we have the same problem as we have solved above (see Fig.1).

The heat conduction equation in this geometry is:

$$\frac{\partial^2 T}{\partial u^2} + \frac{\partial^2 T}{\partial v^2} = -\delta(u)\delta(v) \cdot \frac{q_0}{4\kappa\sqrt{u^2 + v^2}} \quad (7)$$

The coefficients in the row (2) can be defined from the Furie equation: $\vec{q} = -\kappa\nabla T$ written for the each component of heat flux and temperature. So,

$$A_n = \frac{q_0}{\kappa\lambda^2 v_0 \cdot \left\{ \int_0^\infty e^{-\lambda u} \cdot \sqrt[4]{u^2 + v^2} \cdot du \right\}} \quad (8), \quad \text{and the temperature}$$

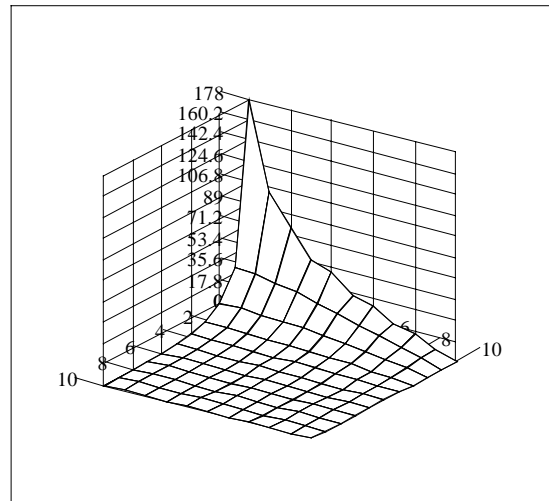
distribution is:

$$T = \sum_{n=0}^{\infty} \frac{q_0}{\kappa v_0} \cdot \frac{1}{\left(\frac{2n+1}{2} \cdot \frac{\pi}{v_0}\right)^2} \cdot \cos\left(\frac{2n+1}{2} \cdot \frac{\pi}{v_0} \cdot 2xy\right) \cdot B_n \quad (9)$$

$$\cdot \exp\left(-\frac{2n+1}{2} \cdot \frac{\pi}{v_0} |x^2 - y^2|\right)$$

$$\text{where } B_n = \int_0^{\infty} \sqrt[4]{u^2 + v_0^2} \exp\left(-\frac{2n+1}{2} \cdot \frac{\pi}{v_0} \cdot u\right) \cdot du$$

The comparison of plane geometry with “hyperbolic” one has shown that the maximum temperatures differs from each other not more than by 10% (see Fig. 3a,b and 4a,b) if the distance between two hyperbolas $d = \sqrt{v_0}$ is equal to the distance between two planes $d=y_0$. In our calculations $y_0 = 1 \text{ cm}$, $v_0 = 1 \text{ cm}^2$



T

step x=0.5 cm; step y=0.1 cm

Fig. 1a. Temperature distribution in plane geometry for point heat source

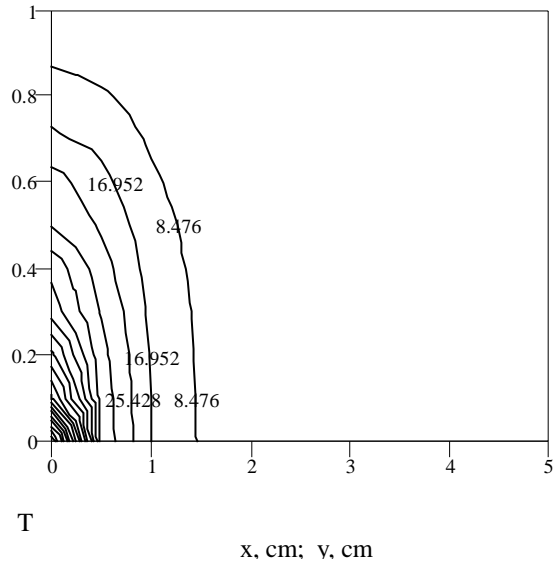


Fig. 3b. Isotherms of temperature distribution in plane geometry for point heat deposition

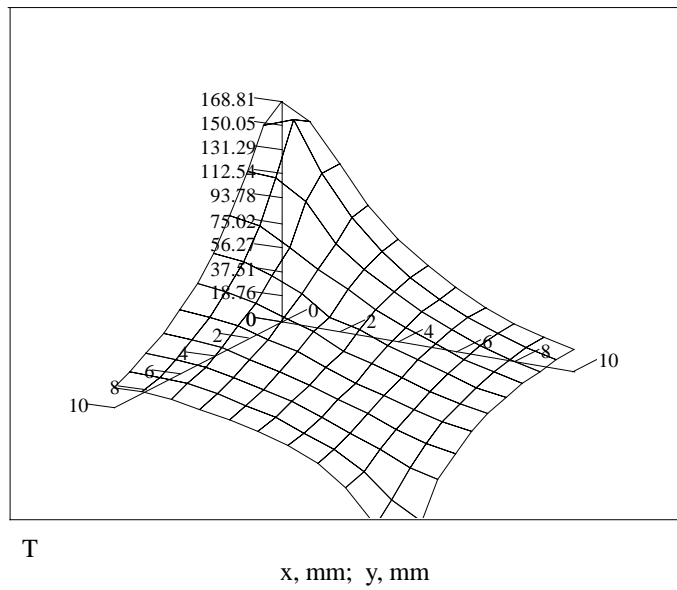


Fig. 2a. Temperature distribution in hyperbolic geometry for point heat source

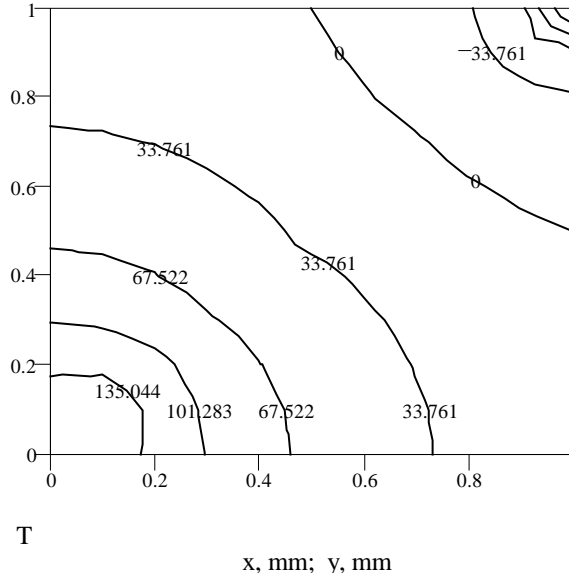


Fig. 4b. Isotherms of temperature distribution in hyperbolic geometry for point heat source

Now we can solve problem in case of plane geometry with any heat deposition distribution on the surface $y=0$. The heat conduction equation is:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -f(x)\delta(y) \cdot \frac{q_0}{\kappa} \quad (10) \text{ -- where } f(x) \text{ is some heat distribution.}$$

After the substitution (2) into (10) we get the equation:

$$T_n'' - \lambda^2 T_n = -\frac{2}{y_0} \cdot f(x) \cdot \frac{q_0}{\kappa} = \tilde{f}(x) \quad (11)$$

We have found above the source function $G(x, \tau)$ of this problem, so the solution can be written as:

$$T_n(x) = \int_{-\infty}^{+\infty} G_n(x, \tau) \cdot \tilde{f}(\tau) d\tau \quad (12)$$

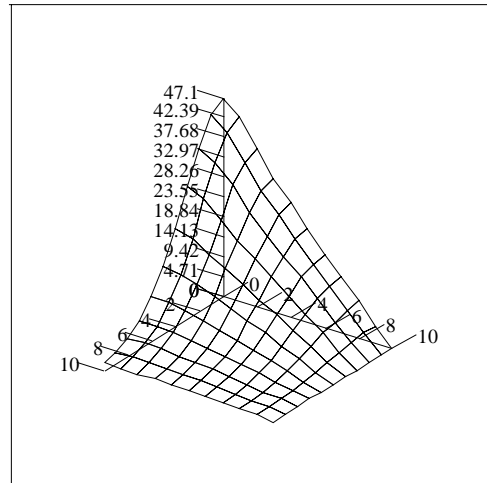
$(G_n(x, \tau))$ is defined by the equation: $G_n'' - \lambda^2 G_n = \delta(x)$,

$$G_n(x, \tau) = -\frac{1}{2\lambda} e^{-\lambda|x-\tau|}$$

And for the Gauss distribution of heat deposition

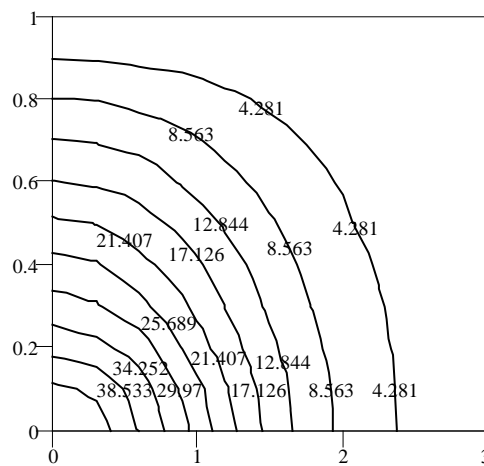
$f(x) = \frac{1}{\sqrt{2\pi} \cdot \sigma} \exp(-x^2/2\sigma^2)$ the temperature distribution is defined as:

$$T = \frac{q_0}{\sqrt{2\pi} \cdot \sigma} \cdot \frac{1}{\kappa v_0 \lambda} \sum_{n=0}^{\infty} \left\{ \int_{-\infty}^{+\infty} \exp(-\lambda|x-\tau| - \tau^2/2\sigma^2) d\tau \right\} \cos(\lambda y) \quad (13)$$



T
step x=0.3 cm; step y=0.1 cm

Fig. 3a. Temperature distribution in plane geometry for Gauss heat deposition



T
x, cm; y, cm

Fig. 5b. Isotherms of temperature distribution in plane geometry for Gauss heat deposition

So, these calculations have shown that at $10 T$ magnetic field the maximum temperature will not exceed 100° , and at $15 T$ field the maximum temperature will not exceed 250° . These are the estimations only, for more precision it is necessary to solve the problem of heat conduction in real geometry by means of computer simulation.