

# Appendix 1

## HYDRODYNAMIC BEHAVIOR OF TARGET MATERIAL

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### **1 Introduction**

The problem of target destroy under the high density energy deposition during the beam spill is the important one, restricting in many cases the particle density in primary beam and thus the achievable value of phase density in secondary beam. The use of liquid metal solves the problem of target destroy, but still remains that of target envelope survival, especially at target faces, where its thickness is restricted. So we are to consider attentively the pressure, arising in a system in result of beam spill.

The hydrodynamic consideration in linear (acoustic) approach is adequate the problem. It permits to find as the stresses, arising in solid or liquid target matter directly in a region of energy deposition, as those in target envelope, where the pressure waves propagate to. For free liquid jet target the conditions for splashing out and drop velocities can be found.

## 2 Pressure calculation

In linear (the acoustic) approximation the pressure in dependence on time and coordinates,  $P(\mathbf{r}, t)$ , is found as solution of wave-like equation:

$$\frac{\partial^2 P}{\partial t^2} - c_0^2 \Delta P = \frac{\partial q(\mathbf{r}, t)}{\partial t}.$$

Here  $c_0$  is the sound velocity, defined through the standard condition values of compressibility  $\chi_0$  and specific volume  $V_0 (V = 1/\rho)$  of matter as  $c_0^2 = \frac{V_0}{\chi_0}$ . The velocities, arising in matter, are connected with pressure as  $\frac{\partial \mathbf{u}}{\partial t} = -V \mathbf{grad} P$ .

Function  $q(\mathbf{r}, t)$  in the right hand side of above equation is a specific power of energy deposition. Its dependence on time may be regarded as being of the  $\delta$ -function form,  $q(\mathbf{r}, t) = \delta(t)Q(\mathbf{r})$ , because of a short duration of beam spill as compared to the characteristic hydrodynamic time of a system, defined as a time of sound propagation through a region of energy deposition. The pressure, arising instantly at  $t = 0$  inside the target, is defined through the deposited specific energy  $Q(\mathbf{r})$  as :

$$P(\mathbf{r}, 0) = \frac{\Gamma Q(\mathbf{r})}{V}.$$

Here  $\Gamma$  is the Gruneisen coefficient, expressed by normal conditions through the volume coefficient of thermal expansion  $\alpha$ , specific heat capacity  $c_v$  and sound velocity  $c_0$  as  $\Gamma = \frac{\alpha c_0^2}{c_v}$ .

In axially-symmetric case,  $P(\mathbf{r}, t) = P(r, z, t)$ , the dependence on radial coordinate is expressed through a superposition of Bessel functions  $J_0(\lambda r)$  with continuous ( $P(r, z, t) = \int_0^\infty P_\lambda(z, t) J_0(\lambda r) \lambda d\lambda$ ) or discrete series of  $\lambda$  values. In both cases for spectrum amplitude  $P_\lambda(z, t)$  one gets an equation:

$$\frac{\partial^2 P_\lambda}{\partial t^2} - c_0^2 \left( \frac{\partial^2 P_\lambda}{\partial z^2} - \lambda^2 P_\lambda \right) = 0. \quad (1)$$

For high energy beam the energy deposition region has rather small radial dimension, much smaller than the characteristic length of longitudinal variation of deposited specific energy magnitude. This permits to consider only the radial propagation of pressure at long enough distances from target faces, that is to neglect the derivative  $\frac{\partial^2}{\partial z^2}$  as compared to  $\lambda^2$ . Function  $P_\lambda$  in this

case is found as  $P_\lambda(z, t) \cong P_\lambda(z, 0) \cos \lambda c_0 t$ , where  $P_\lambda(z, 0)$  is defined by deposited energy density distribution  $Q(r, z)$ :  $P_\lambda(z, 0) = \frac{\Gamma}{V} \int_0^\infty Q(r, z) J_0(\lambda r) r dr$ . If distribution with respect to transverse coordinates may be considered Gaussian, after an integration over  $\lambda$  is fulfilled, the expression for preassure at beam axis reads:

$$P(0, z, t) \cong \frac{\Gamma Q_0(z)}{V} \left( 1 - 2 \frac{c_0 t}{\sigma \sqrt{2}} e^{-\frac{c_0^2 t^2}{2\sigma^2}} \int_0^{\frac{c_0 t}{\sigma \sqrt{2}}} e^{x^2} dx \right),$$

where  $Q_0(z)$  stands for specific energy deposited at beam axis.

The preassure is falling from initial value  $\frac{\Gamma Q_0(z)}{V}$  down to zero at  $c_0 t \cong 1.3\sigma$  and then to minimum value  $P_{\min}(0, z, t) \cong -0.285 \frac{\Gamma Q_0(z)}{V}$  at  $c_0 t \cong 2.1\sigma$ .

The negative preassure is responsible for a tension, arising in solid matter, and for its destruction, when the tension exceeds the limit value. This defines the limit for permissible value of specific energy deposition. For target of tungsten this limit is  $Q_{0,\max} \cong 160$  J/g (fig.1, line 1). For tungsten-rhenium alloy, which has better mechanical properties at high temperature it may be by  $\sim 20\%$  higher, that is  $Q_{0,\max} \sim 200$  J/g. This estimation is valid if an interference of direct and refracted preassure waves near target faces does not result in higher values of negative preassure.

Near target faces the solution of (1), satisfying the initial conditions for  $P$  and  $\frac{\partial P}{\partial t}$ , has a form:

$$P_\lambda(z, t) = \frac{1}{2} [f(z - c_0 t) + f(z + c_0 t)] - \frac{\lambda c_0 t}{2} \int_{z - c_0 t}^{z + c_0 t} f(s) \frac{J_1 \left( \lambda \sqrt{c_0^2 t^2 - (s - z)^2} \right)}{\sqrt{c_0^2 t^2 - (s - z)^2}} ds. \quad (2)$$

Here, evidently,  $f(s) = P_\lambda(s, 0)$ . The terms in square brackets represent the plane waves, propagating if forward and back directions along z-axis, while the third term makes an account for radial propagation of preassure. The expression is valid until the refracted waves from target boundaries arrive.

Let the boundary between target and outlet flange be at  $z=0$ . Let us also

neglect in first approximation the energy deposition in flange<sup>1</sup>. This means that by  $z + c_0 t > 0$  the refracted from boundary wave  $f^*(z + c_0 t)$  substitutes for  $f(z + c_0 t)$ , while a forward propagating wave goes into flange through the boundary. Correlation between these three waves is determined by condition of equality of pressure values and  $z$ -velocities  $u_z$  in both matters by  $z=0$ . This defines the refracted wave  $f^*$  in relation to  $f$  as follows:

$$f^*(s) = -f(-s) \left( \frac{1 - R}{1 + R} \right), \quad (3)$$

where  $R = \frac{V c_0^*}{V^* c_0}$  with star marking the sound velocity and specific volume of flange matter.

With account for (3) made the  $\lambda$ -constituents of pressure and  $z$ -velocity at inner surface of flange by  $t > 0$  are found in a form:

$$P_\lambda(0, t) = \frac{R}{1 + R} \left[ f(-c_0 t) - \lambda c_0 t \int_0^{c_0 t} J_1(\lambda \sqrt{c_0^2 t^2 - s^2}) \frac{f(-s) ds}{\sqrt{c_0^2 t^2 - s^2}} \right] \quad (4)$$

$$u_{z,\lambda}(0, t) = \frac{V}{c_0(1 + R)} \left[ f(-c_0 t) - \lambda \int_0^{c_0 t} J_1(\lambda \sqrt{c_0^2 t^2 - s^2}) \frac{f(-s) s ds}{\sqrt{c_0^2 t^2 - s^2}} \right]$$

With Gaussian transverse distribution of deposited specific energy,  $Q(r, z) = Q_0(z) \exp(-\frac{r^2}{2\sigma^2})$ , the function  $f(s)$  gets a form:  $f(s) = \frac{\Gamma}{V} \sigma^2 Q_0(s) e^{-\frac{\lambda^2 \sigma^2}{2}}$ , which permits to make an integration over  $\lambda$  analytically. In result the expression for pressure by  $z + c_0 t > 0$  reads:

$$P(r, z, t) = \frac{\Gamma}{2V} \exp\left(-\frac{r^2}{2\sigma^2}\right) \left\{ Q_0(z - c_0 t) - \frac{1 - R}{1 + R} Q_0[-(z + c_0 t)] \right. \\ \left. - \frac{c_0 t}{\sigma^2} \left\{ \int_0^{c_0 t - z} Q_0(-s) \exp\left(-\frac{w_1^2}{2\sigma^2}\right) \left[ I_0\left(\frac{r w_1}{\sigma^2}\right) - \frac{r}{w_1} I_1\left(\frac{r w_1}{\sigma^2}\right) \right] ds \right. \right. \quad (5) \\ \left. \left. - \frac{1 - R}{1 + R} \int_0^{z + c_0 t} Q_0(-s) \exp\left(-\frac{w_2^2}{2\sigma^2}\right) \left[ I_0\left(\frac{r w_2}{\sigma^2}\right) - \frac{r}{w_2} I_1\left(\frac{r w_2}{\sigma^2}\right) \right] ds \right\} \right\}.$$

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<sup>1</sup>This is reasonable, when the flange is made of titanium, while the target – of lead or mercury, so that the values of  $Q$  in them differ by about 5 times.

Here  $w_1 = \sqrt{c_0^2 t^2 - (s+z)^2}$  and  $w_2 = \sqrt{c_0^2 t^2 - (s-z)^2}$ ,  $I_0$  and  $I_1$  are Bessel functions of imaginary argument.

At beam axis the expression (5) simplifies to:

$$P(0, z, t) = \frac{\Gamma}{2V} \left\{ Q_0(z - c_0 t) - \frac{c_0 t}{\sigma^2} \int_0^{c_0 t - z} Q_0(-s) \exp\left(-\frac{w_1^2}{2\sigma^2}\right) ds \right. \\ \left. - \frac{1-R}{1+R} \left[ Q_0[-(z + c_0 t)] - \frac{c_0 t}{\sigma^2} \int_0^{z+c_0 t} Q_0(-s) \exp\left(-\frac{w_2^2}{2\sigma^2}\right) ds \right] \right\}. \quad (6)$$

In the case of free target surface ( $R = 0$ ) the pressure magnitude on it is equal zero, while before the surface at beam axis it is:

$$P(0, z, t) = \frac{\Gamma}{2V} \left\{ Q_0(z - c_0 t) - Q_0[-(z + c_0 t)] - \right. \\ \left. \frac{c_0 t}{\sigma^2} \left[ \int_0^{c_0 t - z} Q_0(-s) \exp\left(-\frac{w_1^2}{2\sigma^2}\right) ds - \int_0^{z+c_0 t} Q_0(-s) \exp\left(-\frac{w_2^2}{2\sigma^2}\right) ds \right] \right\}. \quad (7)$$

At a front of refracted wave, that is by  $z + c_0 t = 0$ , the second of integrals in the right hand side of above expression is equal zero, and the pressure is defined in the main by a magnitude of the first of them. If, as it was supposed above, we can neglect the variation of  $Q_0(s)$  through a distance of several  $\sigma$ , one gets:

$$P(0, -c_0 t, t) = -\frac{\Gamma Q_0(0) c_0 t}{V \sigma^2} \int_0^{c_0 t} \exp\left(-\frac{c_0^2 t^2 - s^2}{2\sigma^2}\right) ds \quad (8)$$

Minimum value of this pressure,  $P_{\min}(0, -c_0 t, t)$ , achieved at  $c_0 t \cong 2.1\sigma$ , is equal to:

$$P_{\min}(0, -c_0 t, t) \cong -1.28 \frac{\Gamma Q_0(0)}{V}.$$

This is by 4.5 times higher in absolute value than in the middle of target, and restricts the specific energy deposition in tungsten target by a value (see fig.1, line 2)

$$Q_{0,\max} \sim 80 \text{ J/g.}$$

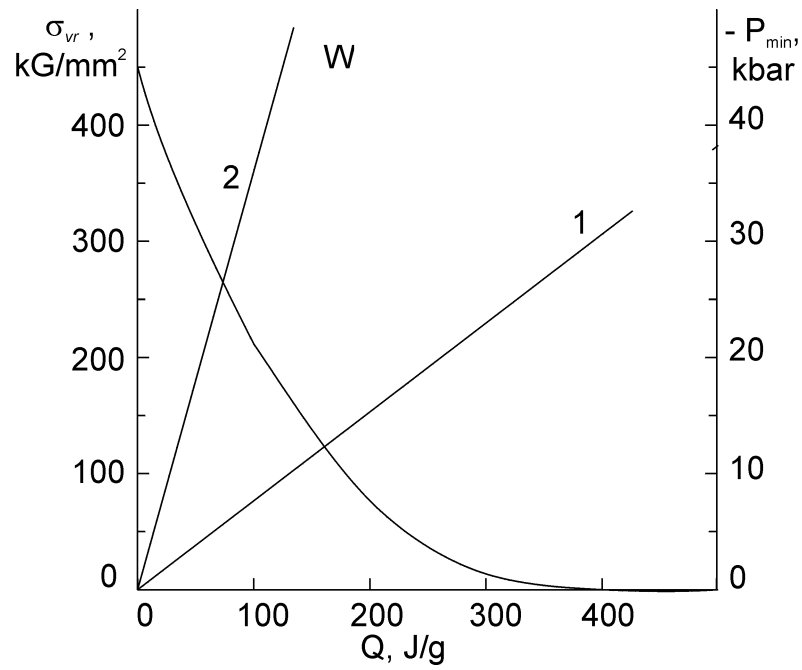


Figure 1: The short time rupture strength  $\sigma_{vr}$  [2] and maximum negative pressure  $-P_{\min}(0, -c_0t, t)$  in tungsten versus the specific energy deposition at beam axis far from (1) and near (2) the target faces by infinitely short duration of beam spill.

Got above high value of negative pressure is a result of short duration  $T$  of beam spill. In dependence on  $T$  the pressure at the front of decompressive wave after the beam spill finish ( $t > 0$ ) is found as follows:

$$P(0, -c_0t, t) = -\frac{\Gamma Q_0(0)}{V c_0 T} \left[ 1 - \exp\left(-\frac{c_0^2 T(T+2t)}{2\sigma^2}\right) \right] \int_0^{c_0t} \exp\left(-\frac{c_0^2 t^2 - s^2}{2\sigma^2}\right) ds$$

When spill duration is big,  $c_0T \gg \sigma$ , one gets:

$$P(0, -c_0t, t) \cong -\frac{\Gamma Q_0(0)}{V c_0 T} \int_0^{c_0t} \exp\left(-\frac{c_0^2 t^2 - s^2}{2\sigma^2}\right) ds.$$

The minimum pressure here is  $P_{\min}(0, -c_0t, t) \cong -0.54 \frac{\Gamma Q_0(0) \sigma \sqrt{2}}{V c_0 T}$ .

## References

1. T. A. Vsevolozhskaya, *Mechanical Stresses in Target Material and Beam Sweeping*, Preprint Budker INP 96-80, Novosibirsk 1996.
2. Reference Book for Properties of Elements, part I - Physical Properties, Metallurgy, Moscow, 1976