

Appendix 3

THE HEAT CONDUCTION CALCULATIONS

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It is obvious that high 10-15 T pulsed magnetic field in the concentrator will result in the target heating. The pulsed magnetic field induces currents in the target, which causes the heat release in the target material. To define the amount of energy releasing in the target the task of field diffusion has been solved.

This task is described by equation (1)

$$\Delta H - \frac{4\pi}{c^2} \cdot \sigma\mu \cdot \frac{\partial H}{\partial t} = 0 \quad (1)$$

Where Δ – is the Laplasian operator, σ – is the conductivity of target material, μ – is the permeability of target material. We will use the cylindrical coordinate system with r, ϕ, z coordinates most suitable for our geometry. Magnetic field is directed along the z -axis and the cylindrical conductor of the infinite length is directed along the z -axis also. Then the field can be written as:

$$\vec{H}(r, t) = (0, 0, H_z(r, t)) \quad (2)$$

So, we can rewrite the equation (1) as:

$$\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial H_z}{\partial r} - \frac{4\pi}{c^2} \sigma\mu \frac{\partial H_z}{\partial t} = 0 \quad (3)$$

We will find the solution as $H_z(r, t) = R(r)e^{i\omega t}$ with two conditions:

$$H_z(r_0, t) = H_0 \cdot e^{i\omega t} ; |H_z(0, t)| < \infty \quad (4)$$

The equation (3) then transforms into the Bessel equation:

$$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} + (\alpha \cdot \sqrt{-i})^2 R = 0 \quad (5),$$

where $\alpha = \sqrt{\frac{4\pi\mu\sigma\omega}{c^2}}$. The solution of this equation is:

$$R(r) = AJ_0(\alpha r \sqrt{-i}) + BN_0(\alpha r \sqrt{-i}) \quad (6)$$

where J_0 and N_0 are the Bessel functions of the first and second kind. A, B constants can be defined from the conditions (4). $B=0$ because $|N_0(\alpha r \sqrt{-i})| \rightarrow \infty$ at $r \rightarrow 0$, so

$$R(r) = AJ_0(\alpha r \sqrt{-i}) \quad (6),$$

where $A = \frac{H_0}{J_0(\alpha r_0 \sqrt{-i})}$ and $J_0(\alpha r \sqrt{-i})$ – is the Bessel function of complex

argument. To calculate the heat deposition the Umov-Poiting vector (7) representing the energy flux can be considered. Taking into account that $\vec{j} = \sigma \vec{E}$ we will obtain:

$$\vec{S} = \frac{c}{4\pi} \cdot [\vec{E} \times \vec{H}] = \frac{c}{4\pi\sigma} [\vec{j} \times \vec{H}] \quad (7)$$

This vector has only one nonzero component along the r-axis. It is defined by equation:

$$S_r = -\frac{c^2}{16\pi^2\sigma} \cdot \frac{\partial H_z}{\partial r} \cdot H_z \quad (8)$$

For the following calculations we should find the energy flux through the surface of cylinder of unit length. The amount of energy penetrating into conductor is defined by the following equation:

$$\hat{S}_r = \int_0^{2\pi} \int_0^1 P_r r d\phi dz = -\frac{rc^2}{8\pi\sigma} \frac{\partial H_z}{\partial r} \cdot H_z \quad (9)$$

where $H_z = H_0 \cdot \frac{J_0(\alpha r \sqrt{-i})}{J_0(\alpha r_0 \sqrt{-i})} \cdot e^{i\omega t}$ (10) and

$$\frac{\partial H_z}{\partial r} = H_0 \cdot \frac{J'_0(\alpha r \sqrt{-i})}{J_0(\alpha r_0 \sqrt{-i})} = -H_0 \cdot \frac{\sqrt{-i} \cdot \alpha \cdot J_1(\alpha r \sqrt{-i})}{J_0(\alpha r_0 \sqrt{-i})} \quad (11)$$

(taking into account the Bessel function property $J'_0(x) = -J_1(x)$).

Substituting (10) and (11) into (9) and shifting from complex values to the real ones we will obtain the value of energy deposition. It is defined as:

$$W = -\frac{r_0 c^2}{8\pi\sigma} \cdot \frac{1}{2} \cdot \operatorname{Re} \left\{ \frac{\partial H_z}{\partial r} \cdot H_z^* \right\} \quad (11), \quad \text{where “*” means the complex}$$

conjugation, and

$$W = \frac{r_0 c^2}{16\pi\sigma} \cdot H_0^2 \cdot \alpha \cdot \operatorname{Re} \left\{ \frac{\sqrt{-i} J_1(\alpha r_0 \sqrt{-i})}{J_0(\alpha r_0 \sqrt{-i})} \right\} \quad (12)$$

here $\alpha = \sqrt{\frac{4\pi\mu\sigma\omega}{c^2}} = 3.86$; $c = 3 \cdot 10^{10} \text{ cm/s}$ – is the light velocity,

$\omega = 2\pi f$; $f = 20 \text{ kHz}$ – is the frequency of half sine current pulses in concentrator (not repetition rate) and for the liquid lead target

$\sigma = 8.9 \cdot 10^{15}$ at $T = 400^\circ\text{C}$ and $\mu = 1$.

So the heat deposition for half sine time interval is:

$$Q = W \cdot \frac{T}{2} = W \cdot \frac{\pi}{\omega} \cong 36 \text{ J/cm} \quad (13),$$

where $T = 50 \mu s$ – is the period of sine pulses. It corresponds to the power of energy deposition $4.3 kW/cm$ for the repetition rate of half sine pulses equal to $120 Hz$. The region of maximum heat deposition corresponds to the region of the highest magnetic field and the length of this region is $1.5 cm$ approximately (it was estimated by computer simulation of magnetic field in concentrator; See chapter 5). Thus, the main heating has place in small region. The energy deposition given by (13) causes the rise of target's temperature by 49° for one pulse of half sine current in concentrator.

$$\Delta t = \frac{Q}{c_{Pb} \cdot m}, \quad m = \rho_{Pb} \cdot V = \rho_{Pb} \cdot \pi r_0^2 \cdot l, \quad \text{where } m \text{ – is the mass of lead}$$

cylinder of unit length, $c_{Pb} = 0.013 \frac{cal}{g \cdot deg}$ – is the heat capacity of the lead,

$\rho_{Pb} = 11.3 g/cm^3$ – is the lead density, $r_0 = 0.4 cm$ – is the target radius, $l = 1 cm$. The energy deposition by pulsed heating is less than the energy deposition caused by the beam. It is $6.5 kW$ and $23 kW$ correspondingly. This fact was taken into account during the calculation of the liquid lead velocity in the target for the effective heat removal.