Appendix 1

HYDRODYNAMIC BEHAVIOR OF TARGET MATERIAL

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Pressure calculation

In linear (the acoustic) approximation the pressure in dependence on time and coordinates, $P(\mathbf{r},t)$, is found as solution of wave-like equation:

$$\frac{\partial^2 P}{\partial t^2} - c_0^2 \Delta P = \frac{\partial q(\mathbf{r},t)}{\partial t}$$

Here c_0 is the sound velocity, defined through the standard condition values of compressibility χ_0 and specific volume V_0 $(V = 1/\rho)$ of matter as $c_0^2 = \frac{V_0}{\chi_0}$. The velocities, arising in matter, are connected with pressure as $\frac{\partial \mathbf{u}}{\partial t} = -V\mathbf{grad}P$.

Function $q(\mathbf{r},t)$ in the right hand side of above equation is a specific power of energy deposition. Its dependence on time may be regarded as being of the δ - function form, $q(\mathbf{r},t)=\delta(t)Q(\mathbf{r})$, because of a short duration of beam spill as compared to the characteristic hydrodynamic time of a system, defined as a time of sound propagation through a region of energy deposition. The pressure, arising instantly at *t=0* inside the target, is defined through the deposed specific energy $Q(\mathbf{r})$ as :

$$P(\mathbf{r},0)=\frac{\Gamma Q(\mathbf{r})}{V}$$

Here Γ is the Gruneisen coefficient, expressed by normal conditions through the volume coefficient of thermal expansion α, specific heat capacity c_v and sound velocity c_0 as $\Gamma=\frac{\alpha c_0^2}{c_v}$.

In axially symmetric case $P(\mathbf{r},t)= P(r,z,t)$, the dependence on radial coordinate is expressed through a superposition of Bessel functions $J_0(\lambda r)$ with continuous ($P(r,z,t)=\int\limits_0^\infty P_\lambda(z,t)J_0(\lambda r)\lambda d\lambda$) or discrete series of λ values.

In both cases for spectrum amplitude $P_\lambda(z,t)$ one gets an equation:

$$\frac{\partial^2 P_\lambda}{\partial t^2}-c_0^2\left(\frac{\partial^2 P_\lambda}{\partial z^2}-\lambda^2 P_\lambda\right)=0 \qquad (1)$$

For high energy beam the energy deposition region has rather small radial dimension, much smaller than the characteristic length of longitudinal variation of deposed specific energy magnitude. This permits to consider neglect the longitudinal propagation of pressure as compared to only the radial propagation of pressure at long enough distances from target faces, that is to neglect the derivative $\frac{\partial^2}{\partial z^2}$ as compared to λ^2. Function P_λ in this case is found as $P_\lambda(z,t) \cong P_\lambda(z,0)\cos\lambda c_0 t$, where $P_\lambda(z,0)$ is defined by deposed energy density distribution $Q(r,z)$:

$P_\lambda(z,0) = \frac{\Gamma}{V}\int_0^\infty Q(r,z) J_0(\lambda r) r dr$. If distribution with respect to transverse coordinates may be considered Gaussian, after an integration over λ is fulfilled, the expression for pressure at beam axis reads (in accordance with [1]):

$$P(0,z,t) \cong \frac{\Gamma Q_0(z)}{V}\left(1 - 2\frac{c_0 t}{\sigma\sqrt{2}} e^{-\frac{c_0^2 t^2}{2\sigma^2}} \cdot \int_0^{\frac{c_0 t}{\sigma\sqrt{2}}} e^{x^2} dx\right),$$

where $Q_0(z)$ stands for specific energy deposed at beam axis.

The pressure is falling from initial value $\frac{\Gamma Q_0(z)}{V}$ down to zero at $c_0 t \cong 1.3\sigma$ and then to minimum value $P_{\min}(0,z,t) \cong -0.285\frac{\Gamma Q_0(z)}{V}$ at $c_0 t \cong 2.1\sigma$.

The negative pressure is responsible for a tension, arising in solid matter, and for its destruction, when the tension exceeds the ultimate value [2]. The presented above value of negative pressure defines the limit for permissible specific energy deposition far from the ends of target, while near the target faces an interference of direct and reflected pressure waves can results in higher values of negative pressure.

Near target faces the solution of (1), satisfying the initial conditions for P and $\frac{\partial P}{\partial t}$ has a form:

$$P_\lambda(z,t) = \frac{1}{2}\left[f_\lambda(z - c_0 t) + f_\lambda\left[z + c_0 t\right]\right] -$$

(2)

$$-\frac{\lambda c_0 t}{2} \int_{z-c_0 t}^{z+c_0 t} f_\lambda(s) \frac{J_1\left(\lambda\sqrt{c_0^2 t^2 - (s-z)^2}\right)}{\sqrt{c_0^2 t^2 - (s-z)^2}} ds$$

Here, evidently, $f_\lambda(s) = P_\lambda(s,0)$.

The expression (2) can be regarded as a superposition of waves, propagating in forward and back directions along z-axis. It is valid until the reflected waves from target boundaries arrive. Let the boundary between target and outlet flange be at $z=0$. Let us also neglect in first approximation the energy deposition in flange.

This means that by $s>0$ the reflected from boundary wave $f_\lambda^*(s)$ is to substitute for $f_\lambda(s)$, while the forward propagating wave goes into the flange through boundary. Correlation between these three waves is determined by condition of equality of pressure magnitudes and z-velocities u_z in both matters by $z=0$. In plane wave approach ($\lambda=0$) this defines the reflected wave f_0^* in relation to f_0 as follows:

$$f_0^*(s) = -f_0(-s)\left(\frac{1-R}{1+R}\right) \qquad (3)$$

where $R = \dfrac{Vc_0^*}{V^* c_0}$ with star marking the sound velocity and specific volume of flange matter.

In reality the relation (3) is valid in a few particular cases, including the case of free target surface (*R=0*), where it means $f_\lambda^*(s) = -f_\lambda(-s)$.

With Gaussian transverse distribution of deposed specific energy, $Q(r,z) = Q_0(z)\exp(-\dfrac{r^2}{2\sigma^2})$, the function $f_\lambda(s)$ gets a form:

$f_\lambda(s) = \dfrac{\Gamma}{V}\sigma^2 Q_0(s) e^{-\frac{\lambda^2\sigma^2}{2}}$, which permits to make an integration over λ analytically. In result the expression for pressure by $z + c_0 t > 0$ in case of free target surface reads:

$$P(r,z,t) = \frac{\Gamma}{2V}\exp\left(-\frac{r^2}{2\sigma^2}\right)\Bigg\{Q_0(z-c_0t) - Q_0\left[-(z+c_0t)\right] +$$

$$+\frac{1}{c_0}\left\{\int_0^{c_0t-z} Q_0(-s)\frac{\partial}{\partial t}\left[\exp\left(-\frac{w_1^2}{2\sigma^2}\right) I_0\left(\frac{rw_1}{\sigma^2}\right)\right]ds - \right. \qquad 4)$$

$$\left. - \int_0^{z+c_0t} Q_0(-s)\frac{\partial}{\partial t}\left[\exp\left(-\frac{w_2^2}{2\sigma^2}\right) I_0\left(\frac{rw_2}{\sigma^2}\right)\right]ds\right\}\Bigg\}$$

where $w_1 = \sqrt{c_0^2 t^2 - (s+z)^2}$ and $w_2 = \sqrt{c_0^2 t^2 - (s-z)^2}$, I_0 is the Bessel function of imaginary argument.

At beam axis this simplifies to:

$$P(0,z,t) = \frac{\Gamma}{2V}\left\{ Q_0(z - c_0 t) - Q_0[-(z + c_0 t)] - \right. \tag{5}$$

$$\left. -\frac{c_0 t}{\sigma^2}\left[\int_0^{c_0 t - z} Q_o(-s)\exp\left(-\frac{w_1^2}{2\sigma^2}\right) ds - \int_0^{z + c_0 t} Q_0(-s)\exp\left(-\frac{w_2^2}{2\sigma^2}\right) ds \right] \right\}$$

At a front of reflected wave, that is by $z + c_0 t = 0$, the second of integrals in the right hand side of above expression is equal zero, and the pressure is defined in the main by a magnitude of the first of them. If, as it was supposed above, we can neglect the variation of $Q_0(s)$ through a distance of several σ, one gets:

$$P(0,-c_0 t,t) = -\frac{\Gamma Q_0(0)}{V}\frac{c_0 t}{\sigma^2}\int_0^{c_0 t} \exp\left(-\frac{c_0^2 t^2 - s^2}{2\sigma^2}\right) ds$$

Minimum value of this pressure, $P_{\min}(0,-c_0 t,t)$ achieved at $c_0 t \cong 2.1\sigma$, is equal to:

$$P_{\min}(0,-c_0 t,t) \cong -1.28\frac{\Gamma Q_0(0)}{V}$$

This is by 4.5 times higher in absolute value than in the middle of target.

Such a high value of negative pressure is a result of short duration of beam spill *T*. In dependence on *T* the pressure at the front of decompressive wave after the beam spill finish ($t>0$) is found as follows:

$$P(0,-c_0 t,t) \cong -\frac{\Gamma q_0(0)}{V c_0}\left[1-\exp\left(-\frac{c_0^2 T(T+2t)}{2\sigma^2}\right)\right]\int_0^{c_0 t}\exp\left(-\frac{c_0^2 t^2 - s^2}{2\sigma^2}\right)ds$$

Here q_0 stands for specific power of energy deposition. When spill duration is big, $c_0 T >> \sigma$, one gets:

$$P(0,-c_0 t,t) \cong -\frac{\Gamma q_0(0)}{V c_0}\int_0^{c_0 t}\exp\left(-\frac{c_0^2 t^2 - s^2}{2\sigma^2}\right)ds$$

The minimum pressure here is $P_{\min}(0,-c_0 t,t) \cong -0.54\frac{\Gamma q_0(0)}{V}\frac{\sigma\sqrt{2}}{c_0}$

The λ - constituent of pressure in flange matter ($z>0$) is found as:

$$P_\lambda^*(z,t) = \frac{1}{2}\varphi_\lambda\left(t - z/c^*\right) -$$

$$-\frac{\lambda c_0^* z}{2}\int_0^{t-z/c_0^*}\varphi_\lambda(\tau)\frac{J_1\left(\lambda\sqrt{c_0^{*2}(t-\tau)^2 - z^2}\right)}{\sqrt{c_0^{*2}(t-\tau)^2 - z^2}}d\tau - \qquad (7)$$

$$-\frac{c_0^* V}{2V^*}\cdot\int_0^{t-z/c_0^*} g_\lambda(\tau) J_0\left(\lambda\sqrt{c_0^{*2}(t-\tau)^2 - z^2}\right)d\tau$$

where functions φ_λ and g_λ denote the pressure $P_\lambda(z,t)$ and its derivative with respect to z in target by the boundary, $\varphi_\lambda(t) = P_\lambda(0,t)$ and $g_\lambda(t) = \left.\frac{\partial P_\lambda(z,t)}{\partial z}\right|_{z=0}$.

To meet the condition of equality of pressure and z - velocity magnitudes in both matters by the boundary, functions φ_λ and g_λ are to satisfy an equation:

$$\varphi_\lambda(t) + \frac{c_0^* V}{V^*}\int_0^t g_\lambda(\tau) J_0(\lambda c_0^*(t-\tau))d\tau = 0 \qquad (8)$$

When expressed through an independent of time power of energy deposition *q(r,z)* the λ - amplitude of pressure in target by $t + z/c_0 > 0$ has a form:

$$P_\lambda(z,t) = \frac{1}{2c_0}\left[\int_0^{c_0 t - z} f_\lambda(-s) J_0\left(\lambda\sqrt{c_0^2 t^2 - (s+z)^2}\right)ds + \right.$$

$$\left. + \int_0^{c_0 t + z} f_\lambda^*(s) J_0\left(\lambda\sqrt{c_0^2 t^2 - (s-z)^2}\right)ds\right] \qquad (9)$$

with $f_\lambda(z) = \left.\frac{\partial P_\lambda(z,t)}{\partial t}\right|_{t=0} = \frac{\Gamma}{V}\int_0^\infty q(r,z) J_0(\lambda r) r dr$, while the reflected wave amplitude f_λ^* is defined from expression (8).

Indeed, the functions φ_λ and g_λ are found from (9) as being equal to:

$$\varphi_\lambda(t)=\frac{1}{2c_0}\int_0^{c_0 t}\left[f_\lambda(-s)+f_\lambda^*(s)\right]J_0(\lambda\sqrt{c_0^2t^2-s^2})ds,$$

$$g_\lambda(t)=\frac{1}{2c_0}\left\{f_\lambda^*(c_0t)-f_\lambda(-c_0t)-\int_0^{c_0 t}\left[f_\lambda^*(s)-f_\lambda(-s)\right]\frac{\partial}{\partial s}J_0\left(\lambda\sqrt{c_0^2t^2-s^2}\right)ds\right\}$$

and after substitutions made in (8), one gets for correlation between f_λ and f_λ^* an integral equation:

$$\int_0^t\left\{\left[f_\lambda(-c_0\tau)+f_\lambda^*(c_0\tau)\right]J_0(\lambda c_0\sqrt{t^2-\tau^2})+\right.$$

(10)

$$\left.R\left[f_\lambda(-c_0\tau)-f_\lambda^*(c_0\tau)\right]\frac{\partial}{\partial\tau}\int_\tau^t J_0(\lambda c_0^*(t-\xi))J_0(\lambda c_0\sqrt{\xi^2-\tau^2})d\xi\right\}d\tau=0$$

With the use of (3) as zero approximation to solution of (10) the first approximation correction addition to f_λ^*, $\delta f_\lambda^{*(1)}$, is found in a form:

$$\delta f_\lambda^{*(1)}(c_0t)=\frac{2R}{(1+R)^2}\cdot\int_0^t f_\lambda(-c_0\tau)\frac{\partial}{\partial t}\left[J_0\left(\lambda c_0\sqrt{t^2-\tau^2}\right)+\right.$$

$$\left.\frac{\partial}{\partial\tau}\int_\tau^t J_0\left(\lambda c_0^*(t-\xi)\right)J_0\left(\lambda c_0\sqrt{\xi^2-\tau^2}\right)d\xi\right]d\tau$$

If $f_\lambda(s)$ is a slow varying function, so that it may be carried out the integral in (11), the expression for $\delta f_\lambda^{*(1)}$ reads:

$$\delta f_\lambda^{*(1)}(c_0t)=2R\frac{f_\lambda(-c_0t)}{(1+R)^2}\sqrt{\frac{\pi}{2}}\sum_{k=1}^{\infty}\frac{(k-1)!!}{2^{2k}(k!)^2}\left(1-\frac{c_0^{*2}}{c_0^2}\right)^k(\lambda c_0t)^{k+\frac{1}{2}}\cdot J_{k-\frac{1}{2}}(\lambda c_0t)$$

When $c_0^* = c_0$ the addition is equal zero, and, hence, the boundary condition (3) appears valid in case of equal sound velocities in both matters.

Besides this the condition (3) is applicable near – at distance much less than σ – the fronts of both the reflected wave and that propagating through the boundary.

In common case the correction addition of higher order $\delta f_\lambda^{*(n+1)}$, $n = 1,2,3,...,$ are found with the use of formulae:

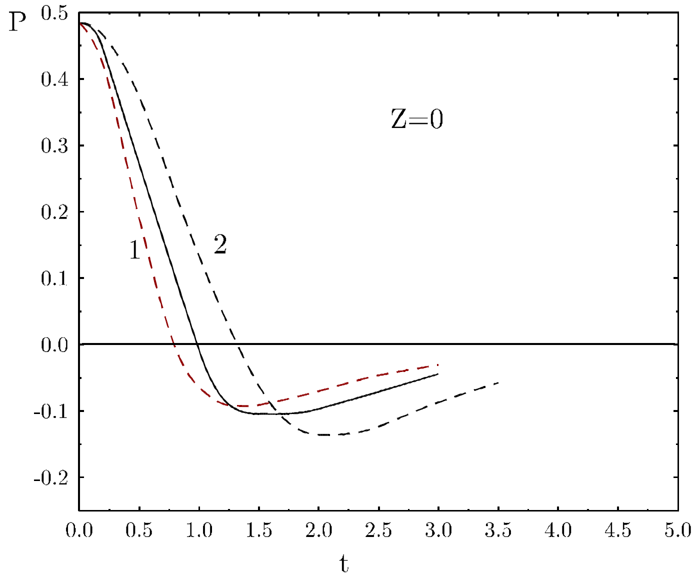
$$\delta f_\lambda^{*(n+1)}(c_0 t) = \frac{\lambda c_0}{1+R}\int_0^t \delta f_\lambda^{*(n)}(c_0\tau)\left\{(t+R\tau)\frac{J_1\left(\lambda c_0\sqrt{t^2-\tau^2}\right)}{\sqrt{t^2-\tau^2}} + \right.$$

$$\left. R\frac{c_0^*}{c_0}\left[J_1\left(\lambda c_0^*(t-\tau)\right) - \lambda c_0\tau\int_\tau^t J_1\left(\lambda c_0^*(t-\xi)\right)\frac{J_1\left(\lambda c_0\sqrt{\xi^2-\tau^2}\right)}{\sqrt{\xi^2-\tau^2}}d\xi\right]\right\}d\tau$$

When $R \cong 1$ and $c_0^*/c_0 \cong 2.4$, which corresponds to the lead target and titanium flange, the convergence of correction addition series is rather slow by large values of ratio $c_0 t/\sigma$.

The numerical solution for $f_\lambda^*(s)$ can be obtained by means of substitution of a sum $\sum_{i=0}^{n} A_i f_\lambda^*(c_0\tau_i)]K_\lambda(t,\tau_i)$ for the integral $\int_0^t f_\lambda^*(c_0\tau)]K_\lambda(t,\tau)$, where $K_\lambda(t,\tau)$ denotes the corresponding function in the equation, got from (10) after derivation with respect to *t*. This creates a system of linear equations for definition of $f_\lambda^*(c_0\tau_i)$ with $\tau_i = 0$ and $\tau_n = t$.

The effect of precise solution for $f_\lambda^*(s)$ is illustrated in figure, where the pressure at z =0 is presented versus time for a case of infinitely short beam spill duration. The dashed lines 1 and 2 are calculated from (7) and (9), accordingly, with the use of condition (3), while the solid curve is described by both of this expressions when $f_\lambda^*(s)$ is the solution of equation (10) .



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