NLC ME NOTE -12		
TITLE: Ground Motion Coupling to NLC Lattice		
No. 3-95 Rev 1	*DATE:* 12 April 1995	**AUTHOR:** Gordon B. Bowden

Introduction

Microseismic waves will propagate continuously in the tunnel floor of NLC. This motion, measured in nanometers, will result in displacement of the quadrupole magnets in the accelerator's focusing lattice. The associated deflection fields will cause jitter in the beam position at the end of the accelerator. Although the motions are extremely small, so too is the beam. The vertical beam height σ_{yf} = *700* nm at the end of the machine. Ground motion acts on 760 quadrupoles spread out over nearly 6 kilometers of the main NLC linac. Beam displacement at the end of the accelerator is the result of misalignment of quadrupoles with respect to the mean axis of the machine rather than the absolute position of any particular magnet or differential motion between any isolated pair. Although it is instinctive to study the motion over time of individual magnets in inertial space what really causes beam jitter is the distortion present in the overall lattice at the instant of beam transport. Lattice distortion is best described by a spatial power spectrum of displacement versus wave number k (radians/m). The lattice sensitivity to ground motion can be characterized by a lattice response function $R(k)$ which gives the beam displacement at the end of the accelerator for a displacement $sin(kx)$ impressed over the entire length of the machine. This note describes a method of estimating the rms magnitude of beam jitter from the lattice response function and the spatial frequencies of ground motion.

Lattice Scaling

The focusing lattice of the accelerator is a FODO array of quadrupoles. The strength and spacing of these quadrupoles is described by Tor Raubenheimer in SLAC Report-387 'The Generation and Acceleration of Low Emittance Flat Beams for Future Linear Colliders', Nov 1991. As the beam gathers energy, quadrupole magnet lengths and FODO cell lengths are increased in proportion to $\sqrt{\gamma}$. The magnet field gradients are kept constant. This results in the following scaling:

Cell length $L_c = L_{c0}\sqrt{\frac{\gamma}{\gamma_0}}$ (meters) Cells get longer

Quad length $L_q = L_{q0}\sqrt{\frac{\gamma}{\gamma_0}}$ (meters) Quads get longer

Quad strength $K_1 = K_{1\,0}\frac{\gamma_0}{\gamma}$ (meters^{-2}) Quads weaken rel. to γ

Beta function $\beta = \beta_0\sqrt{\frac{\gamma}{\gamma_0}}$ (meters) Beta function increases

Here $\gamma_0 = E_0/m_e$, where E_0 is the energy of the injected beam. At each quadrupole (n), numbered from 0 to $N_q - 1$, Raubenheimer uses the scaling:

$$\gamma(n) = \left(\frac{n}{c} + \sqrt{\gamma_0}\right)^2, \text{ where } c = \frac{N_q}{\sqrt{\gamma_f} - \sqrt{\gamma_0}} . \tag{1}$$

Here γ_0, γ_f refer to the initial and final beam energy at the end of the linac. If the magnet lengths are negligible, the location of quadrupole n along the machine, s(n) in meters can be approximated by adding up the distances between quads L_{qsep} in the FODO cells.

$$L_{qsep}(n) \simeq \frac{L_c}{2} = \frac{L_{c0}}{2}\sqrt{\frac{\gamma(n)}{\gamma_0}}$$

$$L_{qsep}(n) \simeq \frac{L_{c0}}{2\sqrt{\gamma_0}}\left(\frac{n}{c} + \sqrt{\gamma_0}\right)$$

$$s(n) \simeq \int L_{qsep} dn = \frac{L_{c0}}{2\sqrt{\gamma_0}}\left(\frac{n^2}{2c} + \sqrt{\gamma_0}n\right) \tag{2}$$

Alternating quadrupole polarity in the FODO array is set by $S_{ign} = (-1)^n$. The strength*length product for magnet n is related to betatron phase advance per cell ψ_c and to γ by:

$$K_1 L_q(n) = \frac{4S_{ign}}{L_{c0}}\sqrt{\frac{\gamma_0}{\gamma(n)}}\sin\frac{\psi_c}{2} \tag{3}$$

The beta function oscillates between maxima $\hat{\beta}$ at the focusing magnets and minima $\check{\beta}$ at the defocusing magnets. In terms of the beta function for the first two magnets of the lattice, the beta functions at all quads can be written:

$$\beta(n) = \sqrt{\frac{\gamma(n)}{\gamma_0}}\left[\frac{\hat{\beta}_0 + \check{\beta}_1}{2} + S_{ign} * \frac{\hat{\beta}_0 - \check{\beta}_1}{2}\right]. \tag{4}$$

Equations (1)-(4) describe the lattice scaling.

Lattice displacement response

Any magnet in the lattice displaced by y_n imparts an angular kick $K_1 L_q(n) y_n$ (radians) to the beam trajectory. This angle is transformed to a displacement y_c at the end of the linac by its lattice transport element $R_{12}(n)$.

$$y_c = R_{12}(n) K_1 L_q(n) y_n \quad \text{where} \quad R_{12}(n) = \sqrt{\beta(n)\beta_f}\sqrt{\frac{\gamma(n)}{\gamma_f}} \sin \Delta\psi(n) \qquad (5)$$

Given uniform betatron phase advance per cell ψ_c, the total phase advance from magnet n to the end of the accelerator is $\Delta\psi(n) = N_{cell}\psi_c - n\psi_c/2$. The ratio of beam motion at the end of the accelerator to vertical motion of any quadrupole in the focusing lattice $\frac{y_c}{y_n}$ is plotted out along the length of the machine as $h(s) \equiv y_c/y_n$ in figure 1 below for the following input values taken from SLAC Report-387:

Energy	16.5 → 500 GeV
# of cells N_{cell}	380
phase advance/cell ψ_c	94°
length of 1st cell L_{c0}	4.4 m
$\hat{\beta}_0$	7.5 m
81	1.3 m

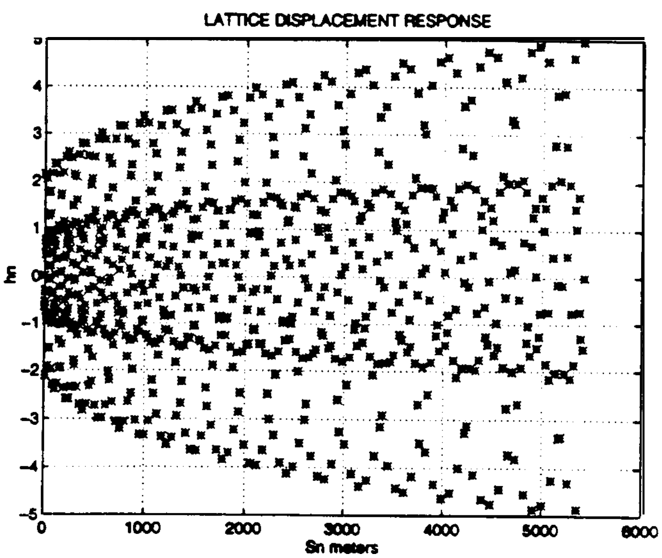


figure 1

In figure **1,** each (*) represents a quadrupole. Focusing quads make up the large envelope sine pattern and defocusing quads form the inner envelope. Maximum sensitivity ranges from about 2 at the beginning of the accelerator to 5 at the end.

Ground motion induced jitter

If the instantaneous position of all quads were known at the time of beam transmission, superposition of all contributions to beam displacement could be summed up using eq (5). Although it is instinctive to visualize ground noise as made up of waves traveling in all directions through time, this motion is effectively frozen over the 20 μ **sec** it takes for the beam to propagate down the machine. Jitter in the output beam position is determined by the instantaneous spatial distortions of the lattice. The beam acts as a strobe sampling the position of all quadrupoles along the machine. At the instant of beam transmission, the lattice has some random distribution of magnet displacements y_n as sketched below.

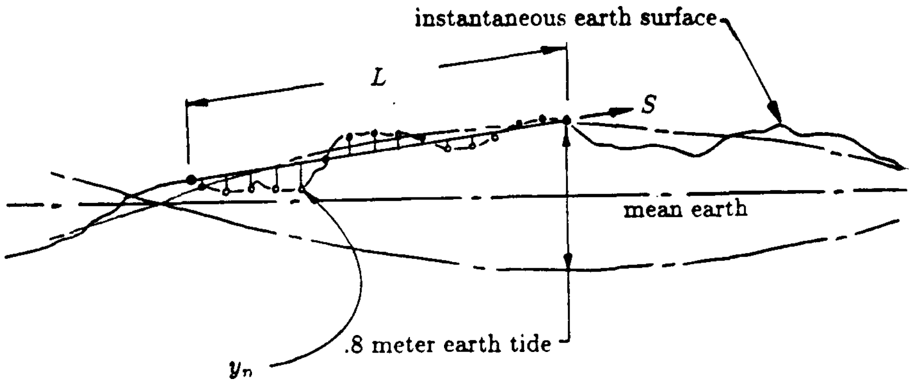


figure 2

The centerline about which all misalignments are distributed is a straight-line fit to the average of all magnet positions. Magnet displacements y_n are measured from the instantaneous axis of the accelerator rather than the long term mean of the earth's surface. To define misalignment, there must be 3 or more magnets in the lattice. Differential motion between two magnets divorced from the rest of the lattice does not define a misalignment. Rigid translations or rotations of the entire machine do not contribute to jitter in the output beam. Indeed earth tides cause large **.8** meter diurnal changes in the earth's local radius while the machine rotates $\simeq 360^\circ$ every 24 hours. Obviously none of these numerous celestial motions has much to do with beam jitter. Magnet displacements are measured from the instantaneous axis of the accelerator rather than the long term mean of the earth's surface. Jitter in beam position is caused by ground distortions with wavelengths on order of the machine's length or shorter.

One of the simplest estimates of beam jitter is for totally random uncorrelated motion of all magnets. The variance of the coupling between beam motion at the end of the accelerator and motion of the typical individual magnet h is:

$$\langle h^2 \rangle = \frac{1}{N_q} \sum_{n=0}^{N_q-1} h^2(n) = 4.31$$

The total beam jitter is then the sum in quadrature of all the contributions from N_q magnets. Beam jitter 'grows with $\sqrt{n}$ as effects **of** uncorrelated quad motion add up. The ratio of standard deviations is:

$$\frac{\sigma_{jitter}}{\sigma_{yn}} = \sqrt{N_q \langle h^2 \rangle} = 57.3$$

Frequency spectra are commonly associated with functions of time but spatial frequencies are more relevant to ground motion induced beam jitter. The sketch of quadrupole displacements (figure 1) plots all magnets versus s at a single instant. On a later acceleration cycle the spatial pattern will have changed due to seismic motion but each displacement pattern is a member of a common stochastic ensemble. These displacement patterns are all composed from a common spectrum of spatial frequencies with wave numbers k (radians/meters). The above estimate of jitter for uncorrelated motion is equivalent to assuming that ground motion is composed of a white noise power spectrum where lattice distortion is distributed uniformly over all spatial wavelengths. Each quad's displacement has no correlation to its neighbors.

Lattice Spectral Response

Ground motion does not show this simple white noise spectra. It decreases as l/k as shorter wavelengths are considered. To estimate the σ of beam jitter for a realistic ground motion spectrum, the total lattice response must be calculated for each spatial wavelength impressed on the entire lattice. This calculation was carried out by T.Raubenheimer in section 38.3 of SLAC Report-387.

Assume that at the instant of beam transmission, magnets are displaced according to : $y_n(k) = \cos(ks, + \phi)$. Here ϕ is an arbitrary phase. The total lattice response to magnet motion with wave number k is defined as:

$$\begin{aligned} R(k, \phi) &\equiv \sum_{n=0}^{N_q-1} \Big[R_{12}(n) K_1 L_q(n) \Big] \cos(ks_n + \phi) \\ &= \sum_{n=0}^{N_q-1} h_n \cos(ks_n + \phi). \end{aligned}$$

The variance of R averaged over all random phases ϕ is computed by separating

s_n from ϕ using the identity

$$\cos(ks_n + \phi) = \cos\phi \cos ks_n - \sin\phi \sin ks_n.$$

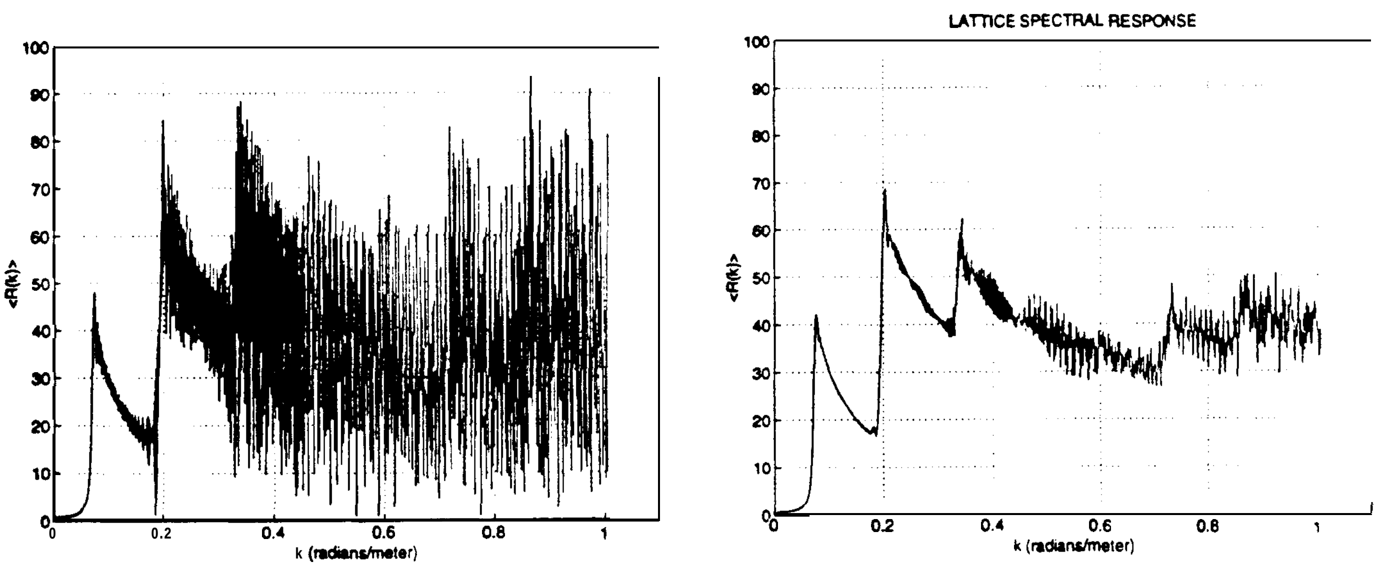
The variance of the lattice spectral response can be averaged over all phases ϕ:

$$\begin{aligned}\langle R^2(k)\rangle = & \left[\sum_0^{N_q-1} h_n \cos ks_n\right]^2 \frac{1}{2\pi}\int_0^{2\pi} \cos^2\phi d\phi \\ & + \left[\sum_0^{N_q-1} h_n \sin ks_n\right]^2 \frac{1}{2\pi}\int_0^{2\pi} \sin^2\phi d\phi \\ & - \left[\sum_0^{N_q-1} h_n \sin ks_n\right]\left[\sum_0^{N_q-1} h_n \cos ks_n\right] \frac{1}{2\pi}\int_0^{2\pi} \sin\phi \cos\phi d\phi\end{aligned}$$

After integration the variance of the lattice spectral response reduces to:

$$\langle R^2(k)\rangle = \frac{1}{2}\left[\left(\sum_0^{N_q-1} h_n \cos ks_n\right)^2 + \left(\sum_0^{N_q-1} h_n \sin ks_n\right)^2\right]. \qquad (6)$$

The standard deviation of *R(k)*, *(R)* is plotted vs wave number *k* in figure 3 below.



unfiltered

smoothed

figure 3

Lattice response to ground motion first becomes large when ground waves become as short as the betatron wavelength at the end of the accelerator; about 100 meters. Secondary resonances show up in the response at shorter wavelengths until for very short wavelengths,(large k), the amplification of ground motion in the output beam jitter is about 40 after traveling the entire length of the accelerator. The long wavelength limit ($k \rightarrow 0$) corresponds to a rigid body displacement of the entire accelerator. For the present lattice the rms value of the response function over all phases is:

$$\lim_{k\rightarrow 0}\langle R(k)\rangle = \frac{1}{\sqrt{2}}\sum_{0}^{N_q-1} h_n = .9067$$

A unit displacement of the machine with respect to the injection axis would displace the transmitted beam by about .9 units.

Transmission of Ground Motion into Beam Jitter

The response function $R(k)$ gives the output beam jitter amplitude associated with a single sinusoidal distortion of the whole lattice with wave-number k. Actual magnet motion is a random lattice distortion containing a full spectrum of wave-numbers or wavelengths. Each beam pulse samples a different pattern of random distortion but all samples in the ensemble have approximately the same spectrum of spatial distortions. The most natural characterization of this is the power spectral density $S_m(k)$, nano meters^2/(radians/m) which measures the square of the Fourier amplitudes inside wave-number band dk of this spectrum. The output beam jitter is related* to input magnet motion by:

$$S_{jitter}(k) = \langle R^2(k)\rangle S_m(k)$$

$$\sigma^2_{jitter} = \int_0^{\infty} \langle R^2(k)\rangle S_m(k)dk. \qquad (7)$$

Many measurements of the spectral density of ground motion have been made for the motion of a single point through time $S_{gr}(\omega)$ but what is needed for the beam jitter problem is power spectral density of instantaneous magnet misalignment along the entire accelerator $S_m(k)$. The spatial power spectrum of magnet misalignments $S_m(k)$ differs slightly from the actual ground motion spatial spectrum $S_{gr}(k)$ in 3 ways:

⋆ chapter 7, Random Vibrations and Spectral Analysis, D.E.Newland, 1975 a Magnets only sample the ground at discrete positions so $S_m(k)$ is a spectrum of discrete lines rather than a continuous function like $S_{gr}(k)$.

b The lattice has only a finite number of magnets so the $S_m(k)$ spectrum is bandwidth limited.

c Because the accelerator is of finite length, its axis from which magnet displacements are measured is redefined from pulse to pulse and is not exactly the same as any inertial coordinates in which total ground motion is measured.

Direct measurement of this spectrum remains to be done. Nevertheless if the directional nature of the waves is 'ignored and it is assumed they all travel with a common velocity v independent of amplitude or wavelength, frequency w and wave-number k are related by $v = \frac{d\omega}{dk}$. Surface or Rayleigh waves travel through the ground with velocities between 300 m/sec and 400 m/sec. As an exercise, the ground motion power spectrum S,,(f) from Report CERN-SL/93-53 Investigations of Power and Spatial Correlation Characteristics of Seismic Vibrations in the CERN LEP Tunnel for Linear Collider Studies, J.V.M.Sery, A.A. Sleptsov, W.Coosemans, G.Ramseier, I.Wilson, Dec 1993 will be converted to *S,(k)* using $k = \omega/(300\text{m/sec})$.

$$S_m(k) \simeq (\frac{1}{2\pi}\frac{d\omega}{dk})S_{gr}(f) = \frac{v}{2\pi}S_{gr}(f)$$

Both the converted magnet motion power spectrum $S_m(k)$ and the lattice spectral response are plotted below in figure 3.

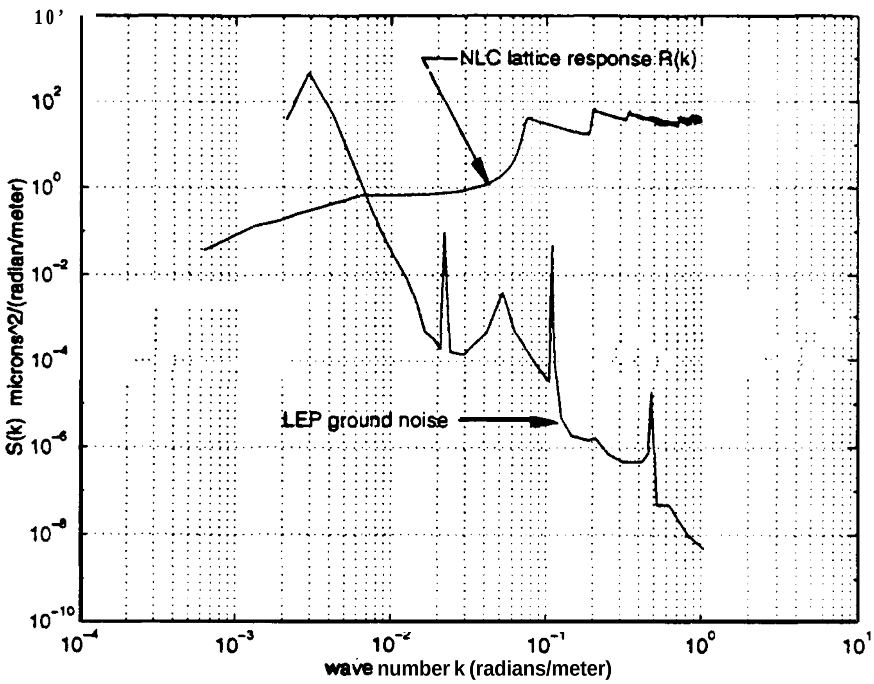


figure 4

For wavelengths longer than 150 meters,(k < .04 radian/m), the lattice response is less than 1. Using the converted LEP seismic data and the the NLC lattice spectral response $\langle R^2(k)\rangle$, the total rms variance of beam jitter can be estimated for wavelengths from 3 kilometers (k = .002) down to 6.28 meters (k = 1.0).

$$\sigma = \left[\int_{.002}^{1.0} \langle R^2(k)\rangle S_m(k)dk \right]^{1/2} = 441 \text{ nano meters.}$$

The actual beam jitter in NLC may be rather different from this number since the calculation is not based on a direct measurement of the spatial spectrum of magnet motion. Never the less lattice response to the spatial power spectrum of ground motion is one way of describing the effect of this distributed disturbance on beam jitter both in the linac and in the final focus of NLC.

Acknowledgement: This note does not yet reach the standards set by Hobey DeStaebler but it has been improved by his careful reading and critique.