NLC Note 11
Distribution: DR
February, 1995

Transverse Coupled Bunch Mode Coupling and Growth Rates for the NLC Main Damping Ring

J. Scott Berg

February 21, 1995

Abstract

Coherent modes in the presense of transverse impedances are computed for the NLC Main Damping Ring. Coupling between the bunches is considered, as well as internal degrees of freedom in the bunches. A modified version of Sam Heifets' model for the impedance [Hei94] is used. The single bunch mode-coupling threshold is found, and growth rates are computed at the expected operating current of the ring.

1 Introduction

The ring is assumed to consist of a series of magnets to guide the beam, plus various devices in the vacuum chamber (as well as the vacuum chamber itself) which generate transverse forces in response to the dipole moment of particles which passed by at an earlier time. These forces that act on a particle are described by a Hamiltonian. The time evolution of a distribution of particles is then described by the Vlasov equation. The resulting equation is Fourier analyzed in time, and the resulting eigenvalue system is solved to find growth rates of coherent modes.

This "distribution" can easily consist of several distributions of particles in various RF buckets. Thus, effects of the bunches upon one another and effects of internal degrees of freedom in the distributions can be combined into one formalism [BR].

2 Descritption of the System

The ring is assumed to be filled with either 1 bunch or 524 identical, symmetrically placed bunches. The bunches are Gaussian. The magnet lattice is assumed to be linear. Only transverse wakefields are considered. Only the $m = 0$ and $m = 1$ multibunch modes are considered. The relevant parameters of the ring are given in table 1.

	ν_x	23.81
	ν_y	8.62
	ν_s	.005
	σ_z	3.5 mm
Circumference	L	220 m
	$\langle\beta_x\rangle$	1.5 m
	$\langle\beta_y\rangle$	4.0 m
At cavities	$\langle\beta_x\rangle_{\text{cav}}$	6.5 m
At cavities	$\langle\beta_y\rangle_{\text{cav}}$	6.5 m
Half aperature	b	1.1 cm
	E_0	2 GeV
	f_{RF}	714 MHz
Harmonic number	h	524

Table 1: Parameters Used in Computation [Rau]

2.1 Impedance Model

The impedance consists of a resistive wall part, an inductive part, a part due to cavity tails, and a part due to cavity higher order modes [Hei94].

The resistive wall piece for 220 m of aluminum (a, = 3.84 x $10^7\Omega^{-1}\text{m}^{-1}$) pipe with radius 1.1 cm is given by [Cha93]

$$Z_\perp^{\text{RW}} = -i\sqrt{2}\frac{0.690}{\sqrt{-i\omega/\omega_0}}\ \text{M}\Omega/\text{m} \tag{1}$$

where $\omega_0 = 2\pi f_{\text{RF}}/h$.

The inductive piece is calculated by scaling a longtudinal inductance of 12 nH [Hei94] by $2c/\omega b^2$.

The piece due to cavity tails is computed by taking the longitudinal cavity tail of [Hei94]

$$\sqrt{2}\frac{R_{\text{cav}}}{\sqrt{-i\omega}}\theta(|\Re(\omega)| - \omega_c) \tag{2}$$

and scaling by $2c/\omega b^2$. Note that we must also scale this impedance by the ratio of the P-function at the cavities to the average ,&function, assuming that the average P-function is used in computing the coherent modes. The function θ is defined by

$$\theta(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases} \tag{3}$$

ω_c is $2\pi \times 5.34$GHz. Sam Heifets computed the loss factor Δk_l due to the cavity tail for a 3.3 mm Gaussian bunch to be 0.153 V/pC [Hei94]. This leads to

$$R_{\text{cav}} = \frac{2\pi\Delta k_l}{\Gamma(1/4) - \gamma(1/4, [\omega_c\sigma_z/c]^2)}\sqrt{\frac{\sigma_z}{c}} \tag{4}$$

where the σ_z here is 3.3 mm. γ is the incomplete gamma function [AS72]. Note that this figure is multiplied by the number of cavities, 4.

The higher-order cavity modes are given in table 2. Again, we must use the β function at the cavities, and must multiply by the number of cavities.

f (MHz)	R/Q (Ω)	$(R/Q)Q_L/(kr_0^2)$ ($k\Omega$/m)
1193.22	10.265	42.5
1693.87	0.249	0.726
1988.17	7.254	18.0
2310.60	2.203	4.71
2397.19	2.700	5.56
2499.79	18.182	35.9
3119.28	0.447	0.707
3175.58	0.039	0.0606
3346.51	0.873	1.28
3423.59	1.329	1.91
3602.96	1.721	2.36
3755.41	2.690	3.54
3961.32	0.409	0.510
3994.04	0.664	0.821
4167.96	0.143	0.169
4293.49	0.043	0.0494
4349.52	0.454	0.515
4440.57	0.556	0.618
4736.23	1.731	1.80
4817.92	0.664	0.680
4861.21	3.887	3.95
4962.20	0.632	0.629
4984.06	0.054	0.0535
5176.96	0.001	0.000954
5213.75	0.164	0.155
5274.69	0.903	0.845
5518.47	0.144	0.129
5545.85	0.452	0.402
5619.50	1.037	0.911

Table 2: Cavity parameters [Hei94]. $Q_L = 100$

3 Results

The vertical plane results are universally worse than those for the horizontal plane, so only the vertical results will be given here.

3.1 Single Bunch

If only a single bunch is considered, transverse mode coupling is seen at around 28 mA, as shown in figure 1. The threshold is almost identical whether the

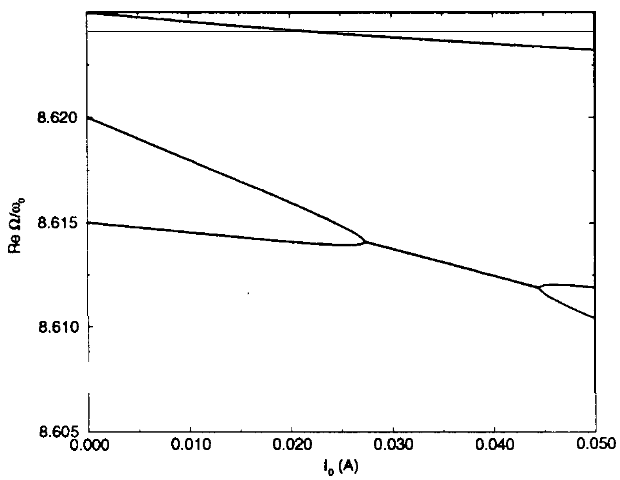

Figure 1: Transverse mode coupling, vertical plane, single bunch.

cavities are de-Qed to 100 or 3000.

3.2 Multiple Bunches

If 524 symmetric bunches are considered, there are 524 sets of modes for each internal-bunch mode. The growth rates for these modes are plotted in figures 2 through 5. Note that 1 A is the total beam current, whereas 1.75 A corresponds approximately to getting the single-bunch current right.

For Q_L = 100, the resistive wall dominates the growth rates, and the m = 1 growth rates are negligible. The growth times are around 0.7 ms at 1 A, 0.4 ms at 1.75 A. It appears that the cavity modes could be de-Qed as high as about 300 before the cavity modes would dominate the resistive wall.

For Q_L = 3000, the cavity modes clearly dominate. The smallest growth times are around 0.11 ms at 1 A, or 0.065 ms at 1.75 A. There is one cavity mode (2499.79 MHz) which is starting to drive an m = 1 mode above radiation damping. The threshold for this to occur is near 1.3 A (see figure 6). In reality, this probably won't be a problem, since the cavity modes won't usually overlay for different cavities, and there are probably other damping mechanisms present.

Note that the curvature of the lines in figure 6 is due to coupling between

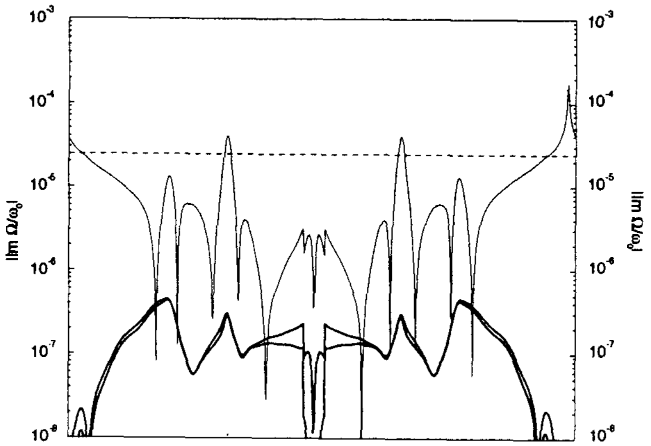

Figure 2: Multibunch modes: vertical plane, cavities Q_L = 100, 1 A. Note that the thinner line is for m = 0 (rigid bunches), and the two thicker lines are are for m = 1. Each line is really 524 separate points corresponding to the multibunch modes. The dashed line is the radiation damping rate (4.8 ms).

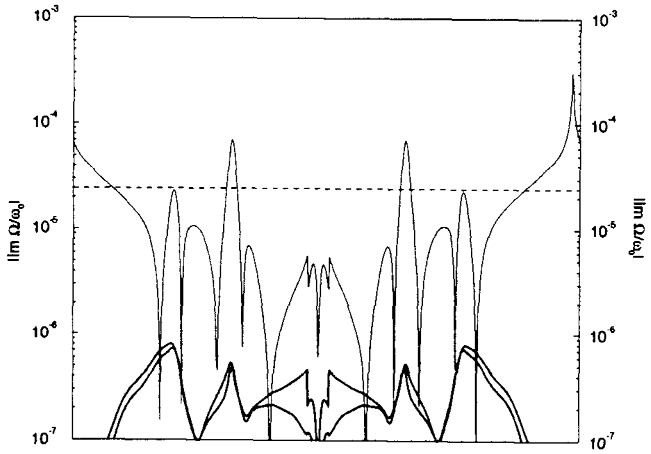

Figure 3: Multibunch modes: vertical plane, cavities Q_L = 100, 1.75 A.

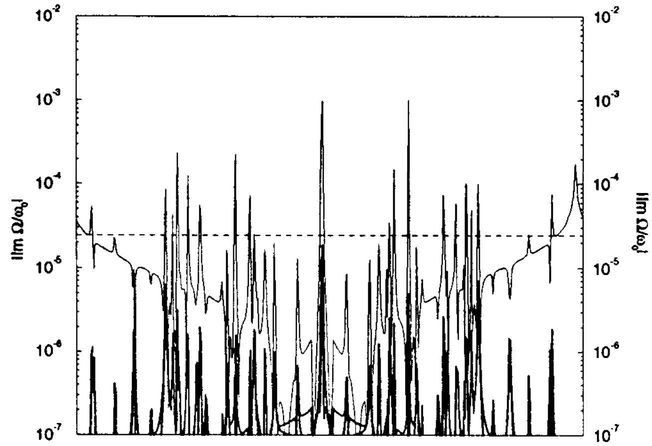

Figure 4: Multibunch modes: vertical plane, cavities $Q_L = 3000$, 1 A.

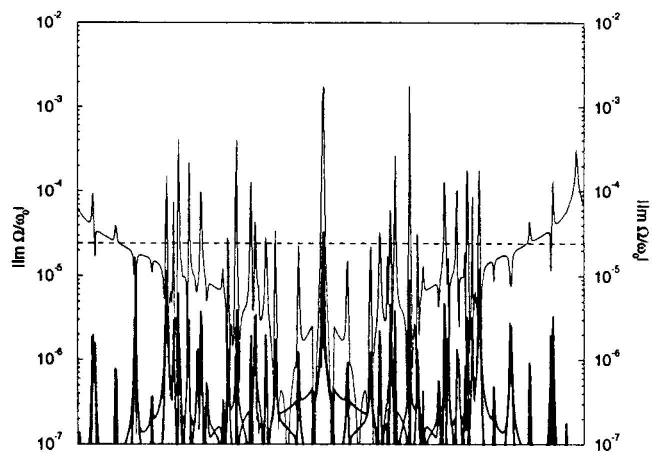

Figure 5: Multibunch modes: vertical plane, cavities Q_L = 3000, 1.75 A.

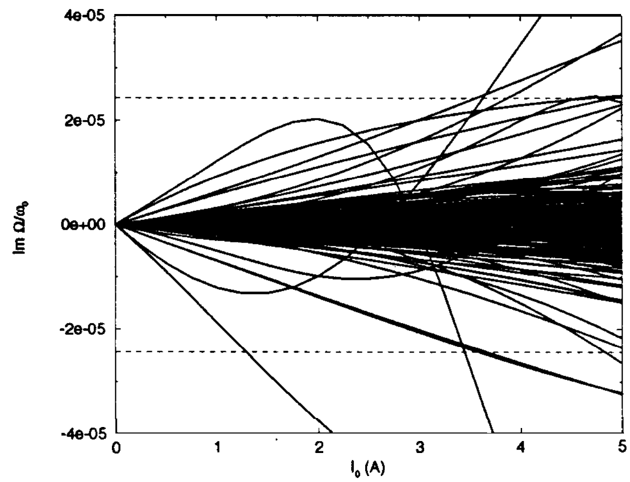

Figure 6: $m = 1$ multibunch modes: vertical plane, $Q_L = 3000$. Dashed lines are radiation damping rate.

the $m = 0$ and $m = 1$ multibunch modes. One can see that this is an extremely strong effect in some cases.

4 Conclusions

Given this model of the impedance, single bunch transverse mode coupling doesn't seem to be a problem for the NLC main damping ring.

Transverse rigid bunch $(m = 0)$ coupled bunch growth rates are significant in the NLC damping ring, and will probably need to be corrected by a feedback system. These growth rates are dominated by the resistive wall growth rate as long as the cavities are de-Qed below about 300; otherwise, the cavity modes begin to dominate the growth rates. As long as the cavities are de-Qed sufficiently, the transverse $m = 1$ growth rates are below the radiation damping rate. This is important since an $m = 1$ transverse feedback system is difficult and costly to build due to the high frequencies at which it must operate.

Several effects are not included here in this calculation:

- Longitudinal impedances are not considered.
- The cavity modes may be different for the 4 cavities due to finite manufacturing tolerances; this paper shows the worst case scenario, where they are all identical

- Chromaticity is not included. Thus, this calculation ignores head-tail damping which has a possibly significant damping effect on the $m = 0$ growth rates.

- Tune shift with amplitude is not included; this could potentially give a Landau-damping type of effect. See [Her69, Zot81] for possible beginnings of ways to compute this effect.

- Bunch-to-bunch tune shifts that would be caused by the gaps in the bunch train are ignored. See [CY86] for a method to calculate this effect.

- The effect of the gaps irregardless of the bunch-to-bunch tune shifts is ignored. However, [Koh85] demonstrates that for a given single-bunch current, the maximum growth rate occurs when the bunches are symmetrically placed.

- Modes are perturbations about Gaussian bunch distribution.

- Higher order internal degrees of freedom are ignored. This effect is typically small. There may be some mode coupling from these, but this usually occurs an much higher currents than the mode coupling of the $m = 0$ and $m = 1$ modes.

References

[AS72] Milton Abramowitz and Irene A. Stegun, editors. *Handbook of Mathematical Functions.* Dover, New York, 1972.

[BR] J. Scott Berg and R. D. Ruth. Transverse multibunch instabilities for non-rigid bunches. Work in progress.

[Cha93] Alexander Wu Chao. *Physics of* Collective *Beam Instabilities in High Energy Accelerators.* John Wiley & Sons, Inc., 1993.

[CY86] Y. H. Chin and K. Yokoya. Landau damping of a multi-bunch instability due to bunch-to-bunch tune spread. DESY 86-097, DESY, August 1986.

[Hei94] Sam Heifets. Study of the NLC DR impedance and collective effects: the first attempt. NLC-Note 8, SLAC, December 1994.

[Her69] H. G. Hereward. Landau damping by non-linearity. CERN/MPS/DL 69-11, CERN, July 1969.

[Koh85] R. D. Kohaupt. On multi-bunch instabilities for fractionally filled rings. DESY 85-139, DESY, December 1985.

[Rau] Tor Raubenheimer. Private communication.

[Zot81] B. Zotter. Longitudinal stability of bunched beams; part ii: Synchrotron frequency spread. CERN-SPS 81-19 (DI), CERN, November 1981.