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THE SHORT RANGE NLC WAKEFIELDS

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Introduction

The NLC linac structure is a detuned, disk-loaded structure consisting of 206 cells that operates at 11.424 GHz. Each cell is slightly different from its neighbors. The cell period is fixed at L = S.75 mm; the cavity radius varies slightly but is roughly b = 11 mm; the iris radius varies from a = 5.90 mm to a = 4.14 mm and the iris thickness from t = 1.26 mm to t = 2.46 mm as one moves from beginning to the end of the structure. For representative cells 1, 51, 103, 154, and 206 the iris radius is, respectively, a = 5.90 mm, 5.21 mm, 4.92 mm, 4.66 mm, 4.14 mm, and the iris thickness t = 1.26 mm, 1.66 mm, 1.86 mm, 2.05 mm, 2.46 mm. Note that for the average cell a/λ = .187, with λ = 26.25 mm the rf wavelength.

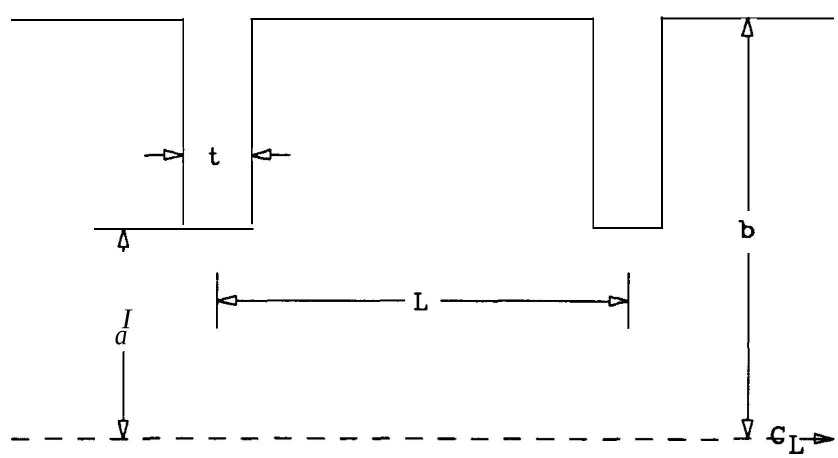

Fig. 1. The model of one cell of the NLC structure that is used in the calculations.

For our wakefield calculations we locally approximate the structure by a periodic model, for which one cell is shown in Fig. 1. A real NLC cell looks the same, except that the irises are rounded. The calculation method is as follows (for details see Ref. 1): For the geometry of one of the five representative cells we use the computer program KN7C[2] to obtain the synchronous frequencies and loss factors of the first 250 or so monopole modes, which give us the low-frequency impedance. We approximate the high frequency contribution using the so-called Sessler-Vainsteyn optical resonator model.[2-4] Fourier transforming these two contributions we obtain the longitudinal wakefield for the given geometry. More precisely, for the real part of impedance (for positive ω) we take

$$R_z(\omega) = \sum_{n=1}^{N} \pi k_n \delta(\omega - \omega_n) + \frac{2Z_0 j_{01}^2}{\pi L \zeta^2} \frac{\sqrt{\nu}+1}{(\nu + 2\sqrt{\nu} + 2)^2} \Theta(\omega - \omega_N) \qquad \omega > 0 \quad , \quad (1)$$

with k_n the loss factor (in units of V/pC/m) and ω_n the frequency of the n^{th} mode, Z_0 = 377 Ω, j_{01} = 2.41 the first zero of the Bessel function J_0, ζ = 0.824, $\nu = 4a^2\omega/(c\bar{L}\zeta^2)$, with c the speed of light and $\bar{L} = \sqrt{L(L-t)}$; $\Theta(x) = 0$ for x < 0, 1 for x > 0. The wakefield becomes

$$W_z(s) = \sum_{n=1}^{N} 2k_n \cos\frac{\omega_n s}{c} + \frac{Z_0 c j_{01}^2 \bar{L}}{\pi^2 a^2 L} \int_{\nu_N}^{\infty} \frac{\sqrt{\nu}+1}{(\nu + 2\sqrt{\nu} + 2)^2} \cos\left(\frac{\zeta^2 \bar{L} s \nu}{4a^2}\right) d\nu \quad , \quad (2)$$

with $\nu_N = 4a^2\omega_N/(c\bar{L}\zeta^2)$. To get the wakefield of the entire NLC linac structure we average the wakes obtained in this way for each of the five representative cells.

The transverse (dipole) wakefield is obtained in the analogous manner, but using the computer program TRANSVRS[5] to obtain the dipole mode frequencies and loss factors. We first obtain the longitudinal *dipole* wakefield $W_z^{(1)}(s)$ using an equation very similar to Eq. (2), except using the transverse mode frequencies and loss factors, and replacing j_{01} by j_{11} = 3.83, the first zero of the first order Bessel function J_1. Then, following the Panofsky-Wenzel theorem[6] we obtain the transverse dipole wakefield: $W_x(s) = \int_0^s W_z^{(1)}(s')\, ds'$.

Results

Consider a periodic structure with the geometry of NLC cell 103. For this case the contribution of the first term in Eq. (1) to the the longitudinal impedance, when averaged over frequency bins, is shown by the histogram of Fig. 2. On the same plot the Sessler-Vainsteyn prediction is given by the dashed curve. Note that this model, in general, agrees well with the numerical results, even at the lower frequencies. This was also found to be true for the four other representative cell geometries. Note also that the area under R,(w) over all frequencies is proportional to the longitudinal wake at the origin $W_z(0)$. We find that for our representative cell geometries the integration of the Sessler-Vainsteyn curve from $(\omega a/c)^2$ = 1000 to infinity gives about a 30% contribution to the total area, which illustrates why the analytic extension is necessary. At high frequencies $R_z \sim w^{-3/2}$. Therefore, if we want to obtain 95% of $W_z(0)$ from the modal sum alone we would need to find the modes over $(0.3/0.05)^2 =$ 36 times the frequency range that we did, which is a practical impossibility.

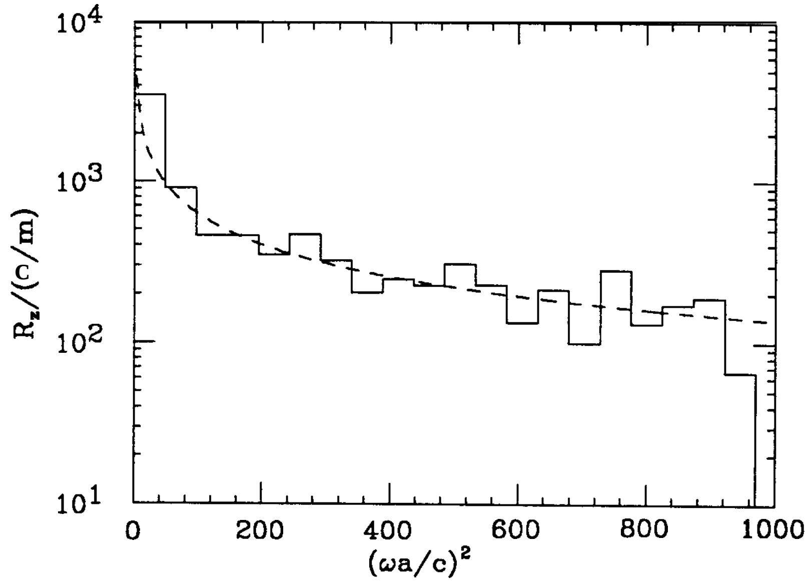

Fig. 2. The real part of the longitudinal impedance, averaged over frequency bins, for the geometry of NLC cell 103. The Sessler-Vainsteyn prediction is given by the dashed curve.

The short range longitudinal wakefields for cells 1, 51, 103, 154, and 206 of the NLC structure, as obtained by Eq. (2), are shown in Fig. 3. The typical rms bunch length in the NLC will be σ_z = 0.1 mm, so the abscissa range of the plot should be sufficient. We should point out that the values at the origin, which should equal[7,8] $W_z(0) = Z_0c/(\pi a^2)$ are about 5%-10% low, indicating some calculation error. The average wake for the entire structure is given by the dashed curve. A fit to the average wake, given by

$$W_z = 1388(V/pC/m) \cdot \exp\left[-1.16(s/mm)^{0.55}\right] \quad , \tag{3}$$

is shown by the dots.

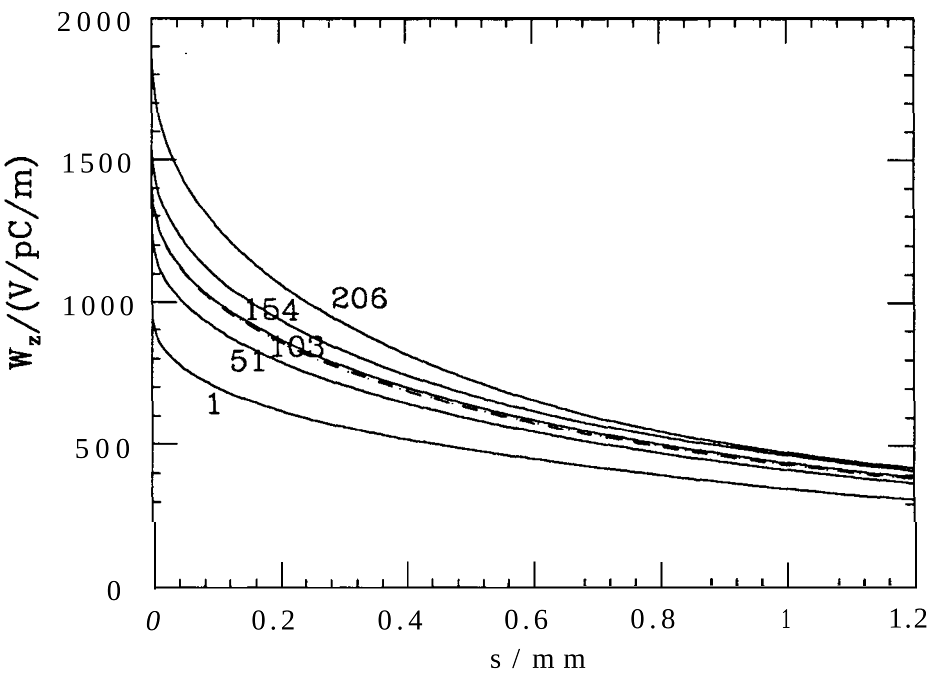

Fig. 3. The longitudinal wakefield of cells 1, 51, 103, 154, and 206 of the NLC structure. The average is the same as that of cell 103. The dashed curve represents the average wake, the dots the model fit, Eq. (3).

The real part of the longitudinal *dipole* impedance $R_z^{(1)}$ for cell geometry 103, averaged over frequency bins, is shown in Fig. 4. The Sessler-Vainsteyn model is given by the dashes. The agreement is fairly good, although it appears to be systematically high, maybe by as much as 10%. This is also true for the results for the other 4 representative cell geometries. The short range transverse wakefields for the representative cells are given in Fig. 5. In this case the slopes at the origin should equal $W_x'(0) = 2Z_0c/(\pi a^4)$; our results are slightly higher, but in no case by no more than 4%. In all cases the Sessler-Vainsteyn contribution to $W_x'(0)$ is about 40%. The average wake for the entire structure is indicated by the dashed curve in Fig. 5. A fit to the average wake, given by

$$W_x = 88(V/pC/\text{mm}/\text{m}) \cdot (1 - \exp\left[-0.89(s/mm)^{0.87}\right]) \quad , \tag{4}$$

is indicated by the dots.

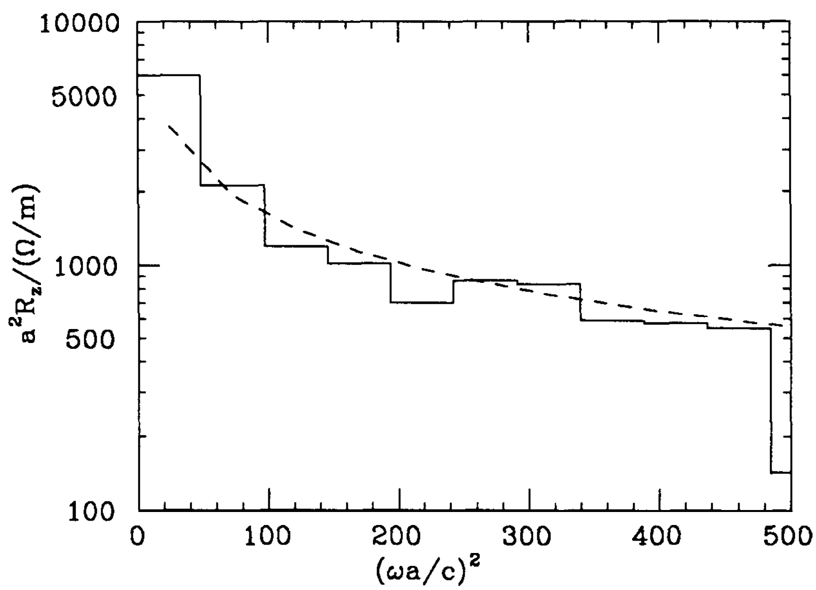

Fig. 4. The real part of the longitudinal *dipole* impedance, averaged over frequency bins, for the geometry of NLC cell 103. The Sessler-Vainsteyn prediction is given by the dashed curve.

Discussion

The wakefields given in this paper can be used as Green functions for an arbitrary distribution of particles, to perform energy spread and beam dynamics calculations,

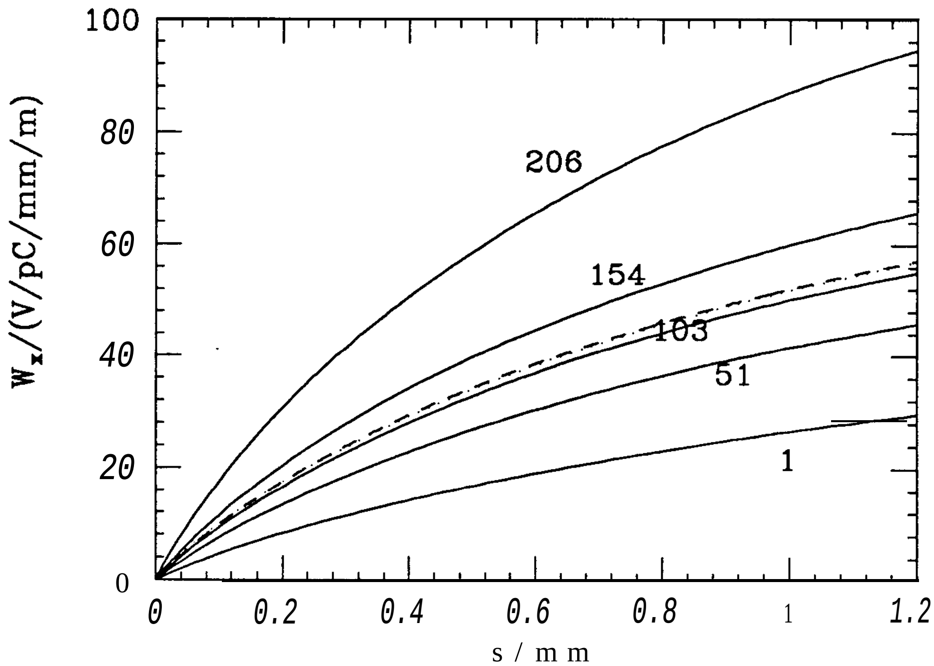

Fig. 5. The transverse (dipole) wakefield of cells 1, 51, 103, 154, and 206 of the NLC structure. The dashed curve gives the average wake, the dots the model fit, Eq. (4).

provided the bunch particles are ultra-relativistic and that the bunch is not extremely short (compared to the nominal 0.1 mm gaussian bunch). These are the asymptotic wakefields, i.e. they do not apply until the bunch has traversed a sufficient number of cells, which must be somewhat larger than $a^2/(4L\sigma_z)$,[7,9] which is 10 cells in the case of the NLC.

Finally, as mentioned above, there is some error in our wakefields, although we expect the extent to be small. The KN7C and TRANSVRS results can have errors that are difficult to analyze, and the Sessler-Vainsteyn model, although appearing to agree reasonably well with the numerical calculations at the higher frequencies, stands on a somewhat weak theoretical foundation. Therefore, estimating the calculation error is a difficult problem. The author is currently working with Gennady Stupakov on just this question. We hope to have an answer soon.

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