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A Spin Rotator System for the NLC

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1 Introduction

This note discusses the problem of controlling the orientation of the electron (or positron) spin vector in the NLC after damping ring extraction. There are seemingly many rotator options such as horizontal and vertical nested bending magnet schemes[1], resonant arcs*, or solenoid systems3 which could he placed at various points in the collider. However, given the extremely small vertical emittance of the NLC and a reasonable constraint on net momentum compaction (R_{56}), it is concluded that vertical bending schemes are not feasible. Finally, a detailed design for an uncoupled low energy solenoid based rotator system is presented.

2 Spin Rotator Design Issues

The spin rotator system must meet several design criteria:

1). *The rotator must be flexible so that longitudinal IP spin orientation can be achieved for various collider energies and different configurations of a multiple* IP Switch[4].
2). *The system must not significantly dilute the transverse emittances.*
3). *The net momentum compaction must be small such that energy fluctuations do not become longitudinal position fluctuations comparable to the 100 μm IP bunch length*[5].
4). *The rotator should be located such that total spin diffusion due to energy spread is held to a reasonably small level.*
5). *The system should be short, simple and as robust as possible.*

3 The Half Serpent

The use of a “half serpent” has been suggested[1] to restore the longitudinal polarization of the damped beam at -10 GeV. Such a system requires nested horizontal and vertical chicanes which inevitably dilute the transverse emittances through synchrotron radiation (SR) fluctuations. In order to estimate this dilution we take the vertical chicane as a fundamental building block of the rotator (fig. 1).

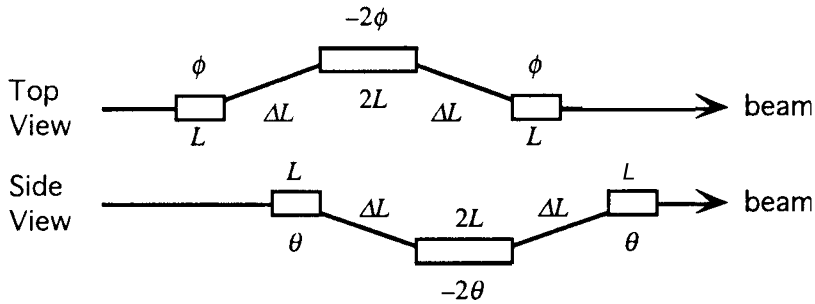

Fig. 1. Half serpent spin rotator of reference 1.

From reference 5, the vertical emittance dilution of only the vertical chicane in fig. 1 is

$$\Delta\gamma\varepsilon_y = 8\times10^{-8}\cdot E^6\,\frac{\theta^5}{L^2}\left[\Delta L + L + \frac{\hat{\beta}+\breve{\beta}}{3}\right] , \tag{1}$$

where E is the beam energy (in GeV), and the magnets are of length L (in meters), bend angle 8 (in radians) and separated by AL (in meters). The maximum and minimum beta function• values, β, in the chicane are in meters and the emittance is in meter-radians. To estimate the emittance dilution we use the following parameters.

$$\Delta L = 2L, \quad \varphi_s = \frac{\pi}{2} = a\gamma\theta, \quad \hat{\beta} = 2\breve{\beta} = L_{tot} = 8L \tag{2}$$

This choice of beta values is approximately the minimum for a drift ($\alpha_i = -\alpha_f = 1$). The spin rotation, $\varphi_s = \pi/2$, is achieved in the first bend, $a = (g-2)/2$ is the anomalous magnetic moment and yis the Lorentz factor. Using (2) we anive at

$$\frac{\Delta\gamma\varepsilon_y}{\gamma\varepsilon_y} = 7.5\times10^{-8}\cdot\frac{E}{\gamma\varepsilon_y L} . \tag{3}$$

Introducing the NLC damped vertical emittance of $\gamma\varepsilon_y = 2\times10^{-8}$ m-rad and requiring an emittance dilution of less than 2%, we find a simple scaling between the length of the bend magnets and the beam energy.

$$\mathrm{L[m]} > 190\cdot\mathrm{E[GeV]} \tag{4}$$

Even at 2 GeV the length of the bend magnets is 380 meters for a chicane type half serpent. This estimate does not take into account the added SR vertical emittance dilution due to vertical dispersion in the nested horizontal bending magnets. Furthermore, the momentum compaction and large dispersion of such a lengthy system are also intolerable. The R_{56} for a chicane-5 is $R_{56} = 2\theta^2(\Delta L + 2L/3)$ which, using the parameters of (2), gives

$$R_{56}[\mathrm{m}] > \frac{480}{\mathrm{E[GeV]}} , \tag{5}$$

which produces an unreasonable 240 meter R_{56} at 2 GeV. In conclusion, due to the extremely small NLC vertical emittance, simple vertical spin rotator chicanes are not possible anywhere after damping in the NLC.

More complicated schemes may be imagined which include quadrupoles to minimize dispersion such that the SR dilution and momentum compaction are reduced, however the rotator system needs to be flexible and for serpents this requires mechanically displacing the heamline components as the bend fields are varied. The tight tolerances on quadrupole alignment and horizontal bend roll angles in the NLC will make such a system mechanically difficult if not impossible to achieve.

4 Solenoid Rotator Solutions

I. Cross Plane Coupling Correction

Solenoid magnets can be implemented to rotate the electron spin about the longitudinal axis by φ_s, however they radially focus the beam and unfortunately introduce a roll about the beam axis[3]. The damped flat beam will be rolled through an angle φ_b, which is one half that of the spin precession.

$$\varphi_s = \left[1 - \left(\frac{g-2}{2}\right)\right]\frac{B_z L_s}{(B_0\rho)} \approx \frac{B_z L_s}{(B_0\rho)} = 2\varphi_b \qquad (6)$$

In (6) above L_s is the effective solenoid length, B_z is the longitudinal magnetic field and $(B_0\rho)$ is the usual magnetic rigidity. For electrons, $(g-2)/2 \approx 1.16\text{x}10\text{-}3$, is the anomalous magnetic moment and can be ignored.

The large x-y coupling introduced in passing the solenoid can potentially destroy the vertical emittance. It can be shown that the vertical projected emittance at the end of a solenoid of strength φ_s is

$$\varepsilon_y^2 = \varepsilon_{x_0}^2 S^4 + \varepsilon_{y_0}^2 C^4 + \varepsilon_{x_0}\varepsilon_{y_0} C^2 S^2\left(\beta_x\gamma_y - 2\alpha_x\alpha_y + \beta_y\gamma_x\right) , \qquad (7)$$

where $C \equiv \cos(\varphi_s/2)$, $S \equiv \sin(\varphi_s/2)$, $\beta_{x,y}$, $\alpha_{x,y}$, $\gamma_{x,y}$, ε_{x_0,y_0} are the beam Twiss parameters and initial emittances at the solenoid entrance. For $\varphi_s = \pi/2$, equal Twiss parameters in the two planes and a horizontal to vertical emittance ratio of 100 as in the NLC, the vertical emittance after the solenoid scales up by

$$\frac{\varepsilon_y}{\varepsilon_{y_0}} = \frac{\varepsilon_{x_0}}{\varepsilon_{y_0}} S^2 + C^2 \approx 50 . \qquad (8)$$

A coupling compensation system needs to be included which is able to reliably correct this large emittance increase. A system of skew quads can be imagined but has the undesirable character of requiring new skew quad settings for each new solenoid setting. A more robust correction scheme can be had by splitting the solenoid in half and introducing a canceling symmetry between the two halves[6]. The first half solenoid rotates the beam (spin) about the longitudinal axis by $\varphi_s/2$ (φ_s). If we follow this by a transfer matrix which is +I in the x-axis and -1 in the y-axis the beam will be reflected about the y-axis where another half solenoid of equal strength will rotate the beam to its flat state again and a net spin precession of $2\varphi_s$ will be accomplished. If $k \equiv \varphi_s/2L_s = B_z/2(B_0\rho)$, the transfer matrix of the solenoid7 is

$$\mathbf{R}_s = \begin{pmatrix} C^2 & SC/k & SC & S^2/k \\ -kSC & C^2 & -kS^2 & SC \\ -SC & -S^2/k & C^2 & SC/k \\ kS^2 & -SC & -kSC & C^2 \end{pmatrix}. \tag{9}$$

Inserting the *reflector* beam-line between the two solenoids produces a full system transfer matrix of

$$\mathbf{R}_s \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \cdot \mathbf{R}_s = \begin{pmatrix} \cos\varphi_s & k^{-1}\sin\varphi_s & 0 & 0 \\ -k\sin\varphi_s & \cos\varphi_s & 0 & 0 \\ 0 & 0 & -\cos\varphi_s & -k^{-1}\sin\varphi_s \\ 0 & 0 & k\sin\varphi_s & -\cos\varphi_s \end{pmatrix}. \tag{10}$$

As long as the solenoids have equal strength, all x-y coupling is canceled independent of solenoid settings. The focusing dependence can be compensated with matching quads.

II. Location of the Rotator

Solenoid rotator systems will be limited to low energy applications due to the necessary scaling of solenoid strength with energy. For the NLC this may seem like a major disadvantage since the second bunch compressor[5] includes a 180 degree arc at 10 GeV which will rotate a transverse incoming spin vector many times potentially depolarizing the beam due to incoming energy spread. However, a calculation of this depolarization shows it to be small. If the arc bends the beam by π, the spin will be rotated by $\varphi_s = a\gamma_0(1+\delta)\pi$ (11.4 turns) and the change in the spin magnitude will be $P(\delta)/P_0 = \cos(a\gamma_0\delta\pi)$. The mean polarization over a beam with a Gaussian energy spread σ_δ is

$$\overline{P} / P_0 = \int e^{-\delta^2/2\sigma_\delta^2} \cos(a\gamma_0\delta\pi)\, d\delta = e^{-(a\gamma_0\pi\sigma_\delta)^2/2} . \tag{11}$$

For σ_δ = 0.25% at 10 GeV[8,9] the relative depolarization is only 1.6%. From this we conclude that a solenoid based rotator system can conveniently be placed at 2 GeV immediately after damping ring extraction with very little net spin diffusion.

III. Rotator Flexibility

A fully flexible rotator can be made by placing a short horizontal bending section between two such split solenoid segments as described above. If the four solenoids are each capable of providing a maximum ±45 degrees of spin rotation around the longitudinal axis and the bend section provides 90 degrees around the vertical axis, the system will provide arbitrary control of the IP spin orientation as long as the spin sign is reversible by some means prior to damping. The net spin rotation through the system is symbolized in (12) below.

$$\Omega_{tot} = \Omega_{sol34} \cdot \Omega_{bend} \cdot \Omega_{sol12} = \begin{pmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{pmatrix} \cdot \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (12)$$

Here $c_i \equiv \cos(\varphi_i)$, $s_i \equiv \sin(\varphi_i)$, where φ_i is the spin rotation of the ith section and $i = 1,2,3$ indicates the first solenoid pair , the bend section, and the second solenoid pair respectively. Since the bend section has a constant rotation of $\varphi_2 = \pi/2$ and the input spin vector from damping ring extraction will be vertical, the spin vector after the full system is

$$\bar{\mathbf{S}} = \Omega_{tot} \cdot \begin{pmatrix} 0 \\ \pm 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \mp\sin(\varphi_1)\cos(\varphi_3) \\ \pm\cos(\varphi_1)\cos(\varphi_3) \\ \pm\sin(\varphi_3) \end{pmatrix} . \quad (12)$$

If the solenoid fields are reversible, then any arbitrary spin orientation is achievable.

IV. Rotator Beamline Optics

The reflector beamline (between solenoids) is built with four FODO cells each with 90” betatron phase advance in x and 45” in y. Three cells of 120°,60° are possible, however the chromaticity of the 120° cells is larger than desirable[10]. The short bend section (mini-arc) is a simple missing magnet scheme[11] containing six horizontal bend magnets and four 90° x and y FODO cells. The net horizontal bend angle at 2 GeV is $\pi/2a\gamma$ = 19.83”. The peak horizontal dispersion is 260 mm and the total R_{56} is 40 mm. The solenoids are 1.39 meters in length with a maximum field strength of ±37.8 kG. There is also a four quadrupole beta matching section between the first solenoid pair segment and the mini-arc and another matching section between the mini-arc and the second solenoid pair segment. These matching sections are used to maintain the periodic beta functions in the mini-arc through all possible solenoid settings. This is necessary since the solenoids radially focus the beam. The reflector sections provide coupling cancellation while the matching sections provide constant beta functions at the output of the entire system. Fig. 2 shows the beta functions of the entire system with solenoids off (fig. 2a) and solenoids full on (fig. 2b). With the solenoids at full strength the x and y beta functions between them are strongly coupled creating very large vertical beta functions due to the locally rolled beam. The dispersion functions are shown in fig. 2c.

V. Chromaticity of the Rotator System

It is important to cancel the large solenoid coupling over a range of particle energies which is comparable to the beam energy spread. The reflector beamline transfer matrix will break down at some level for off energy particles resulting in uncorrected coupling transmitted through the system. If the solenoid rotator system is located immediately after damping ring extraction and before the first bunch compressor rf-section the energy spread will be small at -0.1% rms[5]. This bandpass is easily achieved in the worst case (all four solenoids at maximum strength) by using FODO cells with ≤90° phase advance. Fig. 3 shows the mono-energetic system bandpass with all four solenoids at full strength, $\varphi_S(\text{pair}) = \pi/2$. Fig. 4 shows the emittance as the Gaussian energy spread is increased. The vertical emittance increase at 0.1% rms Gaussian energy spread is <0.5%.

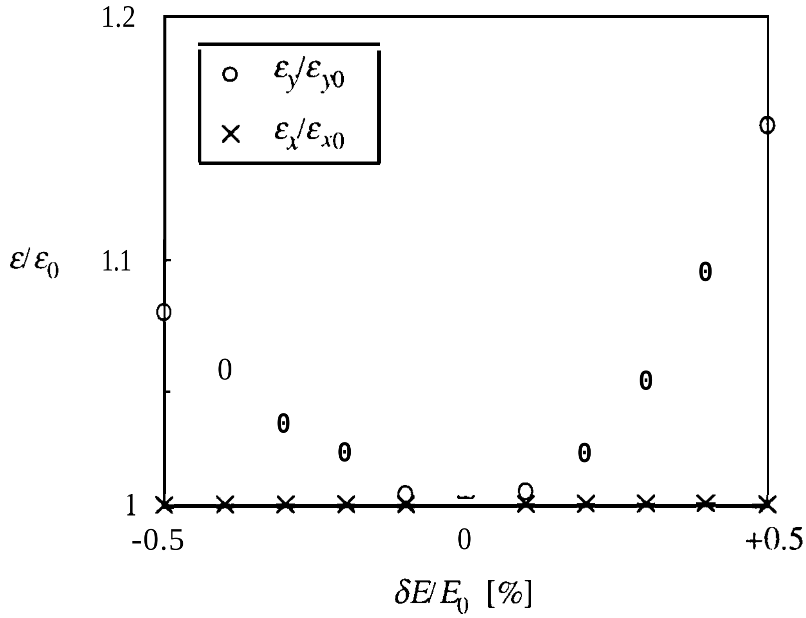

Fig. 3. Energy band-pass of the complete solenoid based rotator system for a mono-energetic beam. The transverse emittance is the single plane projected rms value and the nominal emittance values are ε_{x_0} = 588 μm-μrad, ε_{y_0} = 5.88 μm-μrad.

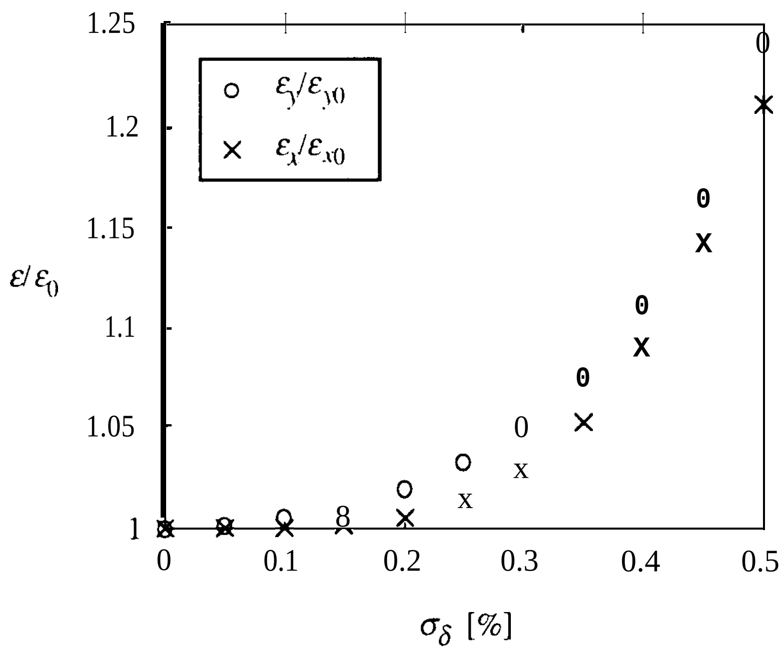

Fig. 4. Gaussian rms energy spread dependence of relative chromatic emittance growth.

VI. Longitudinal Phase Space and Synchrotron Radiation

The rotator system has very little impact on the performance of the first bunch compressor. The longitudinal transfer matrix of the first bunch compressor, not including the spin rotator, is*

$$\mathbf{R}_{bc1} = \begin{pmatrix} 1 + fR_{56} & R_{56} \\ f & 1 \end{pmatrix} . \tag{13}$$

To minimize the final bunch length after compression, the rf parameters, f, are chosen so that $1 + fR_{56} = 0$. In this case, adding the spin rotator system with $\alpha \equiv R_{56}(\mathrm{rot})$ changes only the R_{66} element of the total transfer matrix.

$$\mathbf{R}_{bc1} \cdot \mathbf{R}_{rot} = \begin{pmatrix} 0 & R_{56} \\ f & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & \alpha \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & R_{56} \\ f & 1 + \alpha f \end{pmatrix} \tag{14}$$

The bunch length after first compression is unchanged by the rotator and the energy spread after compression is insignificantly smaller ($f = 2$ m^{-1}, a = -0.04 m).

$$\sigma_{zf} = \sigma_{\delta i} R_{56} \ , \qquad \sigma_{\delta f} = \sqrt{\sigma_{zi}^2 f^2 + \sigma_{\delta i}^2 (1 + \alpha f)^2} \tag{15}$$

The order of magnitude of the emittance increase due to synchrotron radiation in the mini-arc is estimated[5,12] using average parameters in the bend magnets.

$$A\, \gamma\varepsilon_x \approx 4 \times 10^{-8} \cdot E^6 \cdot \frac{4L}{|\rho|^3} \cdot \left(\frac{\eta^2 + (\eta\alpha + \eta'\beta)^2}{\beta} \right) \tag{16}$$

Using bend length and bend radius (L = 1 m, ρ = 17.3 m), average horizontal dispersion and its derivative (η ~ 150 mm, η' ~ ±80 mrad), and the horizontal Twiss parameters (β ~ 3 m, a ~ ∓1) in the four bends with maximum dispersion and using $\gamma\varepsilon_x$ = 2.3x10-6 m-rad, gives the insignificant fractional SR emittance increase at 2 GeV of

$$\frac{\Delta\gamma\varepsilon_x}{\gamma\varepsilon_x} \sim 1 \times 10^{-5} . \tag{17}$$

This result has been verified with a detailed computer calculation.

V. Tuning and Errors

The tightest tolerances on this system are in the construction of the reflector beamline. One of the eight quadrupoles in the reflector must be built with a gradient which meets the absolute specification to an accuracy of -0.2% (2% vertical emittance increase). Most other quadrupoles are significantly looser in this tolerance. In any case, a small tunable coupling correction section is already desirable for minimizing dumping ring extraction coupling[13] and this same corrector section can be used to cancel any residual coupling due to reflector

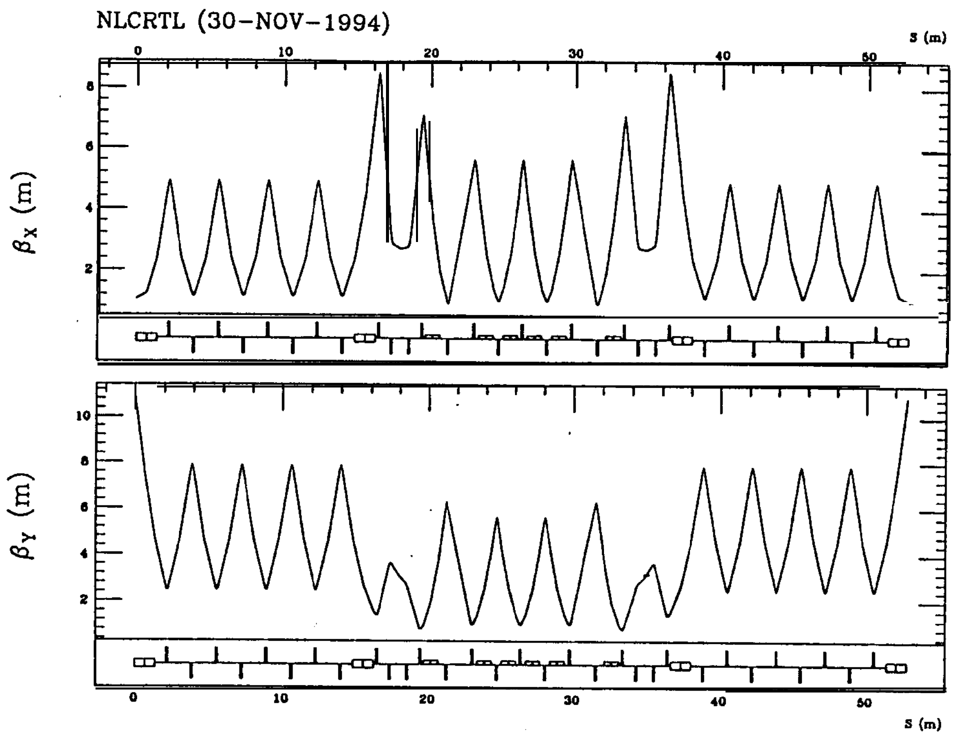

Fig. 2a. Horizontal and vertical beta functions for spin rotator system with all four solenoids switched off.

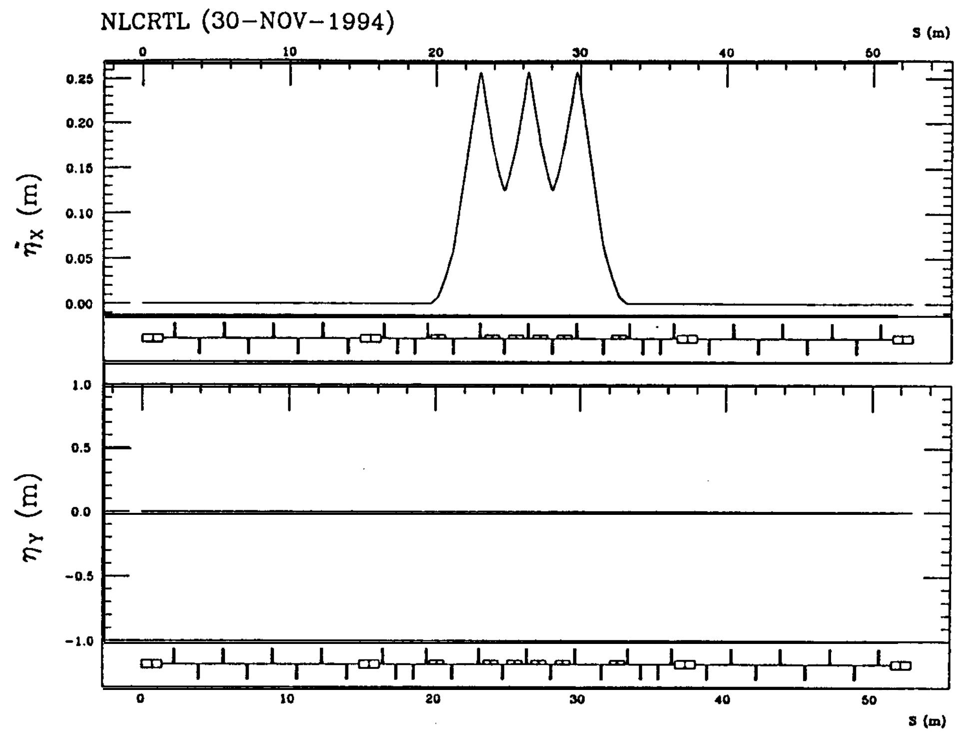

Fig. 2c. Horizontal and vertical dispersion functions for spin rotator system (same for solenoids off or on).

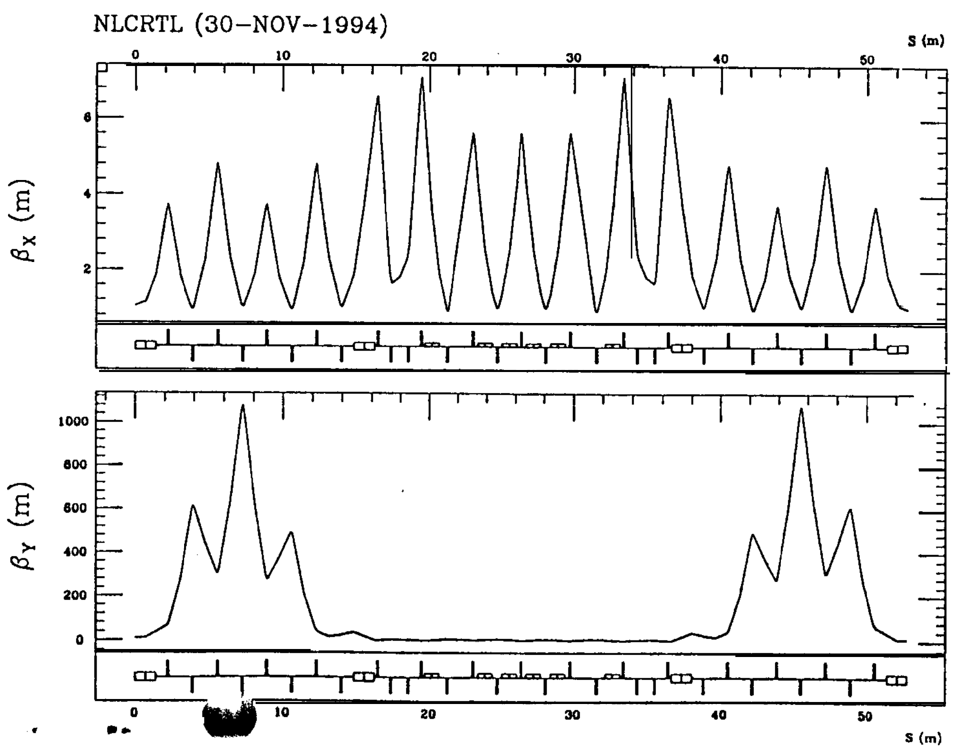

Fig. 2b. "Beta functions" (σ^2/ε_0) with all solenoids at maximum field, ϕ_s(pair) = $\pi/2$. The large coupling between solenoid pairs increases betas locally, however, the matching quads are set so that betas are unchanged in the mini-arc section and after the last solenoid.

imperfections. The tunability of this system has been studied in some detail using the ***Final*** *Focus Flight Simulator*[14] program. Random quadrupole gradient errors of 1% rms have been easily compensated using a four-skew-quad corrector section where the skew quads are placed at appropriate phase advance points to be initially orthonormal to each other[15].

5 Conclusions

The best spin rotator for the NLC appears to be a solenoid system at 2 GeV immediately after damping ring extraction. Such a system can be built to reliably cancel cross-plane coupling independent of rotator settings and has the advantage of being fully flexible so that any IP spin orientation is achievable. It then introduces no constraints on the collider energy or IP switch position. The only disadvantage to the system is a -1.6% relative depolarization at 10 GeV in the second bunch compressor. A rotator based on vertical bend sections does not appear to be a viable option due to the small vertical emittance.

6 References

1. T. Fieguth, *Snakes, Rotators, Serpents and the Octahedral Group*, SLAC-PUB-4195, January, 1987.

2. T. Limberg, P. Emma, R. Rossmanith, *The North Arc of the SLC as a Spin Rotator*, Proceedings of the 1993 Particle Accelerator Conference. Washington, D.C., 1993.

3. T. H. Fieguth, *The Optical Design of the Spin Manipulation System for the SLAC Linear Collider,* Proceedings of the 1989 Particle Accelerator Conference, Chicago, IL., March, 1989.

4. R. Helm, Private Communication,

5. T. 0. Raubenheimer, P. Emma, S. Kheifets, *Chicane and Wiggler Based Bunch Compressors for Future Linear Colliders,* Proc. of the 1993 Particle Accelerator Conference, Washington, D.C., 1993.

6. A similar suggestion has been made previously by W. Spence.

7. K. L. Brown *et al.*, CERN 80-04, March 1980.

8. T. Raubenheimer, *Bunch Compressor Parameters*, NLC-Note 2, SLAC, October, 1994.

9. F. Zimmermann, *Longitudinal Single Bunch Dynamics and Synchrotron Radiation Effects in the Bunch Compressor*, NLC-Note 3, SLAC, October, 1904.

10. The chromaticity reduction was suggested by Ron Ruth.

11. This bend section scheme was suggested by Tor Raubenheimer.

12. R. H. Helm, M. J. Lee, P. L. Morton, M. Sands, *Evaluation of Synchrotron Radiation Integrals,* SLAC-PUB-1193, March, 1973.

13. T. Raubenheimer. Private Communication.

14. M. Woodley, Private Communication.

15. P. Emma, *A Skew Correction Section for Linear Colliders,* to be published.