

Bunch-to-bunch phase variation in the NLC DR

Equilibrium **bunch-to-bunch** phase variation in the DR may **be** translated to the bunch spacing and energy variation in the **linac** and ought to **be** under control. The tolerable variation of the bunch spacing is ± 20 ps (T. **Raubenheimer**). Here is an attempt to make a crude estimate of the effect.

The main reason for the phase variation is the beam loading in the RF cavities, which may depends on the location of a bunch in a train due to transients in the cavities-induced by the gaps between trains. To estimate the effect, let us scale parameters of the HER ring of the B-factory. The nominal parameters of the HER are close to the DR: the average current $I = 1.1$ A and the cavity voltage $V = 0.9$ MeV. A cavity has the shunt impedance $R_s = 3.5$ MOhms and $Q_0 = 3 \cdot 10^4$ at $f_{RF} = 476$ MHz. Hence, the NLC DR may have two cavities of this type and, if R_s scales as $\omega^{-1/2}$, each would have $R_s = 2.86$ MOhms and the same R_s/Q_0 as that of B-factory cavity.

(The broad-band loss factor for the B-factory cavity has been estimated as 0.5 V/pC for the beam pipe radius $b = 5$ cm and bunch length $\sigma = 1$ cm. If it scales as $k_l \propto \frac{1}{b} \sqrt{g/\sigma} \propto \sqrt{\sigma/b}$, it is 8% larger for the DR ($b = 1.4$ cm and $\sigma = 3.3$ mm).

The loss to the radiation in the DR is 0.32 MV/turn in the dipoles. Wigglers increase it to $U = 0.65$ MV/turn. Hence, the **beam** phase $\cos \theta_b = U/V_c^{tot} = 0.43$ for $V_c^{tot} = 1.5$ MV. That gives the coupling

$$\beta = 1 + \frac{2R_c}{V_c} I_{dc} \cos \theta_b$$

equal to $\beta = 4.63$. The loaded $Q_L = 6472$ and the **detuning** angle

$$\tan \delta = \frac{\beta - 1}{\beta + 1} \tan \theta_b$$

is $4 \tan \phi_z = 1.35$. Hence, the detuning for the optimum matching

$$\frac{Aw}{\omega_{RF}} = \frac{\tan \phi_z}{2Q_L} = 1.0410^{-4}$$

or $\Delta f = f_{RF} - f_c = 0.067$ MHz-small compared to the revolution $f_0 = 1.36$ MHz.

Phase of the k-th bunch

$$eV_c^{tot} \cos \theta_k = eU + \sum_{j=k}^{\infty} e^2 N_b W^\delta [s_b(j - k)]$$

gives the phase variation

$$\Delta \theta_b(k) = \frac{eN_b}{V_c^{tot} \sin \theta_b} \sum_{j=k}^{\infty} \kappa(j) W^\delta [s_b(j - k)]$$

where $\kappa = 1$ only within the gaps and

$$W^\delta(s) = \frac{\omega_{rf} R_c^{tot}}{Q_0} \cos(\omega_c s/c) e^{-\omega_c s/2Q_L c}.$$

The phase variation is shown in the Fig. 1. The phase variation is calculated assuming 4 trains of 75 bunches with $N_B = 1.5 \cdot 10^{10}$ particles per bunch in the ring assuming 2 RF cavity with $R_s = 2.8 M\Omega$, $Q_0 = 3 \cdot 10^4$ each, and the accelerating voltage $V^{tot} = 1.5$ MV. The calculation was carried out for all buckets in the ring. The result of Fig. 1 shows monotonic change of the phase within the train of bunches. The phase drops down for empty buckets between trains.

The bunch spacing variation is related to the phase: $\Delta \tau_b(k) = \Delta \theta_b(k) / \omega_{rf} = 223 \Delta \theta_b(k)$ [ps]. The maximum variation is 6.7 ps, what is quite acceptable.

