

Circumference Correction Chicanes for Damping Rings

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1 Introduction

Several large low-emittance damping rings are presently being designed to meet the requirements of future linear colliders. These rings tend to have relatively large circumferences ~ 300 m so that they can damp many trains of bunches at the same time. With the large circumference, the path length around the ring may become quite sensitive to thermal and ground motion effects. In addition, most of the rings include damping wigglers whose path length varies with their strength.

In e^-/e^+ storage rings, the beam revolution time is determined by the rf frequency. Thus, a change in the nominal path length will cause a change in both the beam energy and the closed orbit to restore the proper revolution time. The change in energy is given by:

$$\frac{dE}{E} = -\frac{1}{\alpha} \frac{\Delta C}{C}, \quad (1)$$

where α is the momentum compaction and C is the ring circumference. The change in the closed orbit is simply given by the energy change and the dispersion function. This change in the orbit and energy can decrease the dynamic acceptance of the ring and make it difficult to preserve the ultra-small damped emittances. Because damping rings need strong focusing to attain the small beam emittances and thus tend to have very small values of momentum compaction, they can be very sensitive to changes in their circumference. For example, to limit the energy fluctuations in the NLC damping rings to 10% of the beam equilibrium energy spread, the path length must be controlled to about $10 \mu\text{m}$.

Variation of the circumference has been seen at most storage rings including LEP, the APS at Argonne, the SLC damping rings at SLAC, and the ATF damping ring test facility [1] at KEK. At the APS, typical path length changes are the order of 0.2 mm [2] and can be correlated with seasonal, tidal, and diurnal fluctuations. At the SLC damping rings, the path length changes by many millimeters during the approach to thermal equilibrium when the rings are started, but little variation is seen after equilibrium is reached. At the ATF, variations of up to $\pm 3 \text{ mm}$ over periods of months have been observed [3], however the precise mechanisms responsible for these changes are not, at present, well understood.

Another source of path length variation arises when the strength of the damping wigglers is changed. Assuming a sinusoidal wiggler field, the circumference change is approximately

$$\Delta C = \frac{1}{4} \frac{L_w}{\rho_w^2 \kappa_w^2} , \quad (2)$$

where L_w is the length of the wiggler, ρ_w is the peak wiggler bending radius, and κ_w is the wiggler wavenumber: $2\pi/\lambda$. At the ATF, the wigglers are expected to increase the circumference by about 2 mm when turned to full strength. Similarly, in the NLC damping ring, the wigglers are expected to increase the path length by about 2 mm.

Although some of the variation in path length can probably be reduced by design, the inclusion of a circumference correction method in the design of future damping rings seems prudent. There are a few possible approaches: first, physical displacement of the arc magnetic elements; second, control of the orbit using steering correctors or, equivalently, variation of the arc bending magnets and quadrupoles (the later is necessary to keep the tunes constant); and, third, additional elements dedicated to path length control. In this note, we describe the range of correction available by adding a simple 4-dipole chicane to a straight-section in a damping ring. A chicane has the advantage that it can be varied without significantly affecting the ring optics or trajectory outside of the chicane. Thus, the path length can be varied during nominal operation and the chicane could be used as part of a feedback system to stabilize the circumference. In the following, we will describe the effects of the chicane on critical ring parameters, including the equilibrium emittance and momentum compaction.

2 Chicane Impact on Damping Ring Performance

A standard 4-dipole chicane is shown in **Figure 1** where the symbol definitions are indicated.

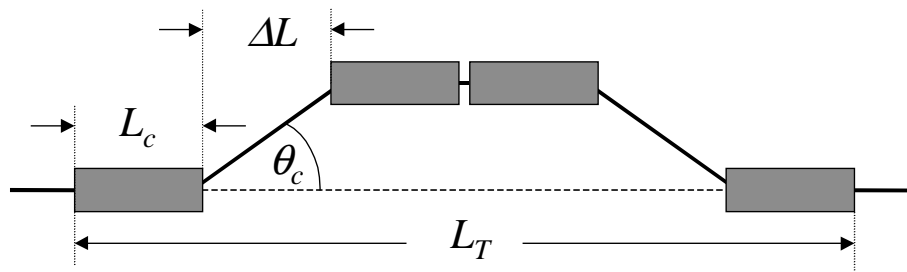


Figure 1: A four dipole chicane with symbol definitions indicated.

2.1 Path Length

For an ‘on-energy’ particle through this chicane, the change in path length (*i.e.* circumference), with respect to the path length without the chicane, and for $\theta_c \ll 1$, is

$$\Delta s \approx \theta_c^2 \left(\frac{2}{3} L_c + \Delta L \right). \quad (3)$$

Note that Δs is always positive so that providing a bipolar circumference correction requires a chicane with a nominal bias at the nominal circumference (*i.e.* $\theta_c = \theta_{c_0} \neq 0$ at $C = C_0$). The path length is then increased (decreased) by an increase (decrease) of the dipole bend angles.

$$\Delta s_b \approx \left(\theta_c^2 - \theta_{c_0}^2 \right) \left(\frac{2}{3} L_c + \Delta L \right) \quad (4)$$

Here the subscript b is added to distinguish a bipolar correction from the unipolar correction of Eq. (3), and $\Delta s_b = \Delta s - \Delta s_{\max}/2$.

2.2 Emittance

The addition of the chicane dipoles will, of course, have some impact on the damped horizontal emittance. The normalized equilibrium emittance for a damping ring, with chicane included, can be written as [4]

$$\gamma \mathcal{E} = C_q \gamma^3 \cdot \frac{I_5}{I_2 - I_4} = \frac{C_q \gamma^3}{J_x I_2} \cdot \frac{I_5 + \Delta I_5}{1 + \Delta I_2 / J_x I_2 - \Delta I_4 / J_x I_2}, \quad (5)$$

where I_j ($j = 2, 4, 5$) are the usual j^{th} synchrotron integrals, ΔI_j are their chicane induced changes, γ is the beam energy factor, J_x is the nominal horizontal partition, and $C_q \approx 3.8 \times 10^{-13}$ m. For an isomagnetic, uncoupled ring with the chicane of **Figure 1** included, the first correction term ($\Delta I_2 / J_x I_2$) in the right-side denominator of Eq. (5) can be expressed as

$$\frac{\Delta I_2}{J_x I_2} = \frac{4}{N_B} \cdot \frac{\tau_x}{\tau_y} \cdot \frac{L_B}{L_c} \cdot \frac{\theta_c^2}{\theta_B^2} \cdot \frac{1}{1 + F_w} \ll 1, \quad (6)$$

where F_w (≥ 0) is the ratio of the energy loss per turn in the wiggler to that in the arcs (typically ~ 1), $\tau_x \tau_y$ ($= 1/J_x$) is the ratio of horizontal to vertical damping times (typically ~ 1), L_B and θ_B are the length and bend angle of an arc dipole, and N_B is the total number of arc dipoles in the ring (typically $\gg 1$). As will be demonstrated, Eq. (6) is ~ 0.01 for a chicane with $\Delta s_{\max} = 5$ mm placed in the 1.54 GeV ATF damping ring with wigglers switched on (see **Table 1**). Thus, this ΔI_2 dependence in the denominator of Eq. (5) can typically be ignored. Similarly, the ΔI_4 term in the denominator is related to Eq. (6) by

$$\frac{\Delta I_4}{J_x I_2} \approx \left(\frac{L_c + \Delta L}{2L_c} \right) \cdot \frac{\Delta I_2}{J_x I_2} \cdot \theta_c^2 \ll 1, \quad (7)$$

which is smaller yet. For the ATF case, described above, Eq. (7) is $\sim 6 \times 10^{-5}$, so this term is completely insignificant. These synchrotron integrals are not significantly changed by a reasonable chicane and, therefore, damping times, losses per turn, and partition numbers are also nearly unaffected. With these results, Eq. (5) becomes

$$\gamma \mathcal{E} \approx \frac{C_q \gamma^3}{J_x I_2} (I_5 + \Delta I_5), \quad (8)$$

where J_x and I_2 are the nominal horizontal partition and 2nd synchrotron integral of the pre-chicane ring, including wigglers and the term proportional to ΔI_5 is the change in the nominal equilibrium emittance due to the chicane.

The change in the 5th synchrotron integral is

$$\Delta I_5 = \frac{L_c}{|\rho_c|^3} \sum_i \langle \eta_i^2 \gamma_i + 2\alpha_i \eta_i \eta'_i + \eta_i'^2 \beta_i \rangle, \quad (9)$$

where $\rho_c (\equiv L_c/\theta_c)$ is the bend radius of the chicane dipoles, η_i and η'_i are the spatial and angular dispersion functions through each dipole, γ_i , β_i , and α_i are the Twiss parameters through each dipole, and the brackets denote the mean value over the length of each dipole. It can be shown that for a small initial α -function (at 1st dipole entrance), $|\alpha_1(0)| < 1$, and a initial β -function which satisfies $\beta_1(0) > L_c + \Delta L$, Eq. (9) reduces approximately to

$$\Delta I_5 \approx \frac{4L_c}{|\rho_c|^3} \bar{\beta} \langle \eta'^2 \rangle = \frac{4|\theta_c|^3}{L_c^2} \bar{\beta} \langle \eta'^2 \rangle, \quad (10)$$

where $\bar{\beta}$ is the mean value of the beta function over all dipoles. (For small $|\alpha|$, the β -function is nearly constant.) In a symmetric chicane, the angular dispersion, $\eta'(s) = s/\rho_c$, is equal in all dipoles and its mean-squared value over L_c is

$$\langle \eta'^2 \rangle = \frac{1}{L_c} \int_0^{L_c} \frac{s^2}{\rho_c^2} ds = \frac{\theta_c^2}{3}. \quad (11)$$

Eq. (3) is now used to substitute for θ_c , and relations for the nominal 2nd synchrotron integral and horizontal partition are introduced.

$$I_2 = \frac{3C_0}{r_e c \gamma^3 \tau_y}, \quad J_x = \frac{\tau_y}{\tau_x} \quad (12)$$

Here r_e is the classical electron radius, c is the speed of light, and C_0 is the nominal ring circumference. The main length of the chicane is defined as $L_T \equiv 4L_c + 2\Delta L$. (This definition does not include a potential small drift between center bends which has no effect on these results.) With these relations added, the change in the nominal emittance from Eq. (8) becomes

$$\Delta\gamma\varepsilon \approx \frac{4}{9} C_q r_e c \cdot \frac{\tau_x \bar{\beta} \gamma^6}{C_0} \cdot \frac{\Delta s^{5/2}}{L_c^2 (L_T/2 - 4L_c/3)^{5/2}}. \quad (13)$$

The 6th power scaling of γ here holds only for a constant τ_x . The denominator in the last fraction of Eq. (13) has a maximum at $L_c = \Delta L = L_T/6$, so the minimum possible emittance increase for a unipolar circumference correction, Δs , is

$$\Delta\gamma\varepsilon \approx 393 \cdot C_q r_e c \cdot \frac{\tau_x \bar{\beta} \gamma^6}{C} \cdot \frac{\Delta s^{5/2}}{L_T^{9/2}}, \quad (14)$$

where the chicane must satisfy

$$|\theta_c| \approx 3 \sqrt{\frac{2\Delta s}{5L_T}}, \quad L_c = L_T/6 = \Delta L. \quad (15)$$

Eq. (14) describes the equilibrium emittance increase produced by adding, to a damping ring, a chicane with additional path length, Δs . Therefore, a bipolar adjustment, as described in Eq. (4), can provide a correction range of $\pm \Delta s_{\max}/2$, with a nominal bend angle of θ_{c_0} at $\Delta s_b = 0$. For the ATF, the emittance increase, with respect to no chicane, at 1.54 GeV is $\sim 8\%$ for an $L_T = 3.4$ -meter long chicane of $\Delta s = 5$ mm placed at $\langle \beta \rangle = 10$ m (see **Figure 2**). The maximum bend angle is $\theta_{c_{\max}} \approx 4.2^\circ$.

2.3 Momentum Compaction

The effect on momentum compaction is

$$\alpha_p = \alpha_{p_0} - 2 \frac{\Delta s}{C_0}, \quad (16)$$

with α_{p_0} the nominal momentum compaction. For the ATF this is a relative decrease of 4% at $\Delta s = 5$ mm. This small change in α_p also indicates that the chicane's effect on bunch length ($\propto \alpha_p^{1/2}$) is insignificant.

2.4 Energy Spread

Finally, the relative change in the 3rd synchrotron integral is given by

$$\frac{\Delta I_3}{I_3} = \frac{4}{N_B} \cdot \frac{L_B^2}{L_c^2} \cdot \left| \frac{\theta_c}{\theta_B} \right|^3 \ll 1. \quad (17)$$

For the ATF chicane described above, Eq. (17) is 0.074. In consideration of the small impact on I_2 and insignificant change in I_4 shown in Eqs. (6-7), for J_x of the order of 1, the chicane's impact on the ring's energy spread can be approximated by

$$\sigma_\delta \approx \sigma_{\delta_0} \sqrt{1 + \frac{4}{N_B} \cdot \frac{L_B^2}{L_c^2} \cdot \left| \frac{\theta_c}{\theta_B} \right|^3 \left[1 - \frac{2}{J_\varepsilon (1 + F_w)} \cdot \left| \frac{\theta_B}{\theta_c} \right| \frac{L_c}{L_B} \right]}, \quad (18)$$

where $J_\varepsilon = 3 - \tau_y/\tau_x$. Eq. (18) shows that the energy spread can increase or decrease, depending on the parameters. For the case above with ATF parameters, Eq. (18) is an insignificant increase in the rms energy spread of $7.20 \times 10^{-4} \rightarrow 7.23 \times 10^{-4}$ at $\Delta s = 5$ mm.

3 ATF Damping Ring

Parameters of the ATF damping ring, with wiggler switched on, are listed in **Table 1**.

Table 1: ATF damping ring parameters with wigglers on and no chicane.

Parameter	symbol	unit	value
Beam energy	E	GeV	1.54
Arc-dipole magnet length	L_B	m	1.0
Arc-dipole bend angle	θ_B	deg	10.0
Total number of arc-dipoles in ring	N_B	—	36
Equilibrium horizontal emittance (rms)	$\gamma\varepsilon_0$	μm	4.3
Horizontal damping time	τ_x	msec	6.8
Vertical damping time	τ_y	msec	9.1
Mean beta at proposed chicane	$\langle\beta\rangle$	m	~ 10
Ring circumference	C_0	m	139
Ratio of wiggler loss to arc losses	F_w	—	1.6
Relative energy spread (rms)	σ_δ	10^{-3}	0.72
Momentum compaction	α_{p0}	10^{-3}	1.9

Figure 2 shows the relative emittance increase, using Eq. (14), for chicane lengths, L_T , of 1, 2, 3, 4, and 5 meters versus unipolar circumference correction, Δs .

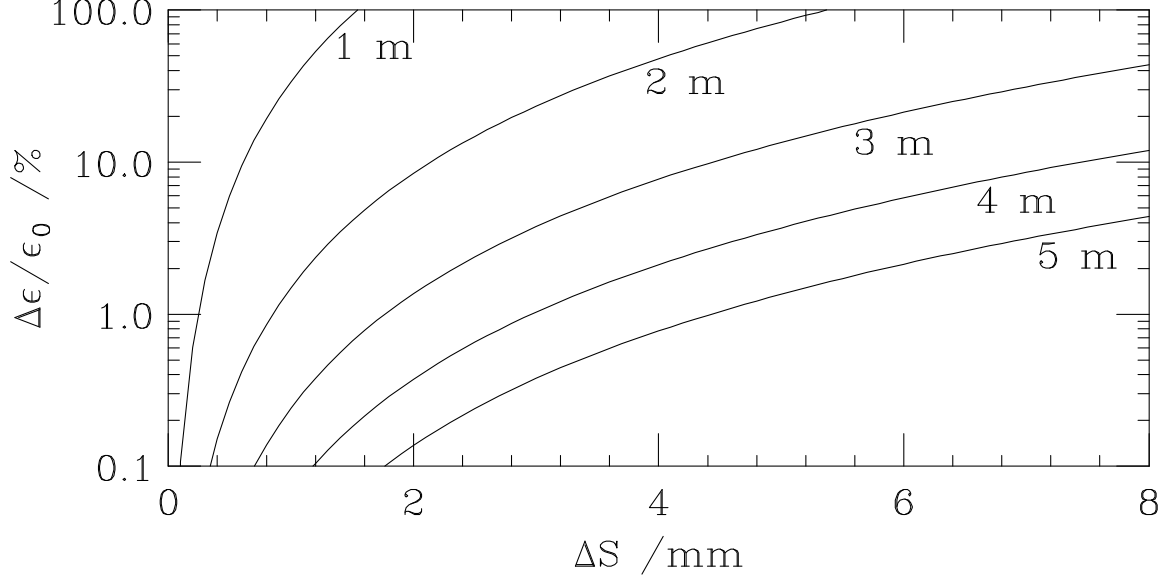


Figure 2: Relative equilibrium emittance increase vs. circumference change for ATF damping ring with chicane length, L_T , of 1, 2, 3, 4, and 5 meters at 1.54 GeV, $\langle\beta\rangle = 10$ m, $\gamma_{E_0} = 4.3 \mu\text{m}$, $\tau_x = 6.8$ msec.

For a bipolar correction, the emittance growth with a nominal chicane bias, $\Delta s_b = 0$ ($\theta_c = \theta_{c_0} = \theta_{c_{\text{max}}}/\sqrt{2}$), with respect to the original ATF, is $2^{-5/2}$ ($\approx 18\%$) of the increase shown in **Figure 2**. The growth reduces to zero for $\Delta s_b = -2.5$ mm ($\theta_c = 0$), and returns to the full values shown for $\Delta s_b = +2.5$ mm ($\theta_c = \theta_{c_{\text{max}}}$).

In order to fit such a chicane into the existing ATF lattice requires a free beamline section of length $\geq L_T$. In the ATF, a 2.1-meter space can be made available by removing one of eight wiggler sections. The removal of two such sections in order to provide up to 4.2 meters is more difficult due to the quadrupole magnet placed between 2.1-meter wiggler sections. In order to limit the complexity of such a modification, only the removal of a single section is considered. In this case, **Figure 2** ($L_T = 2$ m) shows the range of $\Delta s_b = \pm 1$ mm correction is possible with a maximum emittance increase (at $\Delta s_b = +1$ mm) of $\sim 8\%$ at 1.54 GeV. This is a fairly small circumference correction in light of the ± 3 mm variations observed. It may, however, be possible to add two 2-meter chicanes which will double both the Δs_b range and the emittance increase. This should be compared with doubling the Δs_b range using a single, stronger chicane. The latter method multiplies the emittance increase by $2^{5/2} \approx 5.7$. The double chicane, however, requires the elimination of 25% of the ATF wiggler which will increase the vertical damping time by $\sim 20\%$. A simple solution which provides a ± 3 mm correction range does not appear to be viable.

4 NLC Main Damping Ring

Parameters of the 3-bunch train NLC main damping ring used here are listed in **Table 2**. These parameters represent a recent proposal which includes 40-meters of wiggler in order to increase the net momentum compaction to $>5 \times 10^{-4}$.

Table 2: NLC main damping ring parameters with wigglers on and no chicane.

Parameter	symbol	unit	value
Beam energy	E	GeV	1.98
Arc-dipole magnet length	L_B	m	1.07
Arc-dipole bend angle	θ_B	deg	10.6
Total number of arc-dipoles in ring	N_B	—	34
Equilibrium horizontal emittance (rms)	$\gamma\epsilon_0$	μm	2.4
Horizontal damping time	τ_x	msec	5.2
Vertical damping time	τ_y	msec	5.2
Mean beta at proposed chicane	$\langle\beta\rangle$	m	~ 6
Ring circumference	C_0	m	282
Ratio of wiggler loss to arc losses	F_w	—	2
Relative energy spread (rms)	σ_δ	10^{-3}	0.91
Momentum compaction	α_{p_0}	10^{-3}	0.58

In order to provide a ± 1 mm circumference correction range for the NLC ring requires a single chicane of $L_T = 2.6$ m, $\theta_{c_{\max}} = 3^\circ$, with $\Delta\epsilon/\epsilon_0 \approx 6\%$ at $\Delta s_b = +1$ mm. Two such chicanes can provide a ± 2 mm range with a 12% maximum growth, or a ± 1.5 mm range at a 6% maximum growth. **Figure 3** shows the relative emittance increase for the NLC main damping ring, using Eq. (14), for chicane lengths of 1, 2, 3, 4, and 5 meters versus unipolar circumference correction, Δs .

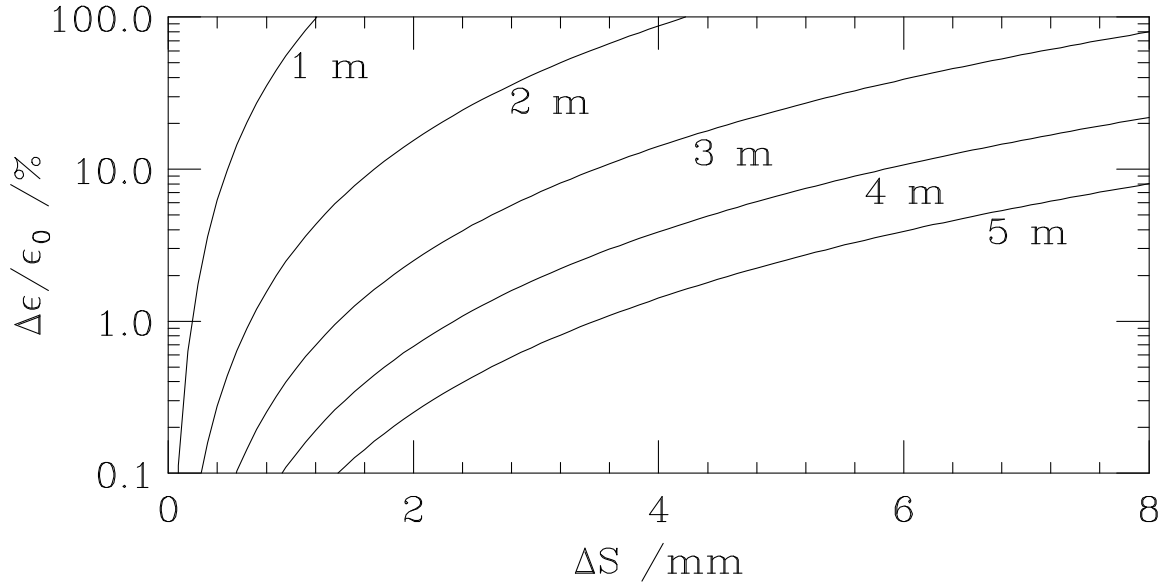


Figure 3: Relative equilibrium emittance increase vs. circumference change for NLC main damping ring with chicane lengths, L_T , of 1, 2, 3, 4, and 5 meters at 1.98 GeV, $\langle\beta\rangle = 6$ m, $\gamma\epsilon_0 = 2.4$ μm , $\tau_x = 5.2$ msec.

The effect of a double chicane for the NLC can be estimated if both horizontal and vertical scales of **Figure 3** are multiplied by 2. In this case Δs is the total path length change and $\Delta\epsilon/\epsilon_0$ is the total emittance increase. Providing more than ± 2 mm correction range for the NLC using only chicanes may be problematic. Larger corrections, although not clearly necessary, can be generated by either moving the ring components or by changing the arc magnet excitation levels.

5 References

- [1] J. Urakawa, "KEK/ATF Damping Ring", PAC97, Vancouver, B.C., Canada, (1997).
- [2] M. Ross, M. Borland, private communication (1998).
- [3] Report on 4th International Collaboration Meeting, July 17, 1998, J. Urakawa editor, internal note.
- [4] R.H. Helm, et. al., "Evaluation of Synchrotron Radiation Integrals", SLAC-PUB-1193 (A), March 1973.