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Disruption Effects in e-e- Linear Colliders

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K. A. Thompson and P. Cher
Stanford Linear Accelerator Center
Stanford, California

Abstract:
We present simple analytic approximations for the "de-enhancement" factor H_D in e^-e^- collisions, as a function of the disruption and hour-glass parameters and the beam aspect ratio σ_x/σ_y. We treat Gaussian beams with essentially arbitrary aspect ratio R, assuming only that the vertical beam size is less than or equal to the horizontal beam size and that the vertical beta function is less than or equal to the horizontal beta function. We also examine the effect of disruption on the calculation of upsilon and n_D for e^-e^- collisions.

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INTRODUCTION

The purpose of this note is to give simple analytic formulas for use in e^-e^- collider physics programs and design studies. Such formulas complement those previously given in the literature [1] for e^+e^-. At this point, we do not give real derivations; we merely use some simple physical arguments to guide us toward empirical fits to beam-beam simulation results. The GUINEAPIG program [2] is used for most of these simulations. As a cross-check, we also did some simulations with the CAIN program [3].

The geometric luminosity per bunch, not taking account of disruption or hour-glass effect, is given by

$$\mathcal{L}_0 \equiv \frac{N^2}{4\pi\sigma_x\sigma_y} \quad , \tag{1}$$

where N is the number of particles per bunch and $\sigma_{x,y}$ are the horizontal and vertical beam sizes. We assume the beam distributions are Gaussian longitudinally and transversely.

The hour-glass effect reduces the undisrupted luminosity unless the parameters

$$A_{x,y} \equiv \frac{\sigma_z}{\beta^*_{x,y}} \tag{2}$$

are much less than 1. Here σ_z is the bunch length and $\beta^*_{x,y}$ are the horizontal and vertical betatron functions at the collision point.

The disruption parameters $D_{x,y}$ are defined by

$$D_{x,y} = \frac{2r_e\sigma_z N}{\gamma\sigma_{x,y}(\sigma_x+\sigma_y)} \quad . \tag{3}$$

We define the luminosity pinch enhancement factor by

$$H_D \equiv \frac{\mathcal{L}_D}{\mathcal{L}_0} \quad . \tag{4}$$

Here $\mathcal{L}_D$ denotes the actual luminosity with disruption and hour-glass effect taken into account. (Caution: H_D is sometimes defined as $\mathcal{L}_D/\mathcal{L}_A$ where $\mathcal{L}_A$ is the luminosity with hour-glass effect taken into account.) For e^-e^- collisions, the

TABLE 1. e-e- IP parameters for round-beam (b) design in Zimmermann,et.al.

E_{beam} [GeV]	500.
N [10^{10}]	0.95
$\gamma\epsilon$ [10^{-6} m-rad]	1.
β^* [mm]	0.25
σ_z [μm]	125.
σ_0 [nm]	16.0
$\mathcal{L}_0$ [10^{33} m^{-2}]	28.12
A	0.500
D	13.4

"enhancement" factor is of course less than 1. For e^+e^- collisions, H_D is greater than one, and if the beams are round the approximate analytic formula given in Reference [4] for the luminosity pinch enhancement factor is

$$H_D \approx 1 + D^{1/4}\left(\frac{D^3}{1+D^3}\right)\left[\ln(\sqrt{D}+1) + 2\ln\frac{0.8}{A}\right] \quad . \tag{5}$$

For flat beams the following approximation holds

$$H_D \approx \left[1 + D_y^{1/4}\left(\frac{D_y^3}{1+D_y^3}\right)\left[\ln(\sqrt{D_y}+1) + 2\ln\frac{0.8}{A_y}\right]\right]^{1/3} \quad . \tag{6}$$

These are empirical formulas obtained by fitting simulation results, and are good to about 10% over reasonable ranges of the parameters A and D.

SIMULATION OF $\mathbf{H_D}$ AND ANALYTIC APPROXIMATION FOR ROUND BEAMS

We begin by focusing on the case of round beams ($\sigma_x = \sigma_y$). We also assume $\beta_x = \beta_y$, so that $A_x = A_y$ (it would be possible to have different horizontal and transverse beta functions if the horizontal and transverse emittances also differed, but this is not generally the case for round beam designs).

First we check that turning beamstrahlung on and off in simulations does not have a large effect on the luminosity. Two round beam parameter sets for the interaction point of an e^-e^- collider were considered in a previous paper [5]. They differ only in the transverse beam size (and beta function). We take the more extreme (i.e. smaller beam, larger disruption and beamstrahlung) case, shown in Table 1. Simulation results with and without beamstrahlung turned on are shown in Table 2, using Guineapig and using CAIN. We see that the luminosity is larger by only a few percent with beamstrahlung turned off.

TABLE 2. Luminosity simulation results with and without beamstrahlung turned on

	Guineapig (disruption and beamstrahlung)	Guineapig (disruption, no beamstrahlung)	CAIN (disruption and beamstrahlung)	CAIN (disruption, no beamstrahlung)
$\mathcal{L}_D$ [10^{33} m^{-2}]	6.9	7.3	6.7	7.2
$H_D \equiv \mathcal{L}_D/\mathcal{L}_0$ (sim)	0.25	0.26	0.24	0.26

We then go on to simulate a number of cases, varying A and D. The results are plotted as the solid curves in Figure 1.

The major features of these curves can be easily understood on physical grounds. For very small disruption, H_D asymptotically approaches the value expected from the hour-glass effect alone:

$$H_D \approx \eta_A \equiv \frac{2}{\sqrt{\pi}A}\int_0^\infty \frac{e^{-u^2/A^2}}{1+A^2u^2}du \quad , \tag{7}$$

For $0 < A < 1$, a reasonably good expansion is $\eta_A \approx 1 - A^2/4$.

One might try to factorize H_D as $H_D = F_D\eta_A$. Note, however, that for very large disruption the divergence of the beam due to the final focus system, represented by A, will be completely overwhelmed, explaining why the simulation curves for different A converge at large D. For $D \gg 1$, the beams disrupt each other away within a distance σ_z/D and the effective value of A becomes

$$\tilde{A} = A/D = \frac{2(\gamma\epsilon)}{r_e N} \quad , \tag{8}$$

depending only on inherent properties of the beam.

To smooth the transition between the regimes of D, we take

$$\eta(A,D) \approx 1 - \frac{1}{4}\left(\frac{A}{1+bD}\right)^2 \quad . \tag{9}$$

where b is an adjustable parameter (we can in addition adjust the parameter 1/4 in front).

A derivation of F_D for round, Gaussian beams and small D [4] goes through for e^-e^- with only a change of sign from that for e^+e^-, and yields

$$F_D \approx 1 - \frac{2D}{3\sqrt{\pi}} \quad . \tag{10}$$

This is just the small-argument expansion of $\exp\left(-\frac{2D}{3\sqrt{\pi}}\right)$ so we might try matching onto that for larger D. One finds that the exponential drops off too quickly, but a modified Bessel function I_0 can be introduced to moderate this drop-off. We try

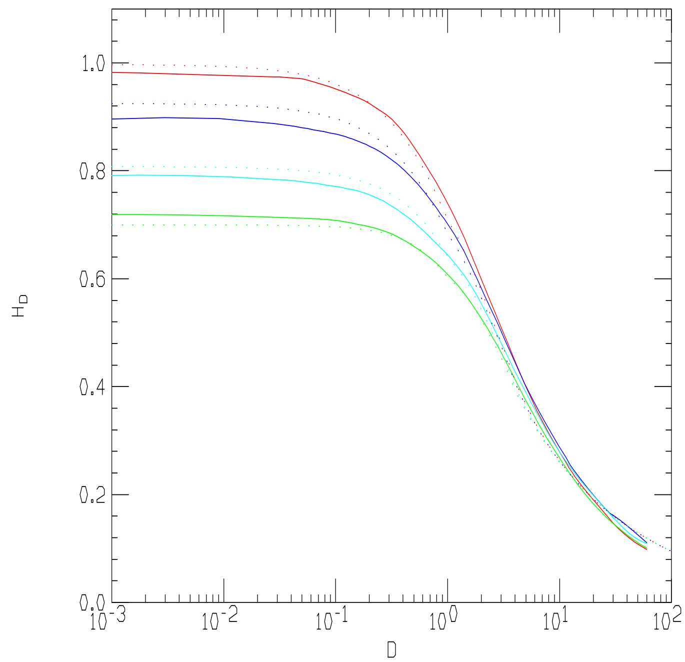

FIGURE 1. Disruption "de-enhancement" factor H_D as a function of D, for round beams. From top to bottom, the curves shown are for $A = 0.1$, $A = 0.5$, $A = 0.8$, $A = 1$. The solid curves are the simulation results and the dotted curves are the analytic approximation.

$$F_D = \exp\left(-\frac{2D}{3\sqrt{\pi}}\right) I_0\left(\frac{2D}{3\sqrt{\pi}}\right) \quad . \tag{11}$$

We could also adjust the coefficient $\frac{2}{3\sqrt{\pi}} \approx 0.376$ that appears in front of D in both the exponential and in I_0. But we did not gain a significant improvement by changing this coefficient in either or both of these places where it appears.

The modified Bessel function has expansions for large and small x that agree well for $x \sim 1$ and thus can be used to cover the entire range of D. These are given by:

$$I_0(x) = \begin{cases} 1 + \dfrac{x^2}{4} + \dfrac{x^4}{64} + \dfrac{x^6}{2304} & (x < 1) \\ \dfrac{e^x}{\sqrt{2\pi x}}\left[1 + \dfrac{1}{8x} + \dfrac{9}{128x^2}\right] & (x > 1) \quad . \end{cases} \tag{12}$$

Finally, to get a good fit for $1 \leq D \leq 100$ we needed to introduce a purely empirical fudge factor f_{choc} given by:

$$f_{choc}(D) = \begin{cases} 1 & (0 \leq D \leq 1) \\ 1 + 0.1 \ln D & (1 \leq D \leq 100) \quad . \end{cases} \tag{13}$$

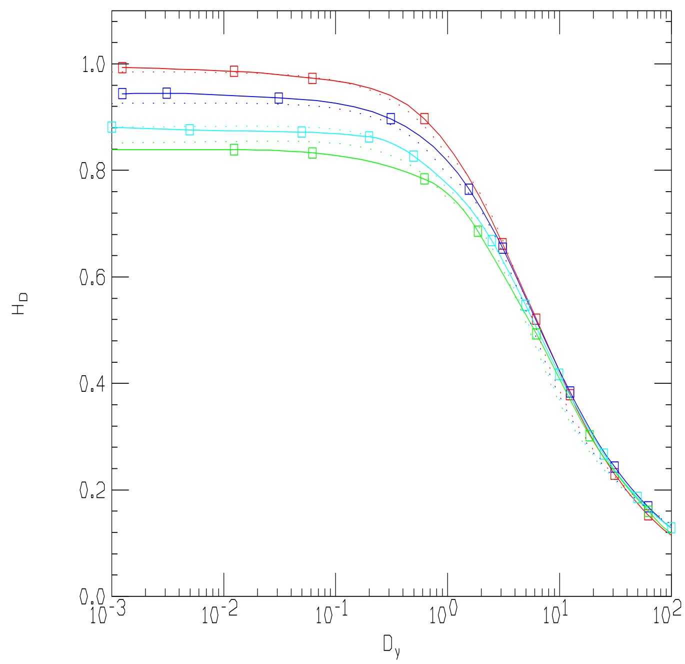

FIGURE 2. Disruption "de-enhancement" factor H_D as a function of D_y, for flat beams ($R = 100$). From top to bottom, the curves shown are for $A_y = 0.1$, $A_y = 0.5$, $A_y = 0.8$, $A_y = 1$. The solid curves are the simulation results and the dotted curves are the analytic approximation.

Our final analytic approximation for H_D for round, Gaussian e^-e^- collisions, is:

$$H_D = \left[1 - 0.3\frac{A^2}{(1+0.4D)^2}\right] \exp\left(-\frac{2D}{3\sqrt{\pi}}\right) I_0\left(\frac{2D}{3\sqrt{\pi}}\right) f_{choc}(D) \quad , \tag{14}$$

where we use the expansion for I_0 given above. This analytic approximation is shown as the dotted curves in Figure 1.

GENERALIZATION OF $\mathbf{H_D}$ APPROXIMATION TO NON-ROUND BEAMS

Let us now try to generalize our results to beams that are not round. Let the vertical dimension be the smaller dimension of the beam, and define the aspect ratio of the beam by

$$R \equiv \sigma_x/\sigma_y \quad . \tag{15}$$

Simulation results for the case of a very flat beam are plotted as the solid curves in Figure 2. The results shown are for the case $R = 100$ but are not very sensitive to R provided it is significantly greater than 1 — for example, we see from Figure 3,

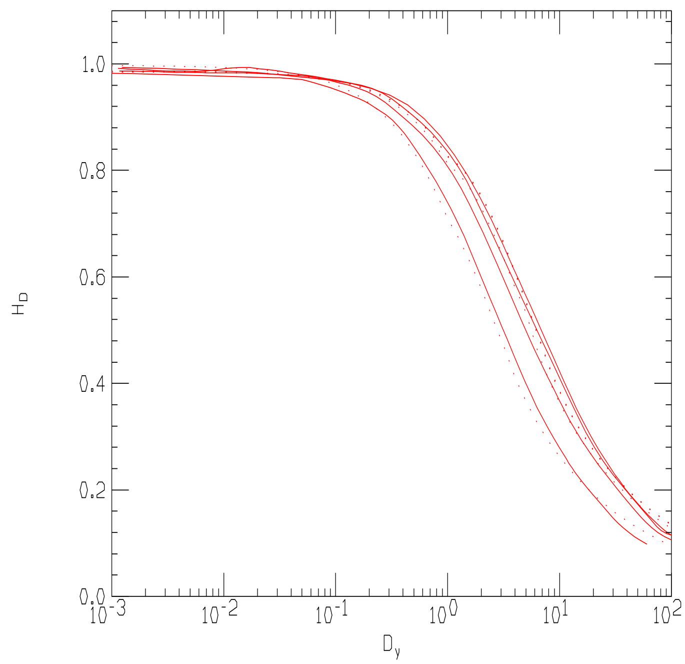

FIGURE 3. Disruption "de-enhancement" factor H_D as a function of D_y, for $A_y = 0.1$ and assuming $A_x \ll 1$. From top to bottom, the curves shown are for $R = 100$, $R = 10$, $R = 3$, $R = 1$. The solid curves are the simulation results and the dotted curves are the analytic approximation.

which shows H_D as a function of D for $A_y = 0.1$ and R varying from 1 to 100 (with $A_x \ll 1$), that the curves for $R = 10$ and $R = 100$ are not very different.

We look for an approximation to H_D that is a function of D_y, A_y, and R, and is valid for beams of any aspect ratio ($R = 1$ to ∞). We will assume A_x is small enough that the hour-glass effect is not significant in the horizontal plane (unless the beam is close to round and A_y is near 1) – if the beam is not round, we will assume $A_x \ll 1$, and if it is round, we will assume $A_x = A_y$. These assumptions generally hold in linear collider designs, since $\beta_x^\star > \beta_y^\star$ for flat-beam designs, $\beta_x^\star = \beta_y^\star$ for round-beam designs, and $A_y \leq 1$.

Let us define

$$f(R) \equiv 1 + \frac{1}{R^2} \quad . \tag{16}$$

Thus $f(R) = 2$ for round beams and $f(R) \approx 1$ for flat beams. We find that the following simple generalization of our round beam expression works very well for arbitrary $R \geq 1$:

$$H_D = \left[1 - 0.15 f(R)\left(\frac{A_y}{1 + 0.4 D_y}\right)^{f(R)}\right] \exp\left(-\frac{f(R) D_y}{3\sqrt{\pi}}\right) I_0\left(\frac{f(R) D_y}{3\sqrt{\pi}}\right) f_{choc}(D_y) \quad , \tag{17}$$

where the expansion for I_0 and the expression for f_{choc} are given in the previous section. This expression is identical to that in the previous section when $R = 1$.

Our analytic approximation for the flat beam case ($R \gg 1$) is shown as the dotted curves in Figure 2. We show also the result of this analytic approximation for R= 1, 3 ,10, and 100, and $A_y = 0.1$, as the dotted curves in Figure 3.

In summary, our expression for H_D agrees with simulations to within about 10% for $0 \leq A_y \leq 1$ and $0 \leq D_y \leq 30$. The agreement is even better over the most interesting ranges of parameters, namely those where H_D is not too much less than 1.

BEAMSTRAHLUNG AND APPROXIMATION OF Υ, N_γ

The strong electromagnetic field of the oncoming beam not only affects the luminosity through the effects of disruption, it also causes particles to radiate beamstrahlung photons as they are bent by the strong field. In this section we simply verify that the same "effective beam size" prescription used for e^+e^- to calculate the beamstrahlung parameter Υ and the average number of beamstrahlung photons n_γ per incoming particle when there is significant disruption works for e^-e^-. We focus on the case of round beams since this is the worst case, i.e. the disruption is highest.

The beamstrahlung parameter Υ is defined by

$$\Upsilon \equiv \frac{e\hbar}{m^3c^4}\sqrt{|F_{\mu\nu}p^\nu|^2} = \gamma\frac{B}{B_c} \quad . \tag{18}$$

Here $p^\nu = (E, \overrightarrow{p})$ is the 4-momentum of the incoming electron or positron, m is the electron mass, $\gamma \equiv E/mc^2$ is the usual Lorentz factor, $F_{\mu\nu}$ is the energy-momentum tensor of the electromagnetic field, $B = |\overrightarrow{B}| + |\overrightarrow{E}|$, and $B_c \equiv m^2c^3/\hbar e \approx 4.4 \times 10^{13}$ Gauss is the Schwinger critical field.

The effective average beamstrahlung parameter Υ, in the analytic approximation of Yokoya and Chen [1] for round, Gaussian beams and small D is:

$$\Upsilon_{eff} = \frac{5}{12}\Upsilon_{max} = \frac{5Nr_e^2\gamma}{12\alpha\sigma_z\bar{\sigma}} \quad . \tag{19}$$

Here $\bar{\sigma}$ is the effective beam size during the collision. If the disruption is not too strong this is not very different from the nominal transverse beam size $\sigma_0 (= \sigma_x = \sigma_y$ for round beams). If the disruption is significant, one may calculate an effective beam size during the collision [6] from

$$\bar{\sigma} = H_D^{-1/2}\sigma_0 \quad . \tag{20}$$

We may then use this effective beam size to calculate Υ_{eff} as shown above, and also n_γ using the following equation from Yokoya and Chen [1]:

TABLE 3. Beamstrahlung simulation results with and without disruption turned on

	Guineapig (beamstrahlung and disruption)	Guineapig (beamstrahlung, no disruption)
$\mathcal{L}_D$ [10^{33} m^{-2}]	6.9	25.1
$H_D \equiv \mathcal{L}_D/\mathcal{L}_0$ (sim)	0.25	0.89
n_γ	4.2	8.7
δ_B	39%	62%

$$n_\gamma \approx 1.06 \frac{\alpha N r_e}{\bar{\sigma}} \frac{1}{(1+\Upsilon_{eff}^{2/3})^{1/2}} \quad , \tag{21}$$

Here α is the fine structure constant and r_e the classical electron radius.

This prescription works in e^-e^- as well as in e^+e^-, as is borne out by our simulations. For example, using the parameters in Table 1, we ran Guineapig with beamstrahlung turned on, in order to calculate n_γ and H_D. We also modified Guineapig to allow us to calculate beamstrahlung with disruption turned off. The results with and without disruption are shown in Table 3.

When disruption is turned off, $\bar{\sigma} \approx \sigma_0$=16 nm. From the above formulas, we then get $\Upsilon_{eff} \approx 2.1$, so that we expect $n_\gamma \approx 8$, in good agreement with the simulation result of 8.7.

When disruption is turned on, $H_D \approx 0.25$ from either the simulation or our analytic approximation of the previous section. Thus $\bar{\sigma} \approx 32$ nm. From the above formulas, we then get $\Upsilon_{eff} \approx 1.05$, so that we expect $n_\gamma \approx 4.5$, again in good agreement with the simulation result of 4.4.

[NOTE: Yokoya and Chen also give an equation for the average beamstrahlung energy loss per particle:

$$\delta_B \approx 0.216 \frac{r_e^3 N^2 \gamma}{\sigma_z \bar{\sigma}^2} \frac{1}{[1+(1.5\Upsilon_{eff})^{2/3}]^2} \quad , \tag{22}$$

but this equation does not take account of multiple photon emissions, so is of limited use for our purposes here.]

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