

Beam-Gas and Thermal Photon Scattering in the NLC Main Linac as a Source of Beam Halo

P. TENENBAUM
LCC-NOTE-0051
12-JAN-2001

Abstract

Scattering of primary beam electrons off of residual gas molecules or blackbody radiation photons in the NLC main linac has been identified as a potential source of beam haloes which must be collimated in the beam delivery system. We consider the contributions from four scattering mechanisms: inelastic thermal-photon scattering, elastic beam-gas (Coulomb) scattering, inelastic beam-gas (Bremsstrahlung) scattering, and atomic-electron scattering. In each case we develop the formalism necessary to estimate the backgrounds generated in the main linac, and determine the expected number of off-energy or large-amplitude particles from each process, assuming a main linac injection energy of 8 GeV and extraction energy of 500 GeV.

1 Introduction

The performance of modern electron storage rings is strongly impacted by the probability of primary-beam scattering. In the SLAC B-Factory, for example, the luminosity during initial operations is limited by detector backgrounds from beam-gas scattering; as the vacuum system is “scrubbed” by the beam, the stored currents may be raised and luminosity is consequently increased [1]. The beam lifetime in the Large Electron-Positron collider (LEP) during noncolliding conditions was dominated by primary beam scattering from blackbody radiation thermal photons [2]. A more complete study of scattering and generation of non-Gaussian tails in LEP has also been performed [3].

The Next Linear Collider (NLC) is a single-pass electron accelerator, which in principle is less sensitive to scattering than a storage ring. In practice, however, the NLC combines an unprecedented beam energy (500 GeV at the end of the main linac), an unprecedented vertical emittance ($\gamma\epsilon \approx 0.03$ mm.mrad), and a relatively high base pressure (1–10 nTorr); this combination implies that scattering mechanisms may be capable of significantly populating the non-Gaussian tails of the beam distribution in position, angle, or energy. Reichel *et al* recently considered the halo population due to scattering in the post-linac beam delivery system of the NLC, based on the 1996 (*ZDR*) design [4].

In this Note we adapt the formalism of [3] and [4] to the NLC main linac, a 10 km linear accelerator with an injected beam energy of 8 GeV, an extracted beam energy of 500 GeV, and an accelerating gradient of 0.05 GeV per meter. We consider four mechanisms for halo population: inelastic scattering off blackbody thermal photons, inelastic scattering off residual gas molecules (beam-gas Bremsstrahlung), elastic scattering off residual gas molecules (relativistic Coulomb or Mott scattering), and inelastic scattering off atomic electrons. Our model of the linac is the CD-1 model, with 3 main linac optical regions:

- ELIN1, which accelerates the beam from 8 GeV to 52 GeV, with a maximum betatron function of approximately 23 meters and a mean betatron function of approximately 11.2 meters
- ELIN2, which accelerates the beam from 52 GeV to 162 GeV, with a maximum betatron function of approximately 45 meters and a mean betatron function of approximately 22 meters

- ELIN3, which accelerates the beam from 162 GeV to 500 GeV, with a maximum betatron function of approximately 65 meters and a mean betatron function of approximately 32 meters.

The horizontal and vertical dispersion is identically zero by design in all parts of the main linac except for a pair of energy-diagnostic chicanes. Because the chicanes are a very small fraction of the main linac length, we will neglect the additional complications of inelastic scattering in a dispersive region which would arise in these chicanes.

2 Review of Relevant Physics

Following the treatment in [3, 4], we will determine the cross-section for a single electron to scatter off of a single photon or atom. We can then convert this to the scattering probability per meter by multiplying this cross-section by the number density of ideal gas particles ($3.22 \times 10^{22} \text{m}^{-3} \cdot \text{P}[\text{Torr}]$) or the number density of thermal photons at room temperature ($0.5 \times 10^{15} \text{m}^{-3}$); for gas molecules we must multiply this result by the number of atoms per molecule.

In order to simplify the mathematics, we will assume that the residual gas population is entirely N_2 , with atomic weight 14, atomic number 7, and 2 atoms per molecule, and the base pressure is a uniform 10 nTorr. We will assume that the blackbody radiation is populated entirely by photons at the mean photon energy of 0.07 eV.

Under what circumstances is a particle scattered out of the collimation aperture of the beam delivery system? The present collimation system design calls for a 1% energy aperture and horizontal and vertical half-gaps which are, at the tightest, 160 and 230 micrometers, respectively [5]. The size of the energy aperture indicates that particles must lose at least 5 GeV in a collision to be considered lost due to energy error. Losses due to large betatron amplitude are slightly more difficult to handle. A particle will encounter a post-linac betatron collimator in the vertical plane if its scattering angle in the linac satisfies:

$$\theta_{y,\text{scatter}} R_{34}(\text{scatter} \rightarrow \text{coll}) > y_{\text{coll}}. \quad (1)$$

We can rewrite $R_{34}(\text{scatter} \rightarrow \text{coll})$ in terms of known betatron functions, phase advances, and design energies as:

$$\theta_{y,\text{scatter}} > \frac{y_{\text{coll}}}{\sqrt{\beta_{\text{scatter}}\beta_{\text{coll}}}} \sin \Delta\psi \sqrt{\frac{E_{\text{coll}}}{E_{\text{scatter}}}}. \quad (2)$$

We can simplify Equation 2 by noting the following:

- If we average over the linac region, we can replace β_{scatter} with $\bar{\beta}$, the mean betatron function in that linac region
- If we assume that the scattering cross-section does not vary with the betatron phase, then we must replace $\sin \Delta\psi$ with the sine of the RMS phase advance from any point in the linac to the collimator which will stop the particle; this value is $1/\sqrt{2}$
- The scattering processes considered here have an isotropic distribution in $x'y'$ space, thus the deflection angle in the yz plane will on average be $1/\sqrt{2}$ of the total deflection angle.

When all of the factors above are included, the minimum scattering angle required to leave the betatron aperture of the collimation system is given by:

$$\theta_{y,\text{scatter}} > 2 \frac{y_{\text{coll}}}{\sqrt{\bar{\beta}\beta_{\text{coll}}}} \sqrt{\frac{E_{\text{coll}}}{E_{\text{scatter}}}}. \quad (3)$$

In this Note we consider only those large-amplitude particles which impact the vertical collimators, where β_y is 500 m and the half gap is 160 μm . In the horizontal the gap size is larger (typically 230 μm) and the beta function is smaller (270 meters); Equation 3 thus indicates that the scattering angle required is larger and the number of particles which will encounter the horizontal collimators is consequently smaller.

Note that, in some cases, the scattered electron will emerge with an energy or betatron error that is so large that it is lost somewhere in the linac and never reaches the beam delivery system collimators. In this analysis we take no credit for such losses – we assume that every particle which undergoes scattering in the linac reaches the beam delivery area.

2.1 Scattering from Thermal Photons

Let us consider an electron with energy E scattering off a photon with energy ω_0 with scattering angle α , where the angle convention is that $\alpha = 0$ is head-on collision. If the outgoing photon energy, ω' , is given by y/E (i.e., $y \equiv -\delta$), then the cross-section is given by:

$$\frac{d\sigma}{dy} = \frac{2\sigma_0}{x} \left[\frac{1}{1-y} + 1 - y - 4r(1-r) \right], \quad (4)$$

where σ_0 is the Thomson cross section of $6.65 \times 10^{-29} \text{ m}^2$; $x \equiv 4E\omega_0 \cos^2(\alpha/2)/m_e^2 c^4$, and $r \equiv y/(x(1-y))$. For a given beam energy, photon energy, and incidence angle the achievable values of y are bounded below by zero and above by $x/(1+x)$. One can invert this expression and demonstrate that for a given value of y , there is a minimum value of x which permits the electron to lose such a large fraction of its energy, and that $x_{\min} = y/(1-y)$. Similarly, for a given electron energy and photon energy, there is a maximum value of x , achieved for head-on collisions: $x_{\max} \approx 15.3E[\text{TeV}]\omega_0[\text{eV}]$. In the event that, for a given set of E , ω_0 , and y , $x_{\min} > x_{\max}$, this may be physically interpreted to mean that such a photon is incapable of making such an electron lose the desired amount of energy.

For a beam with N electrons passing through a region of length L , the distribution of energy loss is given by:

$$\frac{dN}{dy} = NL \int_{\alpha=0}^{\alpha=\pi} \frac{d\Omega}{4\pi} \int_{\omega=\omega_{\min}}^{\infty} d\omega \frac{d\sigma}{dy} (1 + \cos \alpha) n_\gamma(\omega, T), \quad (5)$$

where ω_{\min} is the minimum photon energy capable of producing a fractional energy loss y , given E and α ; and $n_\gamma(\omega, T)$ is the number density spectrum of photons with energy ω at temperature T .

Let us approximate the number density spectrum with a Dirac delta function at the the mean photon energy, i.e. $n_\gamma(\omega, T) \approx \bar{n}_\gamma \delta(\omega - \omega_0)$. In this case, the inner integral in Equation 5 goes to zero when $\omega_{\min} > \omega_0$. This in turn may be re-parameterized as an upper bound on the incidence angle α ; examination of the expression for x indicates that $x_{\min} = x(\alpha = \alpha_{\max})$, or that $\alpha_{\max} = 2 \cos^{-1} \sqrt{x_{\min}/15.3E[\text{TeV}]\omega_0[\text{eV}]}$. We may now rewrite Equation 5 as:

$$\frac{dN}{dy} = \frac{NL\sigma_0\bar{n}_\gamma}{4\pi} \int_{\alpha=0}^{\alpha=\alpha_{\max}} d\alpha \sin \alpha (1 + \cos \alpha) \frac{1}{x} \left[\frac{1}{1-y} + 1 - y - 4r(1-r) \right]. \quad (6)$$

In addition to the energy loss, an electron which scatters off of a thermal photon will emerge with some recoil angle θ_e relative to its initial trajectory. The opening angle of the scattered photons in the laboratory frame is given by [6]:

$$\theta_\gamma = \frac{\sqrt{1 + \gamma\omega_0/m_e c^2}}{\gamma}. \quad (7)$$

For the beam and photon energies of interest, this is approximately equal to $1/\gamma$. The photon emerges double-Lorentz boosted, so $\omega_f \approx \gamma^2 \omega_0$. Assuming that the outgoing photon does not remove a large fraction of the beam energy, the typical electron recoil angle will be given by $\omega_0/m_e c^2$. For 0.07 eV photons, this ratio is 137 nrad.

2.2 Beam-Gas Bremsstrahlung

In beam-gas Bremsstrahlung, an electron collides with an atom and emits a photon which carries away a fraction of the beam's energy. The cross-section for a beam-gas scatter in which the photon carries away a fraction of the beam energy which is greater than some cutoff value y_{\min} is given by:

$$\sigma_{\text{BGB}}(y_{\min}) = \frac{A}{N_A X_0} \left(-\frac{4}{3} \log y_{\min} - \frac{5}{6} + \frac{4}{3} y_{\min} - \frac{y_{\min}^2}{2} \right), \quad (8)$$

where A is the atomic mass (in kilograms per mole) of the residual gas, N_A is Avogadro's Number, and X_0 is the radiation length (in kilograms per square meter) of the residual gas at standard temperature and pressure. The angular distribution of the emerging photon is given by:

$$f(\theta) d\Omega = F(\gamma) \frac{\theta d\Omega}{(\theta^2 + \gamma^{-2})^2}, \quad (9)$$

where $F(\gamma)$ is a function which correctly normalizes the integral of f : $F(\gamma) \int d\theta \theta \sin \theta / 2(\theta^2 + \gamma^{-2})^2 = 1$. Numerical evaluation indicates that $F \approx 0.392/\gamma$. Thus, the cross-section for beam-gas Bremsstrahlung in which the fraction of beam energy lost exceeds y_{\min} and the photon angle exceeds θ_{\min} is given by:

$$\sigma_{\text{BGB}}(y_{\min}, \theta_{\min}) = \frac{AF(\gamma)}{2N_A X_0} \int_{\theta_{\min}}^{\pi} d\theta \frac{\theta \sin \theta}{(\theta^2 + \gamma^{-2})^2} \left[-\frac{4}{3} \log(y_{\min}) - \frac{5}{6} + \frac{4}{3} y_{\min} - \frac{y_{\min}^2}{2} \right]. \quad (10)$$

2.3 Elastic Beam-Gas Scattering

Elastic beam-gas scattering – variously known as Coulomb or Mott scattering – is the simplest to study because, as an elastic process, there is no possibility of the electron becoming a problem for the energy collimation; we may consider only the betatron collimation problem posed by electrons which change their trajectory when encountering a residual gas atom. The cross section for Coulomb scattering through an angle which exceeds some θ_{\min} is given by:

$$\sigma_{\text{Coulomb}}(\theta_{\min}) \approx 6.5124 \times 10^{-8} \frac{Z^2}{(E[\text{GeV}])^2} \left[\frac{1}{1 - \cos \theta_{\min}} \right] \text{ barn}. \quad (11)$$

For small angles we can simplify Equation 11 and recast it to show the parameters used in Equation 3:

$$\sigma_{\text{Coulomb}}(z) = 3.2562 \times 10^{-8} \text{ barn} \cdot \frac{Z^2 \bar{\beta} \beta_{\text{coll}}}{E(z)[\text{GeV}] E_{\text{coll}}[\text{GeV}] y_{\text{coll}}^2}. \quad (12)$$

2.4 Inelastic Scattering off Atomic Electrons

In this case, the beam is scattered not by the nucleus of the encountered atom but by one of the atomic electrons which orbits the nucleus. The total cross section for a relative energy loss greater than y_{\min} is given by:

$$\sigma_{e^-, \delta} = \frac{2\pi r_e^2 Z}{\gamma y_{\min}}. \quad (13)$$

The scattering angle in the laboratory frame is given by:

$$\tan \theta = \frac{1}{\sqrt{2\gamma}} \frac{2\sqrt{y+y^2}}{1-2y}. \quad (14)$$

In [4], the energy loss was estimated to cause 0.1 off-energy particles per bunch train in a 5 km transport line, while the scattering angle was found to be 44 μrad for 0.1% energy loss. This mechanism will therefore generate many more large-amplitude electrons than off-energy ones.

3 Application to NLC Main Linac

3.1 Scattering from Thermal Photons

We can determine the number of off-energy particles generated by the final focus by integrating Equation 6 from the value of y which corresponds to 5 GeV loss per scatter to $y = 1$. Because the beam is accelerated, the lower limit is a function of z along the linac. This in turn means that Equation 6 is transformed to a triple integral over linac z position, fractional energy loss y , and angle α :

$$N_{\text{compton},\delta} = \frac{N\sigma_0\bar{n}_\gamma}{4\pi} \int_0^{L_{\text{linac}}} dz \int_{y_{\text{min}}(E(z))}^1 dy \int_0^{\alpha_{\text{max}}(E(z))} d\alpha \sin \alpha (1 + \cos \alpha) \frac{1}{x(E(z), \alpha)} \quad (15)$$

$$\times \left\{ \frac{1}{1-y} + 1 - y - 4r(E(z), \alpha, y) [1 - r(E(z), \alpha, y)] \right\},$$

where $E(z) = E_0 + Gz$, where E_0 is the injection energy of 8 GeV and G is the loaded gradient of 50 MeV/m; $y_{\text{min}} = 5 \text{ GeV}/E(z)$; and we have explicitly shown functional dependence on $E(z)$, α , and y in all derived variables.

Figure 1 shows the off-energy cross section in millibarns as a function of the beam energy. Note that the cross-section vanishes for beam energies below about 71 GeV: it is at this approximate energy that head-on collision with a 0.07 eV photon can cause a 5 GeV energy loss. The off-energy cross section rises with beam energy because the fractional energy loss required is reduced, and then plateaus because the total compton cross section is falling as the beam energy rises.

When Figure 1 is integrated and appropriate constants applied, the total number of off-energy particles expected from this process is found to be on the order of 40 per bunch train. This is consistent with the result in [4], in which 100 off-energy particles are generated in 5 km by 10^{12} particles at 500 GeV beam energy.

Based on Equation 3, the scattering angle required to exceed the betatron collimator aperture varies from approximately 33 microradians at the low energy end of the linac to approximately 2.5 microradians at the high energy end of the linac. The recoil angles for electrons will typically be 137 nanoradians. Thus, we do not expect a significant number of large amplitude particles from thermal-photon scattering recoils.

3.2 Beam-Gas Bremsstrahlung

The total number of off-energy particles generated by beam-gas inelastic scattering can be computed from Equation 8 in a manner analogous to that used for thermal photon scattering. The relevant equation becomes:

$$\begin{aligned}
N_{\text{BGB},\delta} &= \frac{2NA}{N_A X_0} \cdot 3.22 \times 10^{22} \text{P[Torr]} \text{m}^{-3} \int_0^{L_{\text{linac}}} dz \\
&\times \left[-\frac{4}{3} \log y_{\min}(E(z)) - \frac{5}{6} + \frac{4}{3} y_{\min}(E(z)) - \frac{y_{\min}^2(E(z))}{2} \right],
\end{aligned} \tag{16}$$

where variables are defined as in Equations 8 and 15 and the factor of 2 accounts for the two atoms of nitrogen per molecule. Given values of $A = 1.4 \times 10^{-2}$ kg/mol and $X_0 = 300$ m or 375 kg/m², we find that 1,700 off-energy particles are generated per bunch train.

The beam-gas Bremsstrahlung process can generate large-amplitude betatron oscillations due to the recoil angle of the electron, as shown in Figure 2: a photon takes away a fraction y of the electron's energy and scatters through an angle θ_γ ; the electron, which still has a fraction $1 - y$ of its initial energy, is deflected by an angle θ_e in the opposite direction (in this case, we take the convention that both angles are positive in sign). In the small-angle limit, the relationship between the two angles and the energy loss can be determined by conservation of transverse momentum: $y = \theta_e / \theta_e + \theta_\gamma$. If we replace θ_e with θ_{\min} , the minimum angle required to "lose" the electron at this location, we can use the expression above to determine the minimum y value which permits an electron deflection of θ_{\min} . We can then use Equation 10 to determine the number of large-amplitude particles:

$$\begin{aligned}
N_{\beta,\text{BGB}} &= \frac{N}{N_A X_0} \cdot 3.22 \times 10^{22} \text{P[Torr]} \text{m}^{-3} \int_0^{L_{\text{linac}}} dz F(\gamma(z)) \int_{\theta_{\min}(z)}^{\pi} d\theta \\
&\times \frac{\theta \sin \theta}{(\theta^2 + \gamma^{-2})^2} \left[-\frac{4}{3} \log(y_{\min}) - \frac{5}{6} + \frac{4}{3} y_{\min} - \frac{y_{\min}^2}{2} \right],
\end{aligned} \tag{17}$$

where $y_{\min}(\theta, z) \equiv \theta_{\min}(z) / \theta + \theta_{\min}(z)$. The cross-section for scattering out of the betatron collimation aperture in the vertical is shown in Figure 3 as a function of position along the linac. Equation 17 indicates that the expected number of large-amplitude particles from inelastic beam-gas scattering is approximately 70 per bunch train.

3.3 Elastic Beam-Gas Scattering

Here we can simply integrate Equation 12, using the expression for beam energy as a function of L derived earlier. For 50 MeV/meter, the total number of scatters from this source due to N_2 molecules is given by:

$$N_{\beta,\text{Coulomb}} = NP[\text{Torr}] \cdot 8.031 \times 10^{-3} \text{m}^{-1} \bar{\beta} \log \left(\frac{E_{\text{final}}}{E_{\text{initial}}} \right). \tag{18}$$

In this case, the number of large-amplitude particles is approximately 6,500 per bunch train.

3.4 Inelastic Scattering off Atomic Electrons

The cross-section for generating a particle which has lost more than 1% of the final beam energy can be determined through a suitable rewriting of Equation 13:

$$\sigma_{e^-, \delta} = \frac{2\pi r_e^2 Z}{0.01 \gamma_{\max}}, \tag{19}$$

where γ_{\max} is the Lorentz factor for 500 GeV per beam, or approximately 10^6 . The total number of off-energy particles thus generated is given by:

$$N_{e^-, \delta} = 2 \cdot 3.22 \times 10^{22} \text{P[Torr]} \text{m}^{-3} L_{\text{linac}} N \frac{2\pi r_e^2 Z}{0.01 \gamma_{\max}}, \quad (20)$$

which amounts to a whopping 0.22 off-energy electrons per bunch train.

The large-angle effect is much more serious. Equation 14 can be inverted to determine the minimum energy error required to scatter the beam electron out of the betatron aperture, and this energy is very small: 8.85×10^{-6} in ELIN1, 4.55×10^{-6} in ELIN2, and 3.13×10^{-6} in ELIN3. We can again perform a transformation on Equations 13 and 20 to find that:

$$N_{e^-, \beta} = 2 \cdot 3.22 \times 10^{22} \text{P[Torr]} \text{m}^{-3} N \frac{2\pi r_e^2 Z m_e c^2}{0.05 \text{GeV/m} \cdot \delta_{\min}} \log \left(\frac{E_{\text{final}}}{E_0} \right). \quad (21)$$

Thus, the number of large-amplitude particles generated by atomic electron scattering is approximately 1860 per bunch train.

4 Conclusions

We have investigated the amplitudes for thermal-photon scattering and three forms of beam-gas scattering in the NLC main linac, and compared the results to the collimation amplitudes in the beam delivery system. We find that the number of particles which are scattered through a sufficiently large angle to impact the vertical betatron collimation spoilers is approximately 8,500 per bunch train; an even smaller number of particles is expected to hit the horizontal spoilers, since the horizontal spoiler gaps are larger and the betatron function at the horizontal spoilers is smaller than at the vertical, thus a larger deflection angle is required. The large-amplitude halo from scattering is dominated by Coulomb and atomic-electron scattering, with a small contribution from beam-gas Bremsstrahlung and virtually no contribution from thermal-photon scattering.

The number of particles which are driven sufficiently off-energy in the linac to impact the energy collimators is 1,700 per bunch train, which is dominated by beam-gas Bremsstrahlung. Thermal-photon scattering and atomic-electron scattering are negligible contributors to this portion of the beam halo.

It is important to realize that the population of halo particles from scattering is relatively sensitive to the details of the lattice, the collimation amplitude, and the pressure profile and residual gas composition. As the designs for operation at lower energies become more fully realized, we recommend that these calculations be revisited for those configurations.

5 Acknowledgements

The authors would like to thank I. Reichel and F. Zimmermann, without whom this Note would not have been possible, and T. Raubenheimer for many helpful suggestions and insights.

References

- [1] J. Seeman, private communication.
- [2] H. Burkhardt and R. Kleiss, "Beam Lifetime in LEP," in *Proceedings of the 1994 European Particle Accelerator Conference*, 1353 (1994).

- [3] I. Reichel, *Study of the Transverse Beam Tails at LEP*, CERN-Thesis-98-017, 40 (1998).
- [4] I. Reichel, F. Zimmermann, T. Raubenheimer, P. Tenenbaum, “Thermal-Photon and Residual-Gas Scattering in the NLC Beam Delivery,” SLAC-PUB-8012 (1998).
- [5] P. Tenenbaum *et al*, “Studies of Beam Optics and Scattering in the Next Linear Collider Postlinac Collimation System,” in *Proceedings of the 2000 Linac Conference* (2000).
- [6] P. Tenenbaum and T. Shintake, “Measurement of Small Electron Beam Spots,” in *Annual Reviews of Nuclear and Particle Science*, 49:137 (1999).

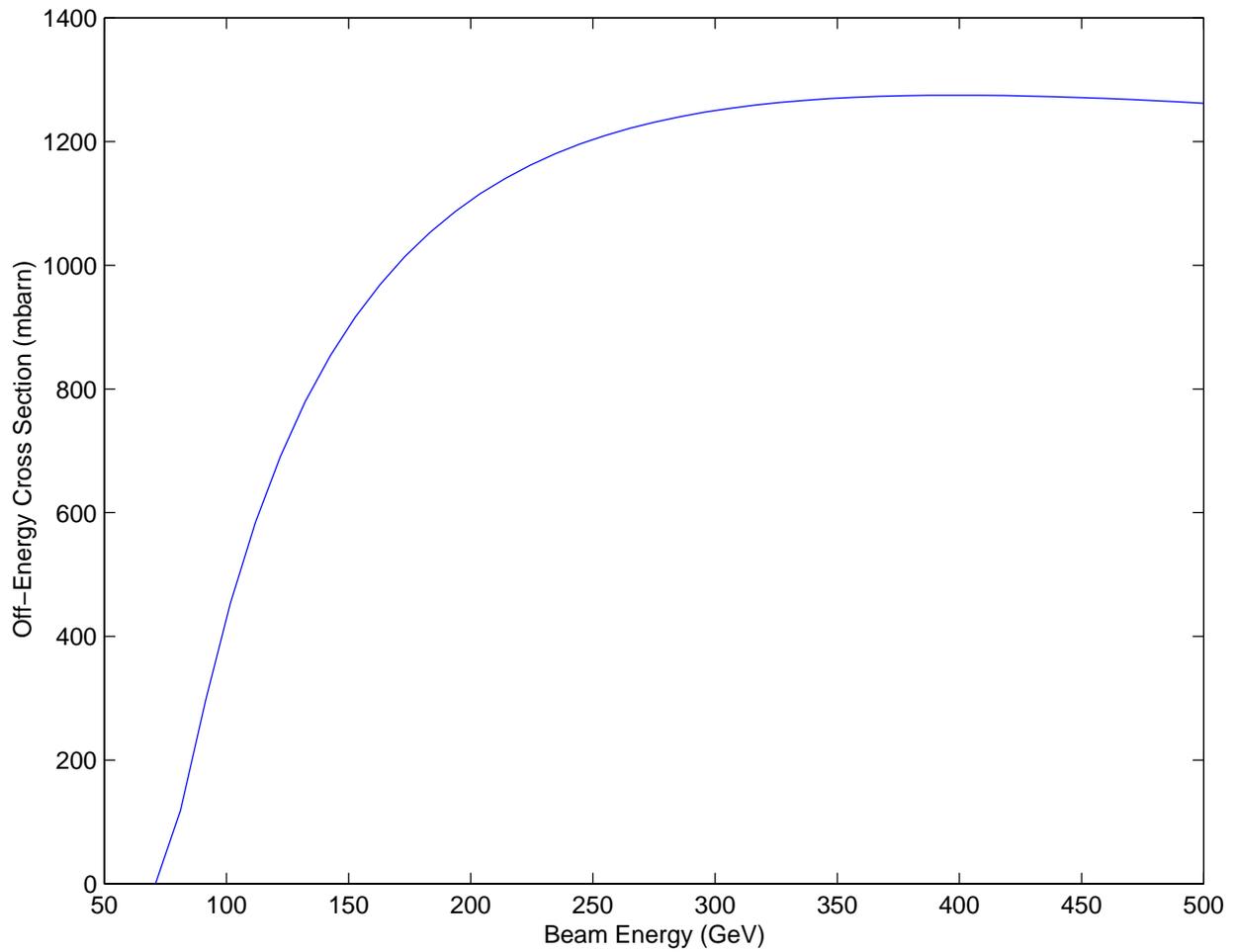


Figure 1: Cross-section for generation of an off-energy particle as a function of beam energy. The “kink” in the distribution at low energy is an artifact of sparse sampling.

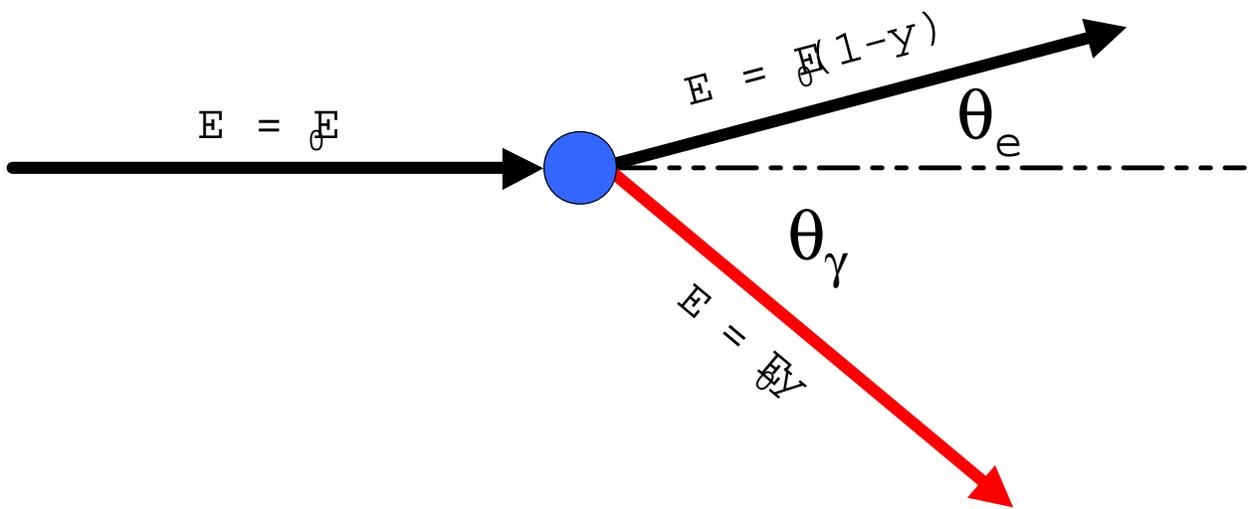


Figure 2: Inelastic beam-gas scattering: the electron loses a fraction y of its initial energy; the photon is emitted at an angle θ_γ , and the electron recoil angle, θ_e , is determined by the requirement of transverse momentum conservation.

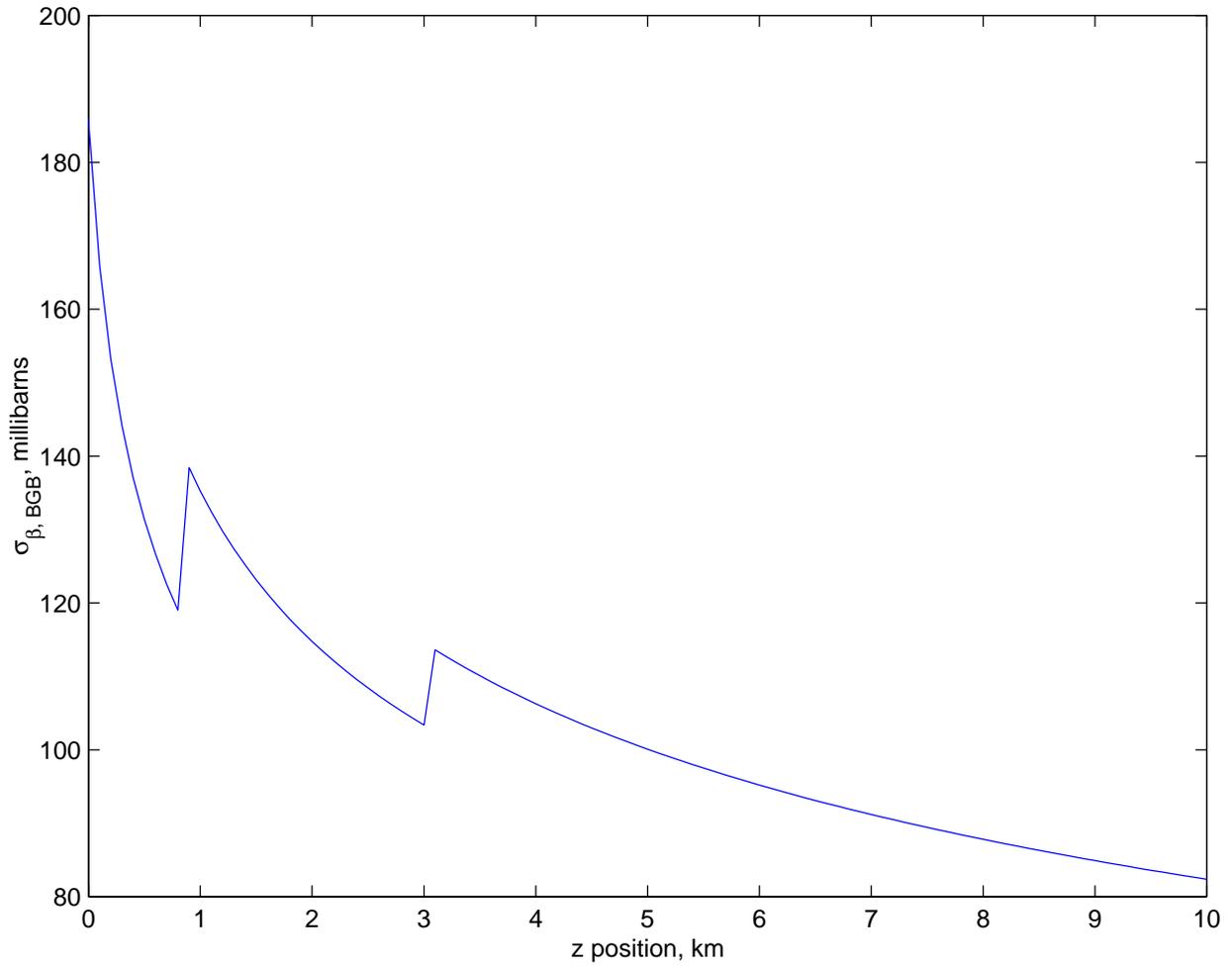


Figure 3: Cross-section for generation of a large-amplitude electron due to beam-gas Bremsstrahlung with a nitrogen atom, as a function of linac z position.