Spectral Content of the NLC Bunch Train due to Long Range Wakefields

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Abstract

The functional specifications of the sub-train position feedback for the NLC Main Linac are determined by the expected amplitude and spectrum of distortions to the bunch train, which in turn arise from the long-range wakefields of the RF structures. We describe a method for estimating the amplitude and spectrum of the distortions due to assorted structure misalignments, and apply the method to both tilted structures and structures with a cell-to-cell misalignment determined by a random-walk model. Some implications for the sub-train feedback, such as the required system bandwidth, are considered.

1 Introduction

In order to achieve its luminosity goals, the Next Linear Collider main linacs must accelerate trains of bunches from 10 GeV to 500 GeV on each machine cycle, and must also maintain the straightness of each bunch train. The primary aberration which acts to distort the bunch train is long range wakefields in the linear accelerator structures; reduction of the long range wakefield, via damping and detuning, has been an intense focus of structure R & D at SLAC for over a decade.

The RDDS scheme of damping and detuning the higher-order modes in the structure relies upon delicate cancellation of the wakefield deflections over the cells of each structure; thus, various internal misalignments of a structure can result in a breakdown of this cancellation. Tilted structures and structures which are on average straight but with cell-to-cell misalignments can thus generate wakefields which deflect later bunches in the bunch train, leading to an increase in the effective emittance.

One of the mechanisms proposed to cure these quasi-static long-range wakefields is a fast kicker system coupled with high-bandwith beam position monitors. Such a system is outlined in the ZDR [1], along with estimated specifications of the system's strength, speed, and resolution requirements. In order to achieve greater insight into the requirements of the high-bandwidth BPM system, we have performed an expanded study of the amplitude and spectrum of bunch-train distortions due to long-range wakefields.

2 Description of the Method

Estimation of the bunch-to-bunch deflections from the RF structures is based upon the wakefield matrix formalism [2]: a *global wakefield matrix*, W_{nc}, is used to find the wakefield (in V/pC/mm/m) due to a unit misalignment of cell c, at a distance n bunch-spacings behind a driving bunch. Assuming that the bunch train contains a string of evenly-spaced bunches with equal charges, and that the train is to good approximation straight (i.e., the RMS bunch position spread is small relative to the train's offset in the cell), the deflection of bunch j is the *sum wake* for that cell:

$$w_{jc} = \sum_{k=1}^{j} W_{kc}. \tag{1}$$

Thus the deflection experienced by each bunch (in V/pC/m) is given by the product of the matrix w_{jc} with a vector which contains the cell misalignments (conventionally in mm):

$$V_j = \sum_{c=1}^{N_{\rm cell}} w_{jc} dx_c. \tag{2}$$

The actual deflection angle, then, is given by:

$$x'_j = V_j N_{\rm bunch} L_{\rm struct}/E. \tag{3}$$

Thus, given a global wakefield matrix W and structure cell positions $\vec{dx}$, the angular deflection at the end of the structure can be computed for each bunch.

In order to understand the spectral properties of the train, we have computed the discrete Fourier transform of the bunch position vector, $X_j \equiv FFT(x_j)$. From this one can compute the power spectrum, $P_j = |X_j|^2$. From the power spectrum, it is possible to compute the *integrated motion*, $M(j)$:

$$M(j) = \left(\sum_{k=j}^{\infty} P_k \right)^{1/2}. \tag{4}$$

The significance of the integrated motion is that it determines the RMS motion which is due to frequencies above the frequency of interest. This is of particular use in this application, in that the $M(j)$ shows the remaining RMS motion if the BPM system has a frequency cutoff at $\nu(j)$.

Assuming that the structures in the linac have uncorrelated misalignments of all kinds, the RMS integrated motion at the end of the main linac is given by:

$$M(j)^{\rm final} = M(j) L_{\rm struct} Q_{\rm part} \left(\sum_{i=1}^{N_{\rm struct}} \left(\frac{1}{E_i} R_{12}^{i \rightarrow {\rm final}} \right)^2 \right)^{1/2}. \tag{5}$$

Rather than explicitly performing the summation on the RHS of Equation 5, we make use of the following simplifications:

- We replace $R_{12}^{i \rightarrow {\rm final}}$ with $\sqrt{E_i/E_{\rm final}}\sqrt{\beta_i \beta_{\rm final}} \sin \Delta\nu$
- we replace $\sin \Delta\nu$ with its RMS value of $1/\sqrt{2}$
- we replace $\sqrt{\beta_i \beta_{\rm final}}$ with its ensemble average over a linac "supersector", $\left(\sqrt{\beta_- \beta_+} + \beta_+\right)/2 \equiv \bar{\beta}$
- we replace $1/\sqrt{E_i E_{\rm final}}$ with its average over 1 "supersector," $2/(E_{\rm final} - E_{\rm initial})(1 - \sqrt{E_{\rm initial}/E_{\rm final}}) \equiv \bar{E}$.

This reduces Equation 5 to:

$$\begin{aligned} M(j)^{\rm final} &= M(j) L_{\rm struct} Q_{\rm part} \left(\sum_{i=1}^{N_{\rm struct}} (\frac{\bar{E}^2 \bar{\beta}^2}{2}) \right)^{1/2} \\ &= M(j) L_{\rm struct} Q_{\rm part} \bar{E} \bar{\beta} \sqrt{N_{\rm struct}/2}. \end{aligned} \tag{6}$$

Note that in Equation 6 we have implicitly projected the bunch train distortions to a location at the end of the supersector where $\beta = \beta_+$. Finally, we take the contributions at the end of each supersector and project to the end of the linac using the R_{11} term; for the NLC main linac optics, the contributions from the three supersectors, combined in quadrature in this fashion, are equal to the contribution from the third supersector multiplied by 1.47.

In the following examples, we consider two bunch train configurations: the nominal high-charge configuration, with 90 bunches spaced 2.8 nsec apart, and 1.1×10^{10} particles per bunch; and a configuration with 180 bunches spaced 1.4 nsec apart, and 0.6×10^{10} particles per bunch. In both cases the mode spacing in the frequency domain is 4.0 MHz; in the former case the Nyquist frequency is 178.5 MHz, while in the latter it is 357 MHz. In all cases we use the CD1.1 NLC optics [3], with 4968 RF structures per linac and a beam energy at the end of the linac of 500 GeV. The wakefield used is *NewGlobalWake.dat,* a wakefield for the RDDS-1 structure with 4 cells decoupled from the higher-order-mode damping manifolds and no other damping applied in these cells [4]. This wakefield is far from optimal for the NLC, and it is expected that a more optimized wake will be used in the final structure design; however, this wakefield is sufficient to gain a basic understanding of the spectrum of the bunch train.

3 Example 1: Cell-to-Cell Random Walk Misalignments

Recently Stupakov and Raubenheimer [5] calculated the tolerance for emittance growth due to the long-range wakefields which arise from a "random walk" cell misalignment model: each cell has an RMS misalignment of σ_c relative to the previous cell, with the end cells forced to zero. Their calculation used an analytic approach. We consider below the spectrum of the bunch train resulting from such a misalignment mode.

We begin by generating 300 structures whose cells are misaligned in the manner described above; σ_c in this case is set to 1. We compute the power spectrum P for each structure, and then average these to get $\bar{P}$, which is the power spectrum we use for this analysis.

Figure 1 shows the integrated motion at the end of the linac, $M^{\rm final}$, for the 90 bunch NLC parameters; figure 2 shows $M^{\rm final}$ for the 180 bunch NLC parameters. The units are meters of RMS train distortion per meter of RMS cell misalignment in each case. Note that $M(1)$, the ratio of all RMS motion to the cell misalignment amplitude, is nearly 1 in both cases. In the analysis above we have assumed that the bunch train is to lowest order straight and on-axis in all the structures, ie, that a downstream structure does not "notice" the deflections from all the upstream structures. This is equivalent to assuming that the $M(1)$ is less than 1. Consequently this analysis may be a moderate underestimate of the higher-frequency components of the bunch motion.

Is Figure 1 reasonable? $M(2)$ is the RMS motion if the deflection of the bunch train average position is neglected; this is the true measure of the effective emittance increase. Stupakov and Raubenheimer calculated that the tolerance for 10% emittance increase (when $\gamma\epsilon_y = 4 \times 10^{-8}$ m.rad) is 3.4μ cell-to-cell misalignments for the 90 bunch case. At the end of the linac, $\beta_+ = 64$ meters, $\sigma_y = 1.618\mu$, and a 10% emittance increase means that the spot is increased by 0.522 μ added in quadrature. Since $M(2)$ is 0.1486, we find that the tolerance is given by 0.522 μ / 0.1486 = 3.5 μ, in good agreement with the earlier treatment.

Let us consider the bandwidth needed in the sub-train feedback system if we wish to limit emittance growth due to the cell-to-cell misalignments to 2% of the nominal damping-ring extraction emittance (3×10^{-8} m.rad), or a 1% spot size growth: what is the bandwidth required to achieve this? This requirement corresponds to 0.200 μ added in quadrature with the nominal spot. Assuming the aforementioned 3.4 μ cell-to-cell misalignments, we need to find $\nu(j)$ such that $M(j) = 0.2/3.4 = 0.059$. From Figures 1 and 2, the cutoff frequency is 119 MHz for the 90 bunch case and 190 MHz for the 180 bunch case.

4 Example 2: Tilted Structures

Figures 3 and 4 show $M^{\rm final}$, in meters of RMS train distortion per radian of RMS structure tilt, for the 90 and 180 bunch parameter sets, respectively. The value of $M(2)$ for the 90 bunch parameters is 6.53×10^{-3} meters per radian. To achieve a 10% emittance growth based on an injected emittance of 4×10^{-8}, the RMS structure tilt tolerance is $0.522\mu/6.53 \times 10^{-3}$, or 80 microradians. This is the value which was estimated by Stupakov using a different technique [6].

It is worth noting that the functional form of $M^{\rm final}$ is quite different from the form of the cell-to-cell misalignments above, in that $M^{\rm final}$ for a tilted structure contains several "plateaus." These correspond to resonances in the bunch spectrum, which in turn arise from the undamped modes at the downstream end of the structure.

In order to limit emittance growth to 2% of the damping-ring extracted emittance, and assuming that the RMS structure tilt is the aforementioned 80 microradians, we need to find $\nu(j)$ such that $M(j)^{\rm final} = 0.2/80 = 2.50 \times 10^{-3}$ meters per radian. For 90 bunches this value of $M^{\rm final}$ occurs at 43.6 MHz. For 180 bunches, $M(2)^{\rm final} = 1.88 \times 10^{-3}$ m/rad, and therefore in this configuration the emittance growth is under 2% without any multibunch correction schemes.

5 Conclusions and Discussion

We have used the matrix formalism of RF structure long range wakefields to analyze the spectral content of the NLC bunch train after acceleration in the main linac. We have considered the examples of a cell-to-cell misalignment modelled by a random walk, and also a structure which is tilted relative to the beam. In each case the technique used to study the frequency-domain bunch train distortions agrees with results which examine the bunch train in the time domain. We also found that:

- An RMS cell-to-cell misalignment of 3.4μ would cause an emittance increase of 14% relative to the damping ring extracted emittance, for a 90-bunch train, if left uncorrected; a multibunch correction system with a bandwidth of 119 MHz would reduce this to 2%.
- For the same cell-to-cell misalignment and a 180 bunch train, the emittance dilution of 9% can be reduced to 2% with a correction system bandwidth of 190 MHz.
- An RMS structure tilt of 80 microradians would cause a 14% emittance increase in the 90-bunch NLC bunch train; the bandwidth needed to reduce this to 2% is 44 MHz.
- The 80 microradian structure tilt produces less than 2% emittance increase for the 180 bunch NLC train.
- At the end of the NLC main linac, at a maximum-beta point ($\beta_+ = 64$ m), the RMS train distortion which increases the emittance by 2% is 0.2μ.

One may ask, "Is it worthwhile to have even one sub-train feedback system in the NLC?" Based upon the above information, the answer is almost certainly, "Yes:" a sub-train feedback system with moderate bandwidth can reduce the emittance dilution from long-range wakefields by a factor of a few. Also, the estimates of emittance dilution assume that the center of mass of the bunch train is at some point steered back to the axis of the accelerator. As shown in Figure 1, $M(1)^{\rm final}$ can be several times as large as $M(2)^{\rm final}$; this implies that the DC offset of the train center is larger by far than the distortions within the train. However, the first bunch in the train never receives a long-range wakefield deflection; therefore, if the train center of mass is re-steered, then the optimum single-bunch orbit and the optimum multibunch orbit are not the same. At the least this will make the accelerator hard to tune up (since some fraction of single-bunch tuning will be made obsolete by the multibunch orbit shift).

Another question is, "How many sub-train feedbacks should the NLC have?" We have seen that one feedback system at the end of the linac is more than adequate for emittance purposes. Furthermore, the magnitudes of M for each of the 3 supersectors are nearly the same (at the 20% level). Thus, for example, an 80 microradian RMS structure tilt will produce an RMS train distortion of 0.447 μ at the end of supersector 1, and a distortion of 0.356 μ at the end of supersector 3; however, the beam size and jitter at the end of supersector 1 are both larger than at supersector 3. As a result the signal-to-noise performance of the system may be better at the end of the linac than at the diagnostic regions within the linac, and the improvement in emittance due to the lower-energy systems will not be as large as for the final system. On the other hand, it is presently envisioned that the conventional steering feedbacks in the linac will attempt to measure the position of the head of the bunch train, in order to minimize the difference between the single-bunch and multibunch orbits. In this case, periodically "straightening out" the bunch train may simplify the conventional feedback problem.

A final consideration is the issue of signal-to-noise versus systematics within the system bandwidth. As noted above, a 0.200 μ RMS train distortion will at the end of the linac will result in a 2% emittance dilution. Conversely, if the sub-train feedback *causes* a 0.200 μ RMS train distortion, it will itself increase the effective emittance by the aforementioned 2% (and the figure of 0.200 μ is at the end of the linac proper; the betatron functions in the diagnostic region are smaller, thus the limit in that region is incrementally tighter). The S/N of the system will probably not be the limiting factor for the feedback since it can average over an arbitrarily large number of pulses. Instead, the achievable total harmonic distortion within the bandwidth of the BPMs and the kicker (due to reflections, for example) will probably be the limiting factor. Ultimately the feedback system design must be a balancing of total harmonic distortion, bandwidth, permitted emittance dilution, and linac construction tolerances. If the bandwidth of the system and its THD are better than those described above, one could consider relaxing the linac construction tolerances, or else correcting a larger fraction of the multibunch emittance dilution.

6 Acknowledgements

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References

[1] *Next Linear Collider Zeroth-Order Design Report*, pages 402-404.

[2] J.W. Wang, PAC 99.

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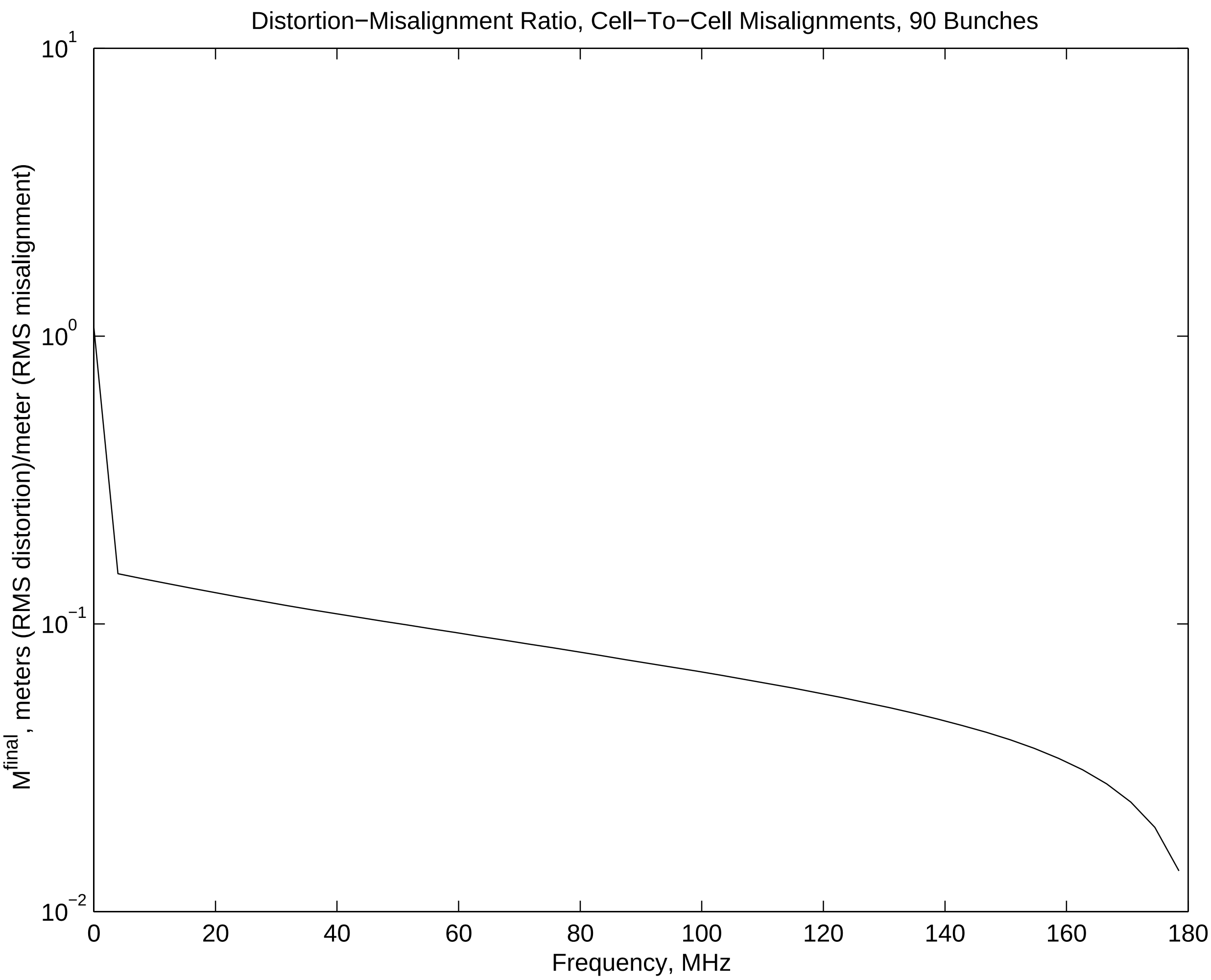

Figure 1: Value of M^{final} as a function of frequency for cell-to-cell misalignments and 90 bunches per train.

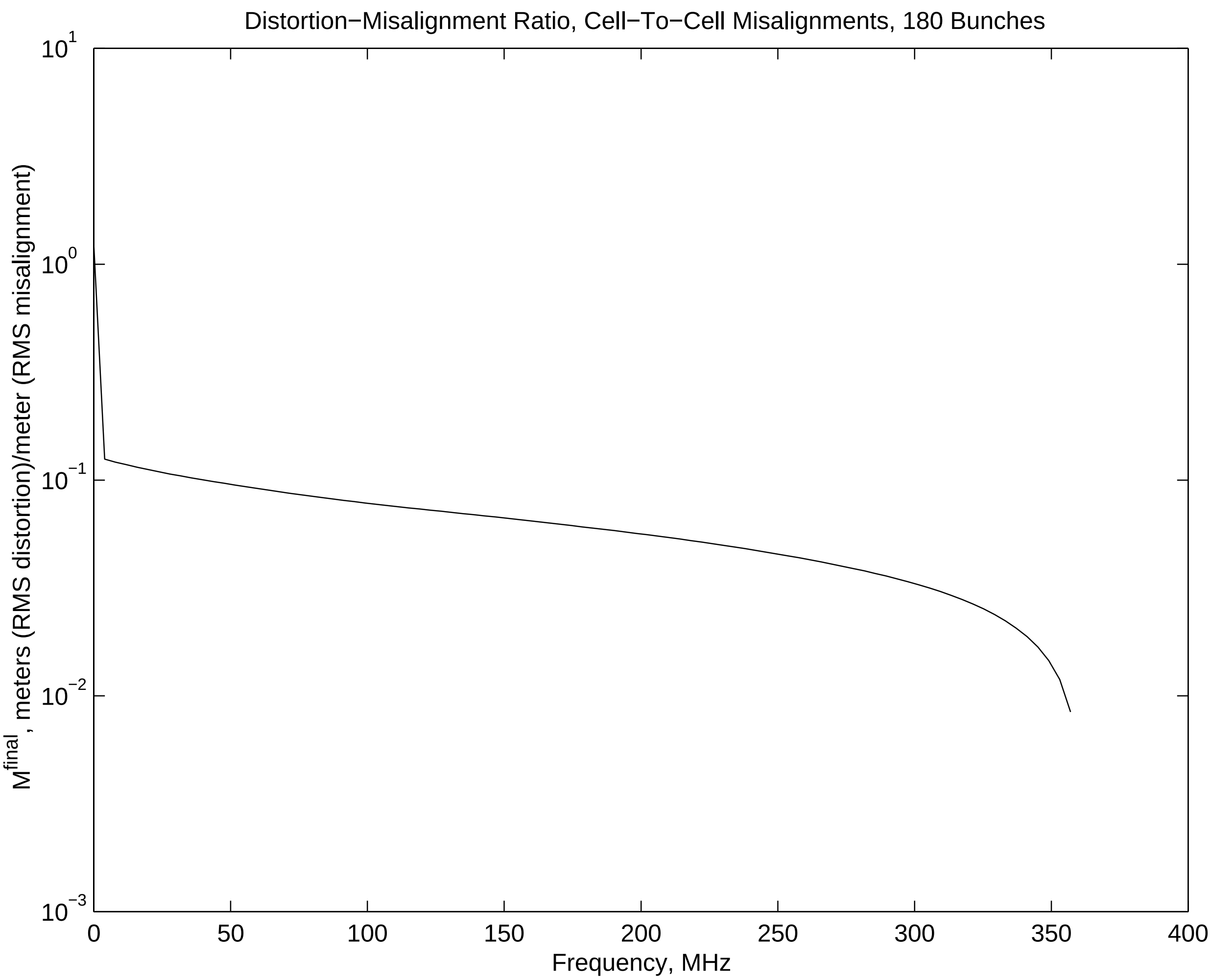

Figure 2: Value of M^{final} as a function of frequency for cell-to-cell misalignments and 180 bunches per train.

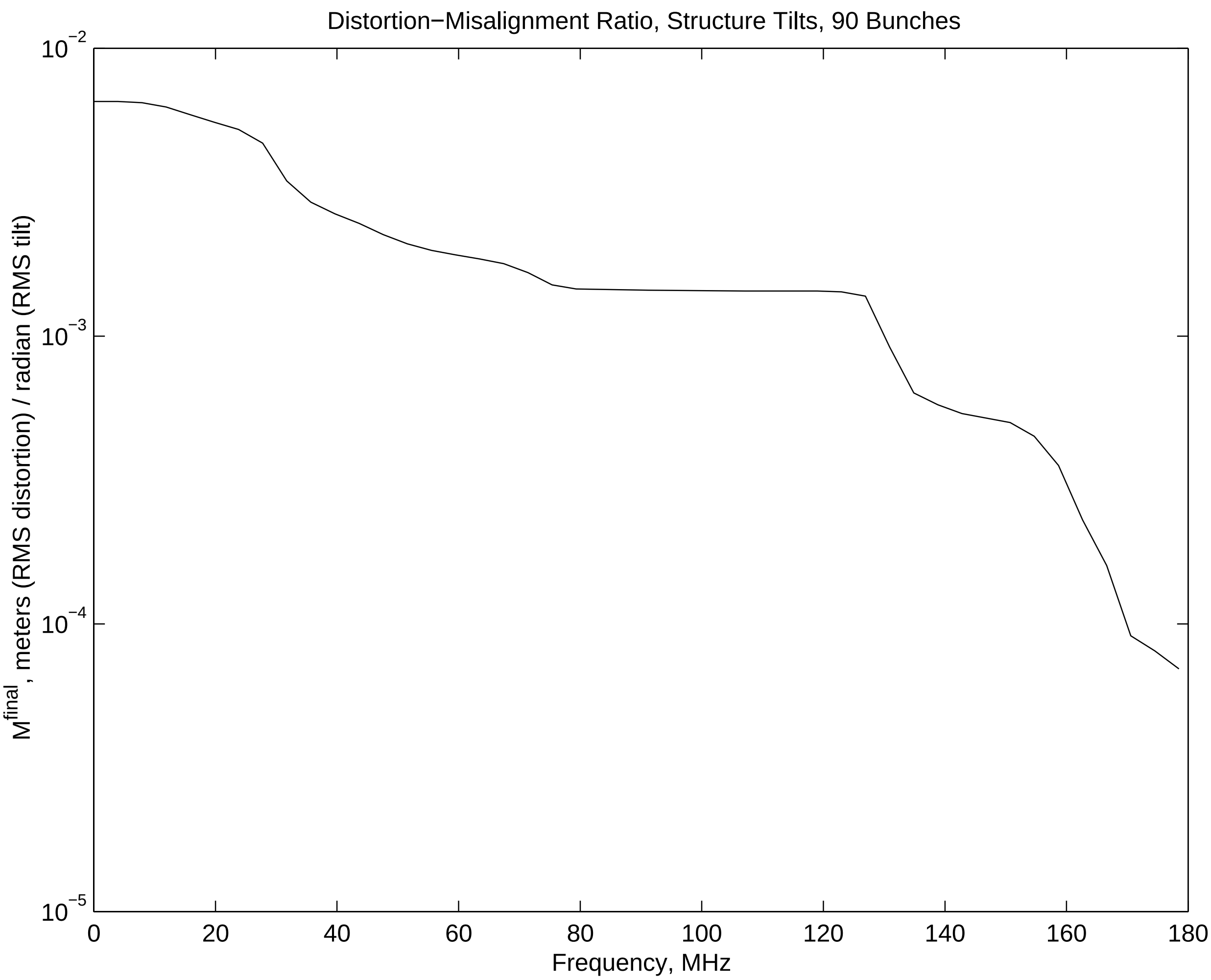

Figure 3: Value of M^{final} as a function of frequency for tilted structures and 90 bunches per train.

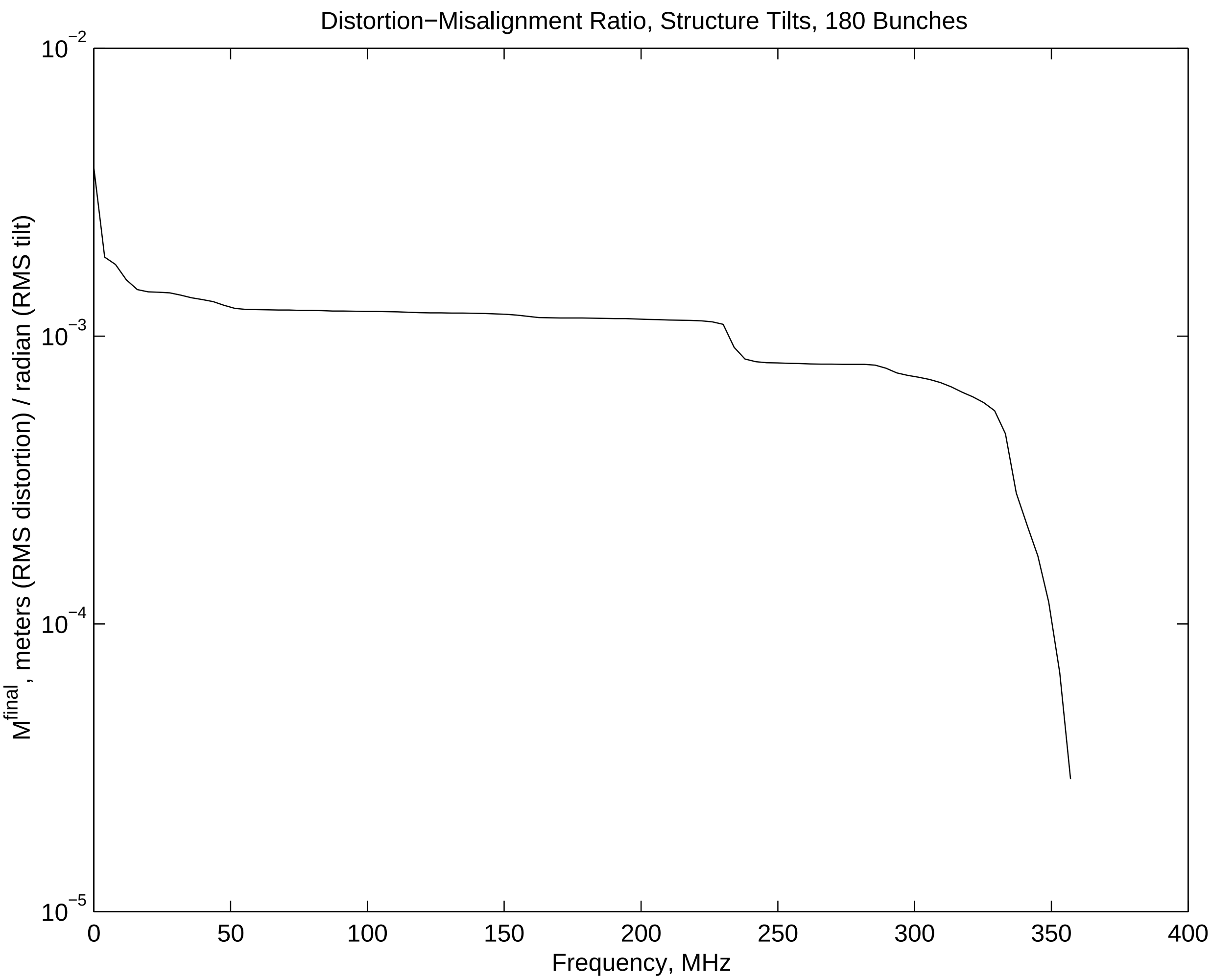

Figure 4: Value of M^{final} as a function of frequency for tilted structures and 180 bunches per train.