Linear Collider Collaboration Tech Notes

The Effect of Intrapulse RF Variation On Bunch Train Energy Spread

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Abstract:

A brief analysis is presented of the effect of small variations in amplitude and phase in the accelerating rf pulse in addition to the deliberate pulse shaping on the energy deviation along a bunch train and the effects of pulse-to-pulse fluctuations.

The Effect of Intra-Pulse RF Variation on Bunch Train Energy Spread

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INTRODUCTION:

In this note, we present a brief analysis of the effect of small variations in amplitude and phase in the accelerating rf pulse in addition to the deliberate pulse shaping on the energy deviation along a bunch train and the effects of pulse-to-pulse fluctuations.

For the sake of mathematical treatment, we approximate the NLC structures as exact constant gradient structures. For constant input power P_0 and no phase variation, the on-crest accelerating field for a constant gradient structure [1] is given by

$$E_a^2 = \frac{r}{L}(1 - e^{-2t})P_0 \tag{1}$$

where r, L, and t are the shunt impedance per unit length, the length, and the attenuation parameter, respectively. This gives a maximum acceleration voltage of

$$V_{a_m} = V_0 = \sqrt{rL\left(1 - e^{-2t}\right)P_0} \tag{2}$$

In the following, we will first discuss the effect of linear and sinusoidal rf amplitude variations and then the effect of linear and sinusoidal variations in the rf phase. Finally, we will estimate the effect of modulator voltage errors that cause large phase deviations in the klystron output.

AMPLITUDE VARIATION:

The accelerating voltage seen by a bunch entering the structure at a phase q relative to the rf crest at time t is

$$V_a(t) = \int_0^L E_a(t + z/c, z)\cos(q)dz$$
$$= \sqrt{\frac{r}{L}(1 - e^{-2t})}\cos(q)\int_0^L \sqrt{P(t + z/c - t_z)}dz \qquad (3)$$

Where E_a depends on power profile in the structure and t_z is the travel time of the rf from the structure input to point z, which, in a constant gradient structure, is given explicitly by

$$t_z = \int_0^z \frac{dz'}{v_g(z')} = -\frac{Q}{w}\ln\left[1 - \frac{z}{L}\left(1 - e^{-2t}\right)\right] \qquad (4)$$

where v_g is the group velocity and Q and ω are the structure quality factor and the rf angular frequency.

Linear:

We begin by considering a linearly varying power, $P_{\mathrm{in}}(t) = P_0 + P' t$. Assuming that the fractional power change is small over both the structure fill time and the bunch train, the voltage can be written:

$$V_a(t) = \int_0^L E_a(t + z/c, z)\cos q\, dz$$
$$= \sqrt{\frac{r}{L}\left(1 - e^{-2t}\right)P_0}\cos q \int_0^L \sqrt{1 + \frac{P'}{P_0}(t + z/c - t_z)}\, dz \qquad (5)$$
$$\approx V_0 \cos q\left[1 + \frac{P'}{2P_0 L}\int_0^L (t + z/c - t_z)dz\right].$$

Inserting the expression in Eq. (4) for t_z and expressing factors in terms of the fill time: $T_f = t_z(L) = 2tQ/w$, we find

$$V_a(t) = V_0\cos q\left[1 + \frac{P' t}{2P_0} + \frac{P' L}{4P_0 c} - \frac{P' Q}{2P_0 wL}\int_0^L \ln\left[1 - \frac{z}{L}\left(1 - e^{-2t}\right)\right]dz\right]$$
$$= V_0\cos q\left[1 + \frac{P' t}{2P_0} + \frac{P' L}{4P_0 c} + \frac{P' T_f}{4P_0 t}\frac{(1 - e^{-2t} - 2te^{-2t})}{1 - e^{-2t}}\right] \qquad (6)$$
$$= V_0\cos q\left\{1 + \frac{P'}{4P_0}\left[\frac{L}{c} + \frac{T_f}{t}\left(1 - \frac{2te^{-2t}}{1 - e^{-2t}}\right)\right] + \frac{P'}{2P_0}t\right\},$$

Over a bunch train duration T_{bt}, we get a fractional induced energy spread of

$$\frac{\Delta V_a}{\overline{V}_a} = \frac{\dfrac{T_{bt}}{2}\dfrac{P'}{P_0}}{1+\dfrac{1}{4}\left[\dfrac{L}{c}+\dfrac{T_f}{t}\left(1-\dfrac{2te^{-2t}}{1-e^{-2t}}\right)+T_{bt}\right]\dfrac{P'}{P_0}} \qquad (7)$$

For the current NLC accelerator structure parameters:

$$L = 1.8\text{m},\quad T_f = 104\,\text{ns},\quad t = 0.533,\quad T_{bt} = 263\,\text{ns}$$

this becomes

$$\frac{\Delta V_a}{\overline{V}_a} = \frac{131.5\text{ns}\,P'/P_0}{1\ +\ 88.7\text{ns}\,P'/P_0}\,.$$

For a small slope, this is well approximated by the numerator alone, as shown in Fig. 1. That is to say, the fractional voltage spread is one half the fractional rf power variation, which follows from the square root dependence.

Sinusoidal:

If instead we assume a sinusoidal variation in rf power, $P_{\text{in}}(t) = P_0 + P_1\cos(w_1 t + f)$, with P_1 small compared to P_0, we have

$$\begin{aligned} V_a(t) &\approx V_0\cos q\left\{1+\frac{P_1}{2P_0L}\int_0^L \cos\left[w_1(t+z/c-t_z)+f\right]dz\right\} \\ &= V_0\cos q\left\{1+\frac{P_1}{2P_0L}\,\text{Re}\left[e^{i(w_1t+f)}\int_0^L e^{iw_1(z/c-t_z)}dz\right]\right\} \end{aligned} \qquad (8)$$

This can be expressed as:

$$\begin{aligned} V_a(t) &= V_0\cos q\left\{1+\frac{P_1}{2P_0L}\,\text{Re}\left[e^{i(w_1t+f)}e^{i\frac{w_1L}{c(1-e^{-2t})}}\frac{L}{1-e^{-2t}}\int_{e^{-2t}}^{1} e^{-i\frac{w_1L}{c(1-e^{-2t})}u}\,u^{iw_1\frac{T_f}{2t}}\,du\right]\right\} \\ &= V_0\cos q\left\{1+\frac{P_1}{2P_0}\frac{c}{Lw_1}\,\text{Re}\left[e^{i(w_1t+f)}\exp\left(i\left\{\frac{L}{c(1-e^{-2t})}+\frac{T_f}{2t}\ln\left[\frac{c(1-e^{-2t})}{Lw_1}\right]\right\}\right)\right.\right. \\ &\qquad \left.\left.\int_{\frac{w_1Le^{-2t}}{c(1-e^{-2t})}}^{\frac{w_1L}{c(1-e^{-2t})}} e^{-i\left(v-w_1\frac{T_f}{2t}\ln v\right)}dv\right]\right\}. \end{aligned}$$

The proper values of time will rotate the enclosed expression along the positive or negative real axis, so that the maximum energy spread, if the bunch train is at least half an oscillation period long, will be twice the absolute value of the deviation term,

$$\left(\frac{\Delta V_a}{\bar{V}_a}\right)_{\max} = \frac{c}{Lw_1}\left|\int_{\frac{w_1 L e^{-2t}}{c(1-e^{-2t})}}^{\frac{w_1 L}{c(1-e^{-2t})}} e^{-i\left(v - w_1 \frac{T_f}{2t}\ln v\right)} dv\right| \frac{P_1}{P_0}, \qquad w_1 \geq \frac{p}{T_{bt}} \qquad (9)$$

which oscillates about a curve falling off inversely with frequency at high frequencies.

For $w_1 > 2p/T_{bt}$, the above expression gives the exact spread (ignoring finite bunch spacing), while below this frequency voltage spread depends on the phase f of the oscillation. For $w_1 < p/T_{bt}$ ($f_1 \equiv w_1/2p < 1.9$ MHz), the full spread cannot be intercepted by the beam. In that case, the maximum intra-train energy error becomes,

$$\left(\frac{\Delta V_a}{\bar{V}_a}\right)_{\max} = \frac{c}{Lw_1}\left|\int_{\frac{w_1 L e^{-2t}}{c(1-e^{-2t})}}^{\frac{w_1 L}{c(1-e^{-2t})}} e^{-i\left(v - w_1 \frac{T_f}{2t}\ln v\right)} dv \, \sin\left(w_1 \frac{T_{bt}}{2}\right)\right| \frac{P_1}{P_0}, \qquad w_1 < \frac{p}{T_{bt}} \qquad (10)$$

but, the pulse-to-pulse fluctuations are still given by Eq. (9). This normalized voltage spread is plotted as a function of f_1 in Figure 2. It reaches a peak of 0.95 P_1/P_0 and by 9.2 MHz has dropped below 0.25 P_1/P_0.

By ignoring the transit time z/c in the above derivation, which, strictly, is only valid at low frequencies when $w_1 << c/L$, the following closed form approximation can be obtained

$$\left(\frac{\Delta V_a}{\bar{V}_a}\right)_{\max} \approx \begin{cases} \dfrac{\sin(w_1 T_{bt}/2)}{1-e^{-2t}}\sqrt{\dfrac{1+e^{-4t}-2e^{-2t}\cos w_1 T_f}{1+(w_1 T_f/2t)^2}}\,\dfrac{P_1}{P_0}, & w_1 < \dfrac{p}{T_{bt}}, \\ \dfrac{1}{1-e^{-2t}}\sqrt{\dfrac{1+e^{-4t}-2e^{-2t}\cos w_1 T_f}{1+(w_1 T_f/2t)^2}}\,\dfrac{P_1}{P_0}, & w_1 \geq \dfrac{p}{T_{bt}}. \end{cases} \qquad (11)$$

where just the lower expression describes the pulse-to-pulse fluctuations while both expressions, with the appropriate frequency regimes, are needed to describe the intra-pulse energy errors. Although, as can be seen in Fig. 2, the oscillations get out of phase with those of the more exact function at higher frequencies, this result preserves the general form and trend which is to decay inversely with frequency.

PHASE VARIATION:

Next, consider filling the structure with an rf pulse of constant amplitude, but varying phase. Assuming that the phase variation is slow enough to ignore dispersion, the local gradient is given by

$$
\begin{aligned}
V_a(t) &= \mathrm{Re}\int_0^L E_a e^{iq(t+z/c-t_z)}\, dz \\
&= \sqrt{\frac{r}{L}\left(1-e^{-2t}\right)P_0}\int_0^L \cos q(t+z/c-t_z)\, dz \\
&= \frac{V_0}{L}\int_0^L \cos q(t+z/c-t_z)\, dz.
\end{aligned} \tag{12}
$$

where $q(t+z/c-t_z)$ is the rf phase.

Linear:

First, we will consider a simple linear phase variation $q(t) = q_0 + q'\,t$:

$$
\begin{aligned}
V_a(t) &= \frac{V_0}{L}\int_0^L \cos\left\{q_0 + q'\left(t + z/c + \frac{Q}{w}\ln\left[1-\frac{z}{L}\left(1-e^{-2t}\right)\right]\right)\right\}dz \\
&= \frac{V_0}{L}\left\{\cos(q_0 + q'\,t)\int_0^L \cos\left(\frac{q'}{c}z + \frac{Qq'}{w}\ln\left[1-\frac{z}{L}\left(1-e^{-2t}\right)\right]\right)dz\right. \\
&\qquad \left. -\ \sin(q_0 + q'\,t)\int_0^L \sin\left(\frac{q'}{c}z + \frac{Qq'}{w}\ln\left[1-\frac{z}{L}\left(1-e^{-2t}\right)\right]\right)dz\right\}.
\end{aligned} \tag{13}
$$

This result can be simplified by assuming that $q' << c/L$, that is, assume the input phase changes negligibly during the time it takes a bunch to traverse the structure (~6 ns for NLC structures). We can then drop the transit time term from the arguments in the integrands:

$$
\begin{aligned}
V_a(t) &= \frac{V_0}{L}\left\{\cos(q_0 + q'\,t)\int_0^L \cos\left(\frac{Qq'}{w}\ln\left[1-\frac{z}{L}\left(1-e^{-2t}\right)\right]\right)dz\right. \\
&\qquad \left. -\ \sin(q_0 + q'\,t)\int_0^L \sin\left(\frac{Qq'}{w}\ln\left[1-\frac{z}{L}\left(1-e^{-2t}\right)\right]\right)dz\right\} \\
&= \frac{-V_0}{\left(1-e^{-2t}\right)}\left\{\cos(q_0 + q'\,t)\int_1^{e^{-2t}} \cos\left(\frac{q'\,T_f}{2t}\ln x\right)dx\right. \\
&\qquad \left. -\ \sin(q_0 + q'\,t)\int_1^{e^{-2t}} \sin\left(\frac{q'\,T_f}{2t}\ln x\right)dx\right\}.
\end{aligned}
$$

These integrals can be evaluated to obtain:

$$V_a(t) = \frac{V_0}{1-e^{-2t}} \; \frac{1}{(q'\, T_f / 2t)^2 + 1}$$
$$\left\{ \left(1 - e^{-2t} \left[\cos q'\, T_f - \frac{q'\, T_f}{2t} \sin q'\, T_f \right] \right) \cos(q_0 + q'\, t) \right. \tag{14}$$
$$\left. + \left(\frac{q'\, T_f}{2t} - e^{-2t} \left[\frac{q'\, T_f}{2t} \cos q'\, T_f + \sin q'\, T_f \right] \right) \sin(q_0 + q'\, t) \right\}.$$

Figure 3 illustrates the fractional spread $\Delta V / \overline{V}$ this presents across the NLC bunch train as a function of q' for different values of q_0 which correspond to the phase at the beginning of the linac ($q_0 \approx -15°$), the middle ($q_0 \approx 0°$), and the end of the linac ($q_0 \approx 30°$).

Sinusoidal:

Lastly, consider a small sinusoidal phase variation $q(t) = q_0 + q_1 \cos(w_1 t + f)$, with $q_1 << 1$. We then have

$$\begin{aligned} V_a(t) &= \frac{V_0}{L} \int_0^L \cos\left\{ q_0 + q_1 \cos\left[w_1 \left(t + z/c + \frac{Q}{w} \ln\left[1 - \frac{z}{L}(1-e^{-2t}) \right] \right) + f \right] \right\} dz \\ &= V_0 \left\{ \cos(q_0) - \frac{q_1 \sin(q_0)}{L} \int_0^L \cos\left[w_1 \left(t + z/c + \frac{Q}{w} \ln\left[1 - \frac{z}{L}(1-e^{-2t}) \right] \right) + f \right] dz \right\} \end{aligned} \tag{15}$$

This integral is the same as that solved in Eq. (8) with the substitution of $q_1 \sin(q_0)$ for $P_1 \cos(q_0) \;/ \; 2P_0$. With this substitution, we can use Eqs. (9) and (10) for the long and short bunch train regimes and the approximate result in Eq. (11) becomes:

$$\left(\frac{\Delta V_a}{V_a} \right)_{\max} \approx \begin{cases} \dfrac{\sin(w_1 T_{bt}/2)}{1-e^{-2t}} \sqrt{\dfrac{1+e^{-4t} - 2e^{-2t}\cos(w_1 T_f)}{1+(w_1 T_f / 2t)^2}}\, 2q_1 \tan(q_0) & w_1 < \dfrac{p}{T_{bt}} \\ \dfrac{1}{1-e^{-2t}} \sqrt{\dfrac{1+e^{-4t} - 2e^{-2t}\cos(w_1 T_f)}{1+(w_1 T_f / 2t)^2}}\, 2q_1 \tan(q_0) & w_1 \geq \dfrac{p}{T_{bt}} \end{cases} \tag{16}$$

where, again, just the lower expression describes the pulse-to-pulse fluctuations while both expressions, with the appropriate frequency regimes, are needed to describe the intra-pulse energy errors. Equation (16) neglects the transit time term but this will not change the qualitative behavior, which is to oscillate about a curve falling off inversely with frequency as is illustrated in Fig. 2.

MODULATOR VOLTAGE VARIATION:

Even with a very clean and steady low-level rf drive system, one needs to remember that the klystron output phase is very sensitive to modulator voltage variation [2]. The phase is determined by the transit time of the klystron beam between the input cavity and the output circuit, and because the beam is not very relativistic, this will depend on voltage. We can quantify this effect by the following derivation, where T_t, L_k, and V_m are the above-mentioned transit time and klystron length and the modulator voltage.

$$\begin{aligned} \frac{db}{dV_m} &= \frac{db}{dg}\frac{dg}{dV_m} = \frac{e}{mc^2}\frac{1}{bg^3}, \\ T_t &= \frac{L_k}{cb}, \quad \rightarrow \quad \frac{dT_t}{db} = \frac{-L_k}{cb^2}, \\ \frac{df}{dt} &= w\frac{dT_t}{dt} = w\frac{dT_t}{db}\frac{db}{dV_m}\frac{dV_m}{dt} \\ &= -\frac{we}{mc^3}\frac{L_k}{b^3g^3}\frac{dV_m}{dt}. \end{aligned} \tag{17}$$

So, even though the klystron may be in saturation and its output power sufficiently flat, a slope or ripple in the modulator pulse will translate into a phase slope or ripple in the high power rf. Taking the NLC PPM klystron design parameters, $V_m = 490\,\text{kV}$ and $L_k \cong 0.72\text{m}$, we get

$$\frac{df}{dt} \cong 7.06\times10^{-5}\frac{dV_m}{dt}, \quad \text{or} \quad \frac{df}{dV_m} \cong 4\,\text{degrees/kV}.$$

Thus, a 1% voltage variation will give about 20° phase variation. This is potentially the largest source of intra-pulse rf induced energy spread. However, provided the modulator pulse shape is constant from pulse-to-pulse, the effect can be corrected for by manipulating the phase profile of the drive signal.

CONCLUSION:

The purpose of this note is to catalogue some equations useful for setting emittance budgets and calculating tolerances on the rf systems. We have considered linear and sinusoidal variations in power and phase across an rf pulse. We have also pointed to a source of the latter that will need to be controlled, namely, the modulator voltage fluctuations. The intra-pulse spread must be considered in conjunction with jitter-induced, inter-pulse energy spread – fortunately, the tolerance expressions are similar except for the very low frequency regime. Correlations among rf stations and feeds for particular variations will enter into relaxing particular rms spread allowances to individual source tolerances. If the rms energy spread allowance for a random effect, uncorrelated between rf stations, is 0.1%, we can multiply by the square root of the number of stations and arrive at a ~1.4% tolerance requirement for a single station consisting of a klystron 8-pac.

REFERENCES:

[1] Perry B. Wilson, "High Energy Electron Linacs: Applications to Storage Ring RF Systems and Linear Colliders," SLAC-PUB-2884, February 1982.

[2] E. Wright and R. Miller, "Klystron Phase Variation due to Fluctuations in Beam Voltage," NLCTA-Note #43, March 8, 1995.

FIGURES:

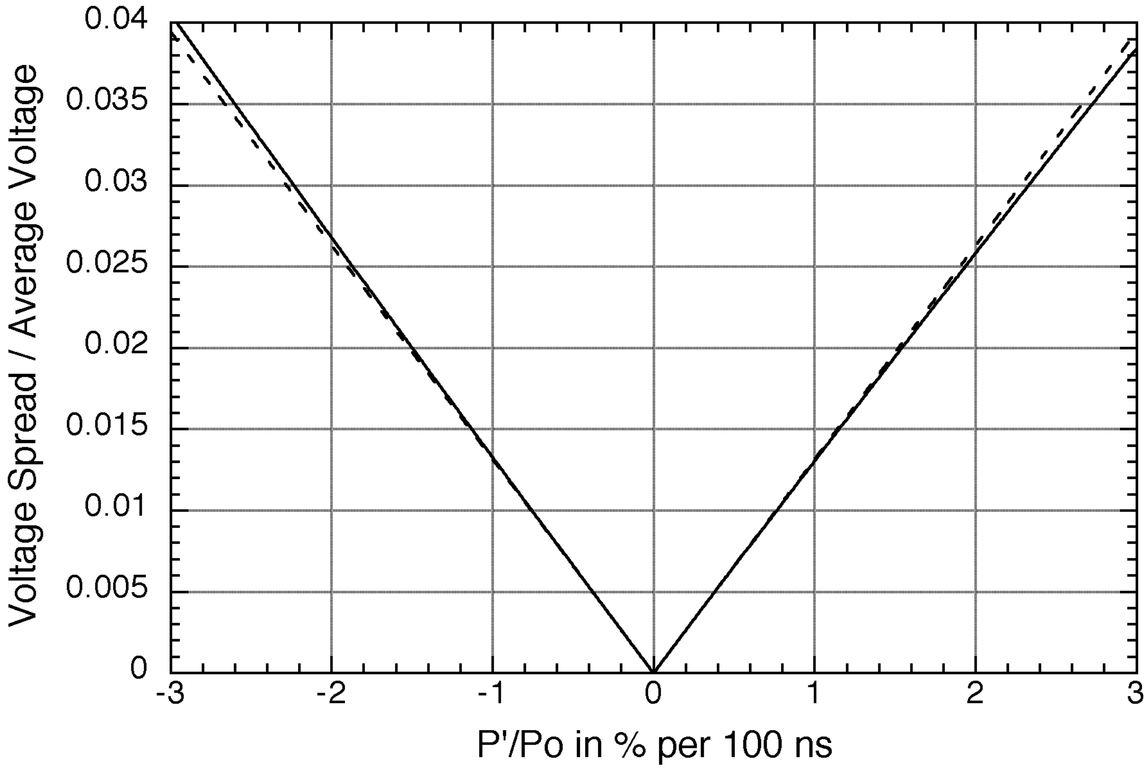

Figure 1. Fractional spread in voltage seen by bunches in an NLC bunch train accelerated by an NLC structure filled with an rf pulse with a slight linear slope in power level. P_0 is the power level entering the structure simultaneously with the first bunch. The dotted line shows the small error caused by taking $(1/2)(P'/P_0)T_{bt}$ and ignoring the change in average voltage.

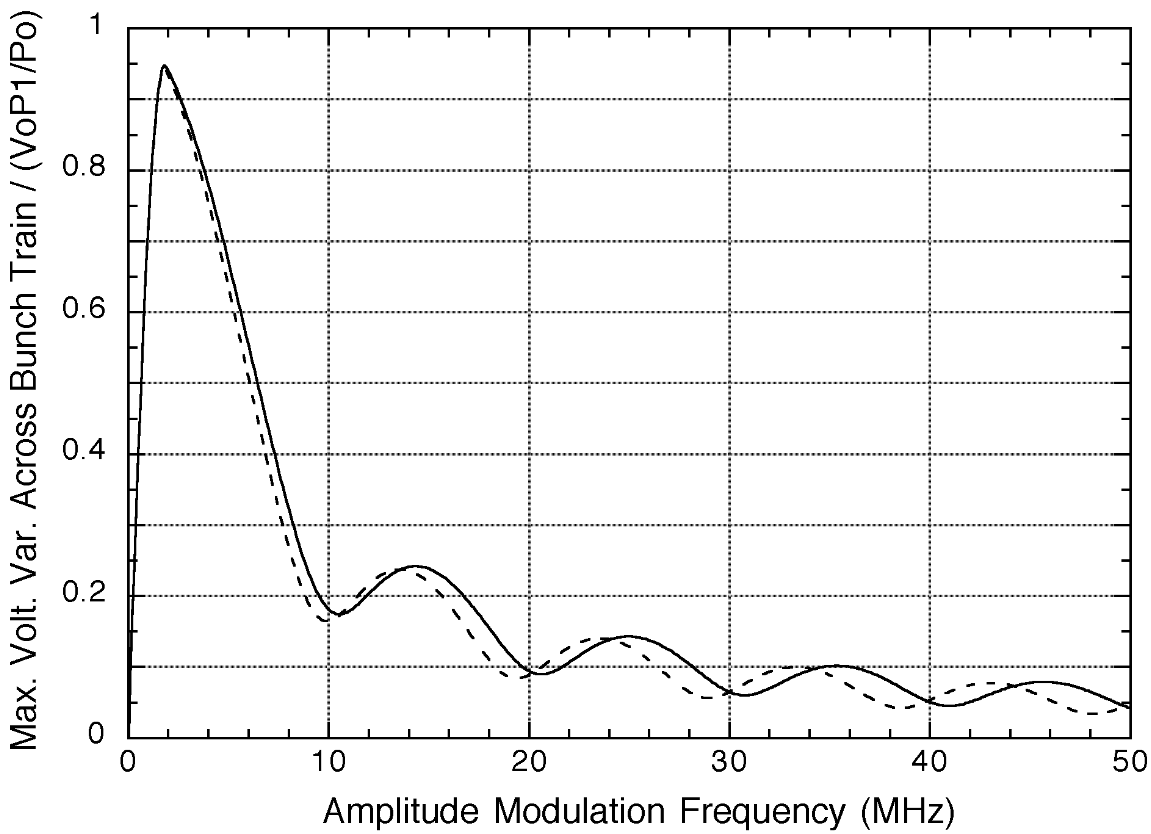

Figure 2. Coefficient of P_1 / P_0 which gives the fractional spread in voltage seen by bunches in an NLC bunch train accelerated by an NLC structure filled with an rf pulse with a slight sinusoidal variation in power level about the average P_0 as a function of modulation frequency. The dotted line is the closed form approximation, for which bunch transit time is ignored.

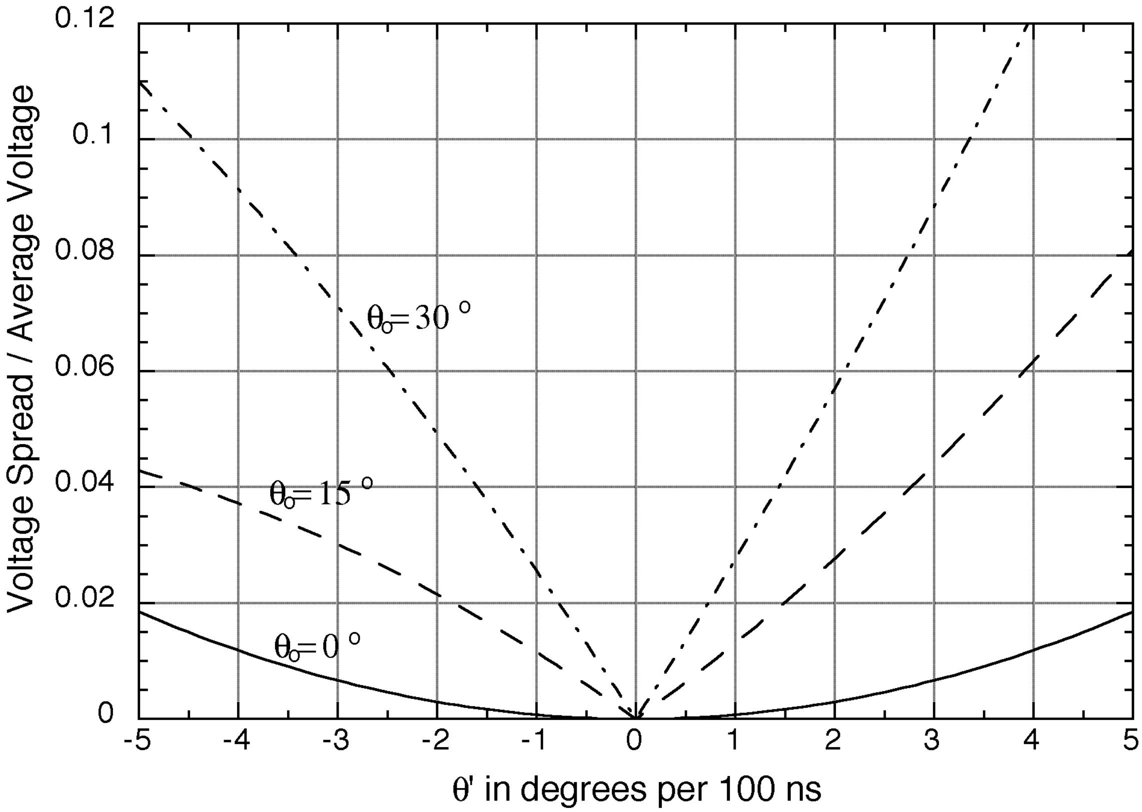

Figure 3. Fractional spread in voltage seen by bunches in an NLC bunch train accelerated by an NLC structure filled with an rf pulse with a slight linear slope in phase for bunches riding at nominal phases q_0 of 0° (solid), 15° (dashed), and 30° (dot-dashed).