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Spin-Tracking Studies for Beam Polarization Preservation in the NLC Main Damping Rings

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Abstract

We report results from studies of spin dynamics in the NLC Main Damping. Our studies have been based on spin tracking particles through the lattice under a range of conditions. We find that there are a number of spin resonances close to the nominal operating energy of 1.98 GeV; however, the effects of the resonances are weak, and the widths are narrow. We do not expect that any significant depolarization of the beam will occur during the store time.

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1 Introduction

The electron beam provided by the source for the NLC has a nominal beam polarization of 80%, which must be preserved in the damping rings. It is known that spin resonances in storage rings can lead to rapid depolarization of the beam. This is the basis of a technique commonly used to determine the beam energy to high precision; see, for example, reference [1]. The nominal operating energy of the damping rings (1.98 GeV) has been chosen to avoid the principle spin resonances, which occur at intervals of 440 MeV. However, other spin resonances (associated, for example, with the betatron motion of particles in the lattice) may be present, and these can be found by tracking studies.

Calculations performed for the ILC-TRC Second Report [2] found that the depolarization time at 1.98 GeV was longer than the store time by several orders of magnitude. However, the conclusions were based on semi-analytical calculations requiring some approximations. Tracking studies were not carried out at that time, but were recommended as part of more thorough investigations. Also, a new lattice design has been produced [3] since the ILC-TRC studies were performed.

We have implemented spin tracking in the beam dynamics simulation code MERLIN 3 [4]. We begin with a brief description of this implementation, and then proceed to report results of tracking studies in the NLC Main Damping Rings. In particular, we show the results of single-particle tracking, which we use to identify the beam energies at which spin resonances occur. We confirm the effects of the spin resonances by tracking a bunch of particles with a distribution corresponding to the expected distribution of particles in an injected bunch. We find that there are some resonances close to the nominal operating energy of 1.98 GeV. Even though the effects of these resonances are extremely weak and no significant depolarization is expected, it seems prudent to design the damping rings to allow the possibility of a small energy variation.

2 Spin Tracking in MERLIN3

The spin dynamics of an electron in a high-energy storage ring are described by the Thomas-BMT equation [5]. The precession of the spin vector $\vec{S}$ is given by:

$$\frac{\mathrm{d}\vec{S}}{\mathrm{d}t} = \vec{\Omega} \times \vec{S} \tag{1}$$

The vector $\vec{\Omega}$ is determined by: the local electric and magnetic fields; and the local co-ordinate system, i.e. whether the reference trajectory is straight, as in a steering magnet or a quadrupole, or curved, as in a dipole. We assume that the electric field is zero. For a straight reference trajectory, $\vec{\Omega}$ is given by:

$$\vec{\Omega} = -\frac{e}{\gamma m}\left[(1+G\gamma)\vec{B}_{\perp} + (1+G)\vec{B}_{//}\right]$$

where γ is the relativistic factor, and G is the anomalous magnetic moment (0.00115965 for electrons). $\vec{B}_{\perp}$ and $\vec{B}_{//}$ are the components of the magnetic field perpendicular and parallel to the motion of the electron, respectively. In a curved reference trajectory, following the motion defined by the field $\vec{B}_\perp$, $\vec{\Omega}$ is given by:

$$\vec{\Omega} = -\frac{e}{\gamma m}\left[G\gamma\vec{B}_\perp + (1+G)\vec{B}_{//}\right]$$

In either case, we can write the solution to (1) as:

$$\vec{S}(t) = \vec{S}_0\cos(\Omega t) + \vec{\Omega}\frac{\vec{\Omega}\cdot\vec{S}_0}{\Omega^2}\left[1-\cos(\Omega t)\right] + \frac{\vec{\Omega}\times\vec{S}_0}{\Omega}\sin(\Omega t) \qquad (2)$$

where $\vec{S}_0 = \vec{S}(0)$ and $\Omega = \left|\vec{\Omega}\right|$.

MERLIN tracks particles through a lattice. If spin tracking is required, the three components of the spin vector are tracked in addition to the usual six components of the phase-space vector for each particle. In this case, the local magnetic field at the particle is calculated from the field description of the component and the particle position; $\vec{\Omega}$ is then calculated according to the geometry of the component (straight or curved reference trajectory); finally, the spin vector is advanced using (2).

In some cases, the magnet fringing fields may be important. In particular, the fringing fields of dipole magnets have longitudinal components that can change the vertical component of the spin vector of a particle. From Maxwell's equations, the integral of the longitudinal component of the field in the region of the fringing field satisfies:

$$\frac{\partial}{\partial y}\int B_z \mathrm{d}s = \int \frac{\partial B_y}{\partial z}\mathrm{d}s$$

where the integral is along the reference trajectory of a particle in the lattice (assumed parallel to the *z*-axis of the magnetic field). If we assume a simple linear increase in the strength of the vertical field component of the region of the fringe field, then we find:

$$\int B_z \mathrm{d}s = \tfrac{1}{2} y\hat{B}_y$$

where $\hat{B}_y$ is the field strength in the body of the dipole. This model of the fringe fields is used for spin tracking in MERLIN. In the particular case of the NLC MDR, we expect the fringe fields to have a small effect, since the dipoles are short compared to the vertical betatron wavelength, and the effects of the entrance and exit fringe fields will nearly cancel – note that the longitudinal fields at the entrance and the exit are in opposite directions, and the vertical position of the particle changes little inside the magnet.

3 Spin Tracking in the NLC MDR

3.1 Spin Tune at Nominal Energy

The injected beam in the NLC MDR will have vertical polarization. A horizontal bending magnet has no effect on the vertical component of the polarization of a particle, if the particle is traveling in the horizontal plane; the horizontal component (horizontal transverse and longitudinal) of the polarization rotates around the vertical axis by an angle proportional to the bending angle of the magnet, and the energy of the particle. The *spin tune* of a storage ring is the number of rotations around the vertical axis made by the polarization vector of a particle in one complete turn of the lattice, and is equal to $G\gamma$. The energy of the NLC MDR has been chosen so that the spin tune is a half-integer (the spin tune is 4.5 at 1.98 GeV). In principle, this minimizes depolarizing effects, since the effects of horizontal and longitudinal magnetic fields (which rotate the polarization vector away from the vertical) cancel on successive turns. Conversely, at an integer spin tune, the effects of horizontal magnetic fields add up on successive turns. However, the betatron motion of a particle means that the horizontal component of the field seen by the particle at a particular location in the lattice changes from turn to turn; this can lead to large changes in the polarization over many turns, even if the spin tune is not close to an integer.

As a simple test, we tracked a single particle through the NLC MDR lattice, starting with spin oriented along the *x*-axis, i.e. in the horizontal transverse direction. No alignment or field errors were applied to the lattice, and the particle followed the design orbit. The angle of the spin vector to the *x*-axis is shown in Figure 1; note that the vertical spin component remained zero throughout, since the only fields seen by the particle were the vertical fields of the bending magnets.

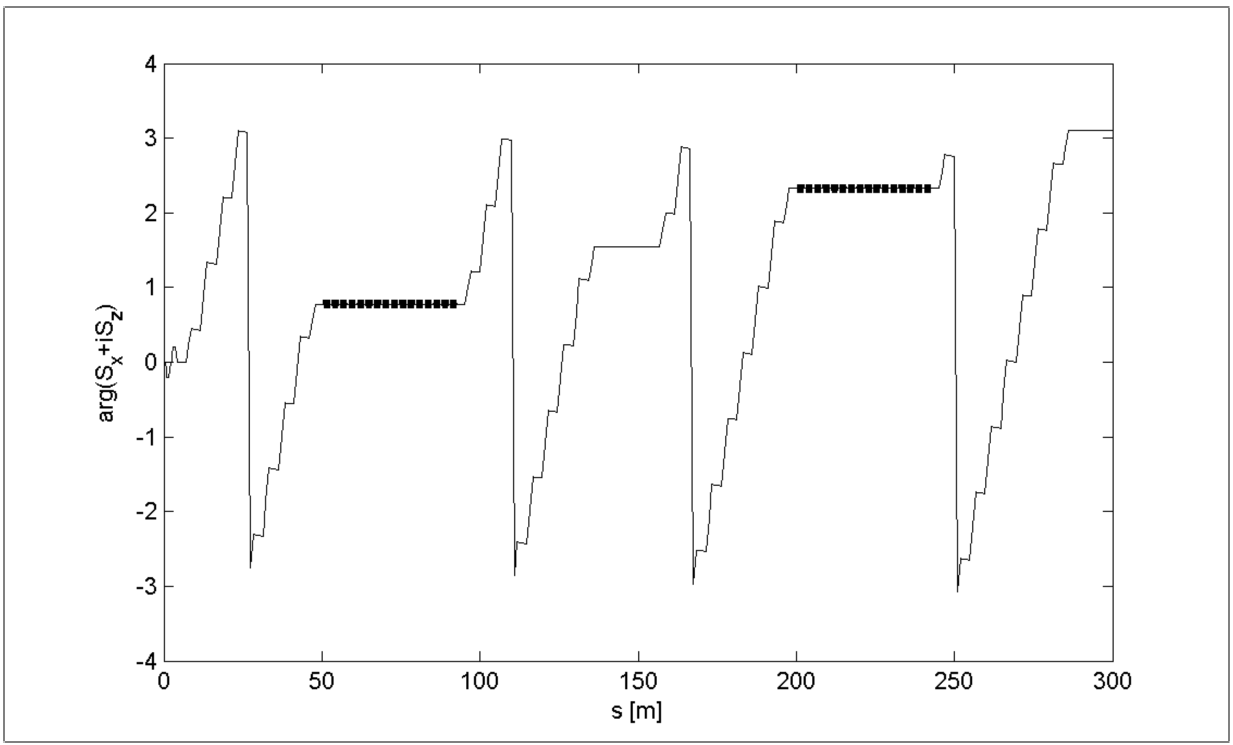

Figure 1

Angle of the spin vector of a single particle in the horizontal plane during tracking through the NLC MDR lattice. The initial direction of the spin vector is along the *x*-axis.

The spin tune is almost exactly 4.5, as expected. The large steps are from the bending magnets in the arcs, while the rapid oscillations are from the wigglers. There is a small initial 'oscillation' of the direction of polarization from the circumference correction chicane. Also note that there is a small change in the direction of the polarization between the dipoles, coming from the fact that the central quadrupole in each arc cell has a small horizontal offset by design.

3.2 Spin Resonances

To understand the principal features of the spin dynamics, we first look at the turn-by-turn variation of the vertical component of the spin vector of a single particle, tracked at different energies through the NLC MDR lattice. For these studies, we applied 20 μm rms vertical misalignment applied to the quadrupoles, which generated a vertical closed orbit distortion of around 400 μm. The particle is launched on the closed orbit, so there are no betatron oscillations. The particle sees a horizontal field component at each quadrupole, and this will change the vertical component of the spin vector.

Figure 2 shows the tracking results at three different energies, corresponding to spin tunes close to 4.5, 4.1 and 4.0. Note the different scales on the different plots. Close to the half-integer spin tune, the vertical component of the spin executes very rapid oscillations of small amplitude. The period and amplitude of the oscillations grow as the energy is shifted towards an integer spin tune.

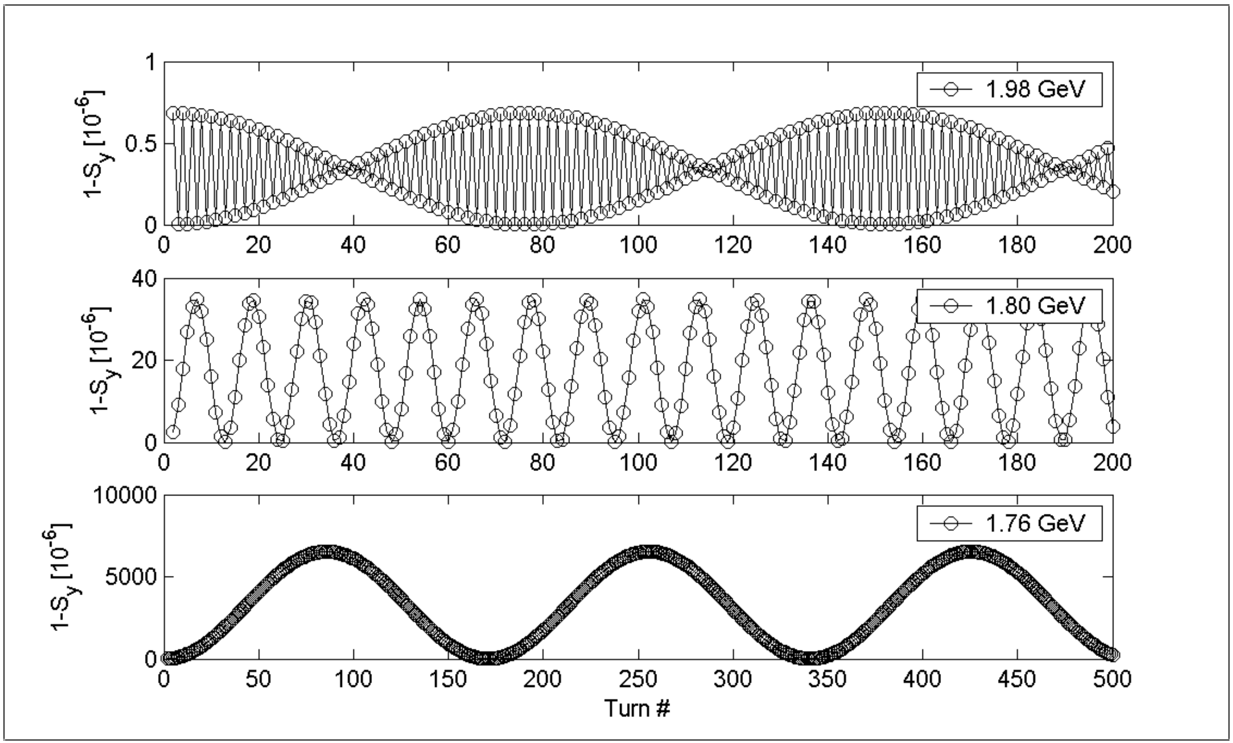

Figure 2

Tracking a particle with initial vertical spin polarization through the NLC MDR lattice at three different energies: top, spin tune 4.5; middle, spin tune 4.1; bottom, spin tune 4.0. The lattice has a vertical closed orbit distortion of 400 μm rms, generated by a random vertical misalignment of the quadrupoles of 20 μm rms. The particle is launched on the closed orbit, so there are no betatron oscillations.

The behavior seen in Figure 2 is what we expect from very general arguments. Close to the half-integer spin tune, the effects of the horizontal field from the quadrupoles almost cancel out from one turn to the next; close to the integer tune, the effects tend to add up.

The resonance behavior is more complicated if there are betatron oscillations: the betatron motion means that the horizontal field seen by the particle at each quadrupole varies turn-by-turn. A simple model suggests that we should expect to see a spin resonance when the fractional parts of the betatron tune and the spin tune are equal. Note that horizontal closed orbit distortion and horizontal betatron motion do not have the same effects as vertical closed orbit distortion and vertical betatron motion, since a particle that is horizontally offset from the center of a normal quadrupole sees a vertical magnetic field, which does not affect the vertical spin component.

To locate the spin resonances in the NLC MDR lattice, we track particles with a range of different energies over a number of turns, and calculate the standard deviation of the vertical spin component. This gives a rough indication of the amplitude of the oscillations seen, for example, in Figure 2. (Fourier analysis could provide a more precise technique). The results are shown in Figure 3, where we plot the standard deviation of the vertical component of the spin vector over a number of turns, under various conditions of horizontal and vertical betatron oscillations and synchrotron oscillations. The amplitude of the betatron oscillations was chosen so that the betatron action was equal to half the nominal emittance of the injected beam.

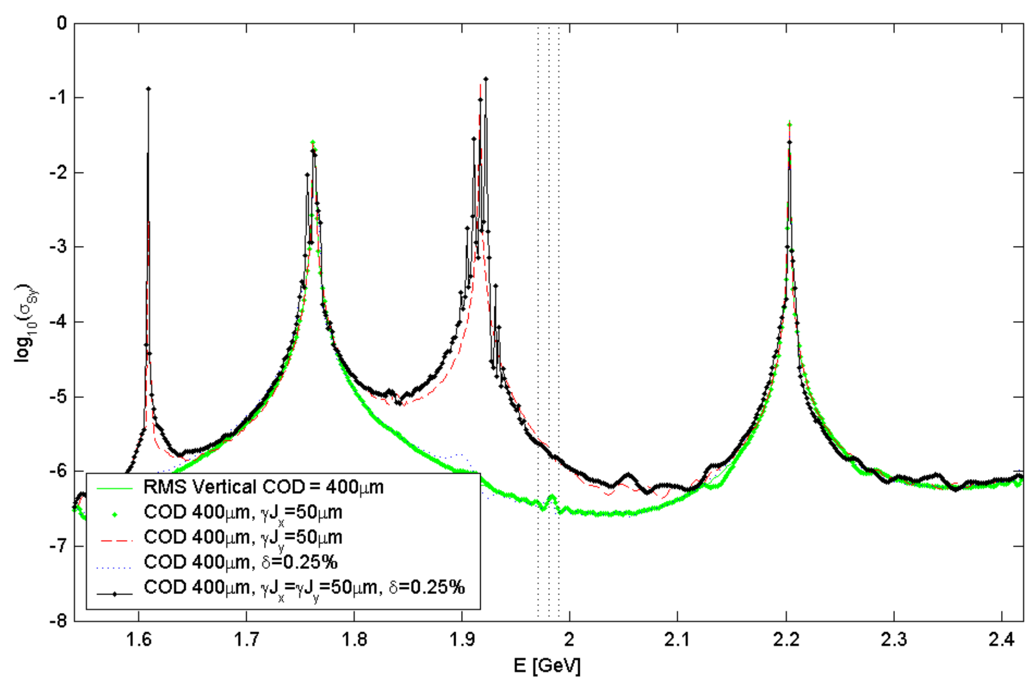

Figure 3

Spin resonances in the NLC MDR. The vertical axis is the standard deviation of the vertical component of the spin vector of a single particle over a number of turns. The vertical dotted lines indicate the nominal energy and injected energy spread of ±0.5%.

Figure 3 shows the expected structure, with strong resonances at 1.76 GeV and 2.20 GeV corresponding to the integer spin tunes of 4 and 5 respectively; these resonances are visible with closed orbit distortion only, i.e. without betatron oscillations. The other strong resonance, driven by the vertical betatron oscillations, occurs at 1.92 GeV; at this energy, the fractional part of the spin tune (4.357) is close to the fractional part of the vertical betatron tune (10.347). The horizontal betatron oscillations do not introduce any new features, and the synchrotron oscillations appear to have little effect.

There is some dependence of the resonant structure on the seed of the random errors on the quadrupole positions. Figure 4 shows the resonances in the NLC MDR with six different seeds of random errors on the quadrupole misalignment.

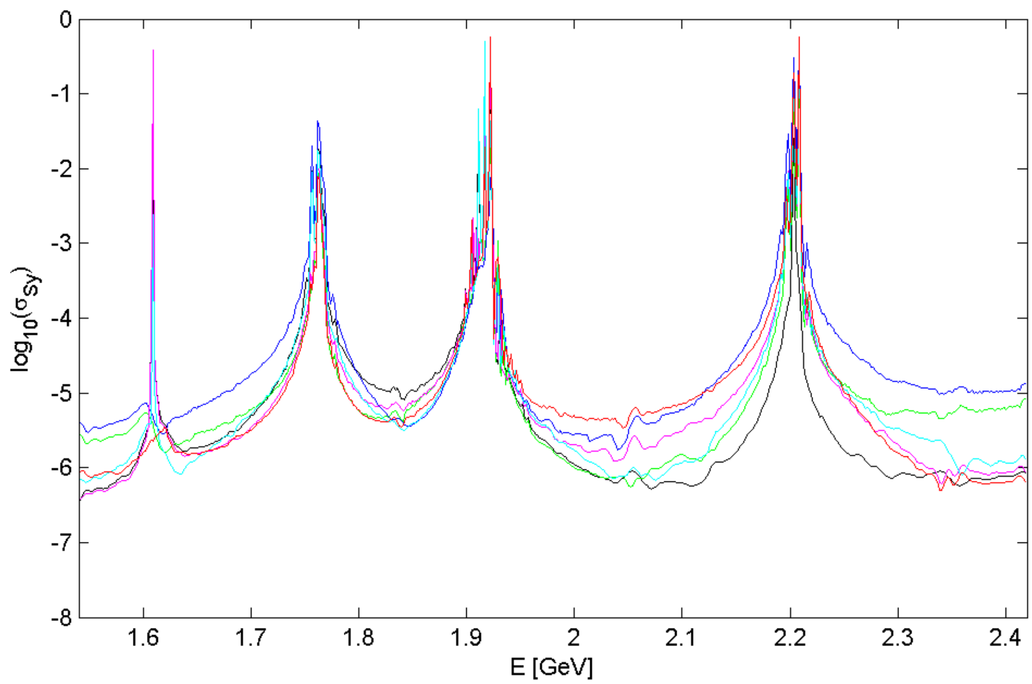

Figure 4

Spin resonances in the NLC MDR, with closed orbit distortion, horizontal and vertical betatron oscillations and synchrotron oscillations. The different color lines correspond to different seeds of random quadrupole misalignment.

In addition to the main resonant peaks, there is some evidence in Figure 3 for much weaker resonances. For example, there appears to be a small peak at 2.05 GeV, where the spin tune is 4.652: this corresponds to the fractional parts of the spin tune and the vertical betatron tune summing to an integer. Perhaps of more interest is the possibility of a resonance at 1.98 GeV, the nominal operating energy of the lattice. Figure 5 shows the variation in the vertical component of the spin vector in the region around 1.98 GeV, on a finer scale than Figure 3 or Figure 4. For some seeds of random misalignments of the quadrupoles, there does appear to be some resonance around 1.98 GeV, but the effect is very weak.

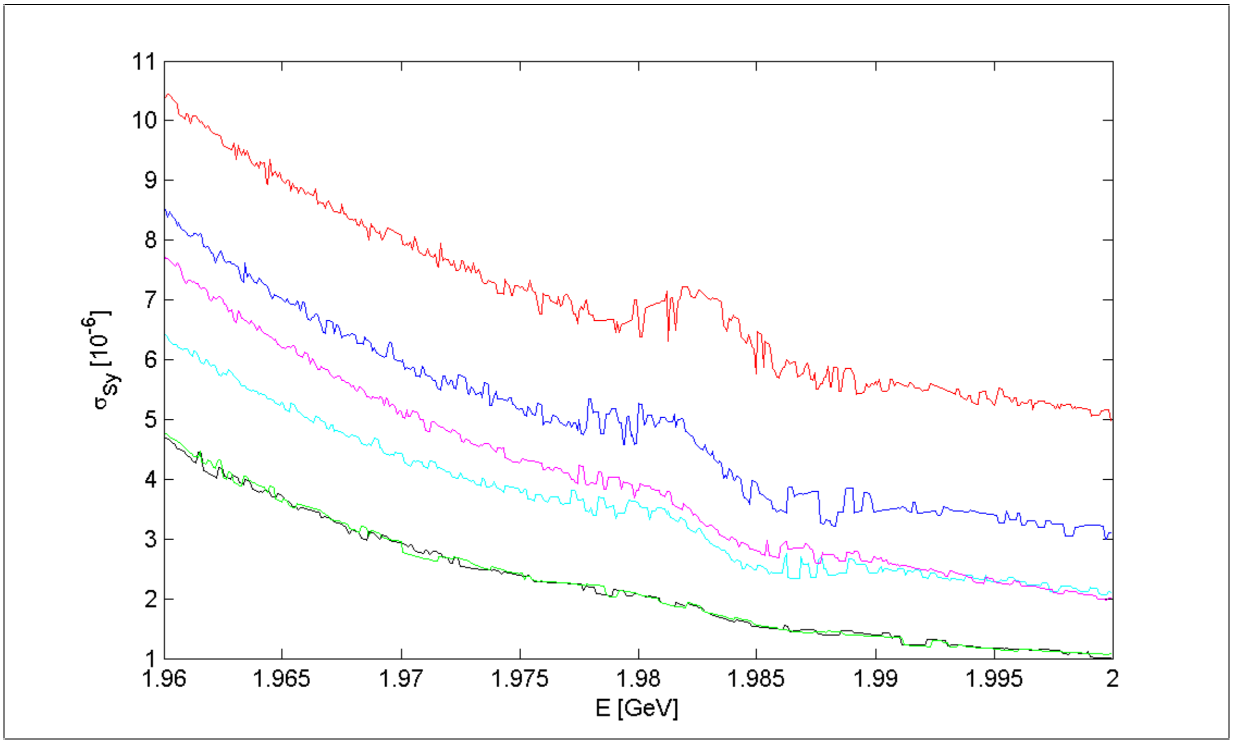

Figure 5

As Figure 4, but with a finer energy resolution, and a linear vertical scale. The different color lines correspond to the same random seeds as in Figure 4.

3.3 Multiparticle Tracking

Finally, we look at the variation in polarization of a bunch of particles as the particles circulate in the NLC MDR. Figure 6 shows the results of tracking a bunch of 1000 particles through 2000 turns of the NLC MDR lattice at four different energies. Initially, all particles in the bunch have vertical polarization. The normalized emittances are 100 µm horizontally and vertically, and the rms energy spread is 0.25%. Clearly, there is some depolarization at 1.92 GeV, corresponding to the spin resonance driven by vertical betatron motion. More surprisingly, there is some slow depolarization at 1.98 GeV; this may be associated with the resonance peak seen in Figure 5: note that the quadrupole misalignment for the multiparticle tracking was generated from the random seed giving the strongest resonance around 1.98 GeV. To investigate this further, we repeated the tracking at energies in a narrow range around 1.98 GeV. The results are shown in Figure 7. There appears to be a gradual loss of polarization at 1.975 GeV and 1.980 GeV, but little change over time at 1.985 GeV and 1.965 GeV. Although the effects are extremely small at 1.98 GeV, it seems prudent to design the damping rings so that some variation in the operating energy is possible.

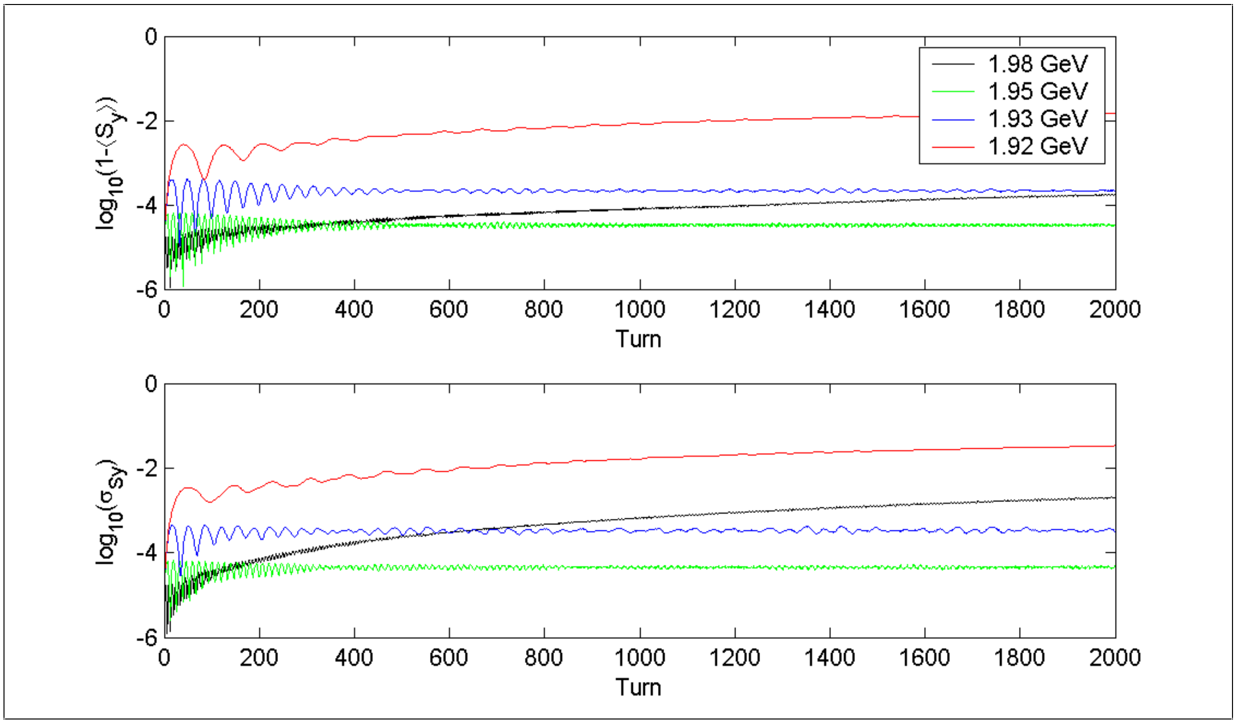

Figure 6

Multiparticle tracking through 2000 turns of the NLC MDR at four different energies. Top: average vertical component of the spin vector over all particles in the bunch. Bottom: standard deviation of the vertical component of the spin vector over all particles in the bunch.

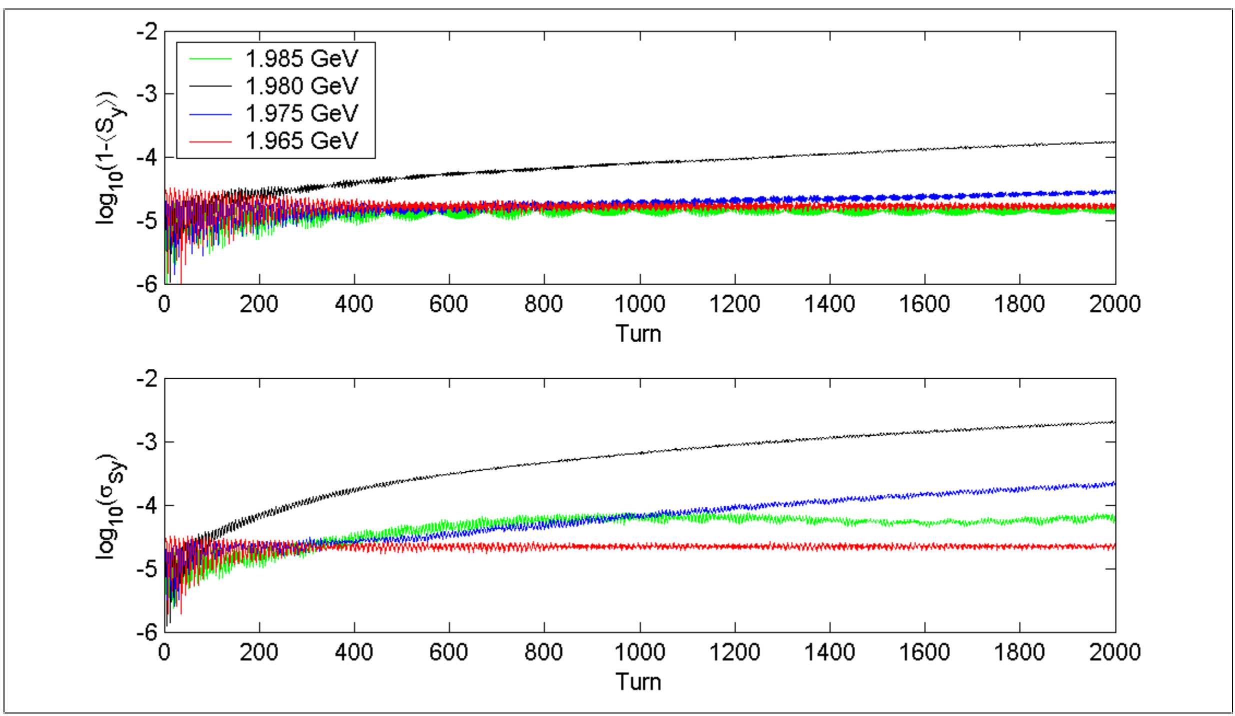

Figure 7

As Figure 6, but tracking at energies close to the nominal operating energy of 1.98 GeV.

4 Conclusions

Spin tracking in the NLC MDR confirms the expected spin resonances at energies corresponding to integer values of the spin tune. These resonances could rapidly depolarize an injected polarized beam. The nominal operating energy of 1.98 GeV (corresponding to a half-integer spin tune) has been chosen to avoid these resonances, and also appears a safe distance from the spin resonance at 1.92 GeV driven by vertical betatron motion of the particles. We note that all these resonances are extremely narrow, and would be expected to have a significant effect on the beam only if the beam energy is within a few MeV of the peak of the resonance.

There is some evidence for a resonance close to 1.98 GeV; however, this resonance appears only for some seeds of random quadrupole misalignment, and its effects are extremely weak. It is not likely that any depolarization of the beam will be observed if operating at 1.98 GeV; nonetheless, it is advisable to design the damping rings to allow some variation in operating energy in case of unexpected effects.

Acknowledgements

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