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Linear Collider Collaboration Tech Notes

Tunnel Wall Heat Transfer

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Abstract: Transient heat conduction from a warm accelerator tunnel to the earth 's surface is a straight-forward analytic heat transfer problem. The problem is 2 dimensional with the tunnel long in comparison to its diameter and depth. All heat lost by the tunnel walls is conducted to the surface .is fixed at reference temperature 0° .The problem is approximately solved in 2 steps. First, the steady state heat conduction through the earth is calculated. This heat source is then abruptly turned on and the transient heat diffusion equation solved.

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1 Introduction

Transient heat conduction from a warm accelerator tunnel to the earth's surface is a straight forward analytic heat transfer problem. The problem is 2 dimensional with the tunnel long in comparison to its diameter and depth. All heat lost by the tunnel walls is conducted to the surface fixed at reference temperature 0°. The problem is approximately solved in 2 steps. First the steady state heat conduction through the earth is calculated. This heat source is then abruptly turned on and the transient heat diffusion equation solved.

2 Steady State

The steady state heat loss rate $\dot{q}$ (watts/m) is set by the conductivity of the earth, the tunnel temperature and geometry. This is a 2 dimensional solution of $\nabla^2 U = 0$ with boundary temperature $U = 0$ at the earth's surface and $U = U_o$ (°C) on the tunnel wall radius r (meters) at depth d (meters) below the surface. The problem is identical to the electrostatic problem of a charged rod above a ground plane. The solution is most simply written as the superposition of fields from a line charge and its image on the opposite side of the ground plane. In the thermal case line charges are replaced by a line heat source $+\dot{q}$ watts/meter at at location $y = -a$ and a sink $-\dot{q}$ at $y = +a$ above the earth's surface. The superposition of this source and its image sink give the temperature U at any point in the earth with conductivity k (watts/m °C).

$$U = \frac{\dot{q}}{4\pi k} \ln\left[\frac{x^2 + (y-a)^2}{x^2 + (y+a)^2}\right] \quad {}^\circ\mathrm{C} \tag{1}$$

Isotherms of this temperature field shown in figure 1 are all circles increasing in radius as they approach the earth's surface.

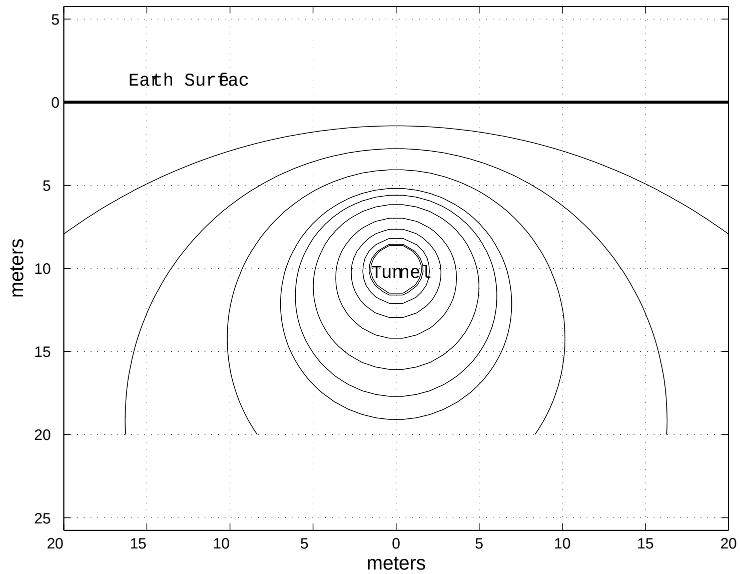

Figure 1: Tunnel cross section temperature field

The line source locations $\pm a$ needed to set the circular tunnel wall to temperature U_o are determined from

$$\exp(4\pi k U_o/\dot{q}) \equiv C(\dot{q}) = \frac{x^2 + (y-a)^2}{x^2 + (y+a)^2} \tag{2}$$

Eq.2 represents a circle of radius $r = 2a\sqrt{C}/(1-C)$ centered at depth $d = a(1+C)/(1-C)$. Eliminating a between r and d yields a quadratic equation for C in terms of d/r.

$$C^2 + (2 - 4\left(\frac{d}{r}\right)^2)C + 1 = 0 \tag{3}$$

The positive root of this quadratic is

$$C = 2\left(\frac{d}{r}\right)^2 - 1 + 2\frac{d}{r}\sqrt{\left(\frac{d}{r}\right)^2 - 1} = \exp(4\pi k U_o/\dot{q}) \tag{4}$$

Solving for the source strength q:

$$\dot{q} = \frac{4\pi k}{\ln\left[2\left(\frac{d}{r}\right)^2 - 1 + 2\left(\frac{d}{r}\right)\sqrt{\left(\frac{d}{r}\right)^2 - 1}\right]} U_o \quad \text{watts/m} \tag{5}$$

In a more compact form, heat loss is related to tunnel wall temperature U_o by:

$$\dot{q} = \left[\frac{2\pi k}{\cosh^{-1}(d/r)}\right] U_o \quad \text{watts/m} \tag{6}$$

The thermal properties of various tunnel and ground materials[1] are tabulated below in table 2.

Material	density ρ kg/m^3	specific heat c_p $\frac{\text{watt*hr}}{\text{kg°C}}$	conductivity k watt/m°C	diffusivity $\alpha = k/\rho c_p$ m^2/hr
Concrete	$1.906 - 2.307 \times 10^3$	0.244	.813-1.402	$1.765 - 2.508 \times 10^{-3}$
Earth,course	2.050×10^3	0.512	.519	4.945×10^{-4}
Sandstone	$2.163 - 2.307 \times 10^3$	0.198	1.626 - 2.076	$3.797 - 4.545 \times 10^{-3}$

The SLAC tunnel was built by cut and cover of a 3 meter square cross section 10 meters below the surface in earth and sandstone. The heat loss from this tunnel ($d/r \simeq$ 10 meter/1.5 meter) at temperature $U_o = 45 - 25 = 20$ °C above ambient depends on the thermal conductivity of the soil or rock its buried under. Using $k \simeq 1.5$ watts/m °C,

$$\dot{q} = \left[\frac{2\pi(1.5 \text{ watts/m °C})}{\cosh^{-1}(10/1.5)}\right](20 \text{ °C}) = 73 \text{ watts/meter} \tag{7}$$

Values of $\dot{q}$ for table entries Earth,course and Sandstone are 25 watts/meter and 79-101 watts/meter respectively. During operation this heat loss is easily supplied by hot accelerator water cooling pipes. To maintain temperature when the rf is off, the water cooling system must be heated. For the 10000 ft long SLAC tunnel $\dot{Q} = (73\text{watts/m})(3 \text{ km}) = 219$ kw.

3 Transients

The transient details of the convective heat transfer problem for the air inside a tunnel do not have a simple analytic solution. The thermal gradients inside the tunnel are very important for understanding the thermal distortions of the accelerator but I assume these gradients become small in the steady state compared to the total temperature drop from inside the tunnel to the earth's surface. The tunnel walls are close to the tunnel air temperature during steady state operation. If the wall surfaces come up to operating temperature within hours or a few days after turn on, how this heat spreads into the rock or soil overburden around the tunnel is a straight forward diffusion problem. The simplest approximation to this diffusion problem is to replace the tunnel by a line heat source. Consider a 2D slice of the tunnel & earth cross section 1 meter thick. In the case of infinite slice, an impulse of heat Q (watt*hr/meter) at the origin diffuses radially out from the origin over time as

$$U(r,t) = \frac{Q}{\rho c(4\pi\alpha t)} e^{-r^2/4\alpha t} \quad \text{°C}. \tag{8}$$

This impulse response or Green's Function can be used to compose a solution to the problem of a heat source buried below the fixed temperature earth's surface. The dynamics are largely determined by the diffusion constant α (see table 2). To meet the boundary condition of $U = 0$ °C at the earth surface, both positive and negative images of the source at tunnel depth d are needed at $r^2 = x^2 + (y-d)^2$ and $r^2 = x^2 + (y+d)^2$. If uniform heat deposition $\dot{q}$ (watts/meter) starts at $t = 0$

$$U(x,y,t) = \frac{\dot{q}}{\rho c(4\pi\alpha)} \int_0^t \frac{1}{\tau} \left(e^{-\frac{x^2+(y-d)^2}{4\alpha\tau}} - e^{-\frac{x^2+(y+d)^2}{4\alpha\tau}} \right) d\tau \quad {}^\circ\mathrm{C}. \tag{9}$$

The time history (eq. 9) for earth temperature above and below the tunnel centerline ($x = 0$) is shown below in figure 2 for a tunnel 10 meters deep in Sandstone.

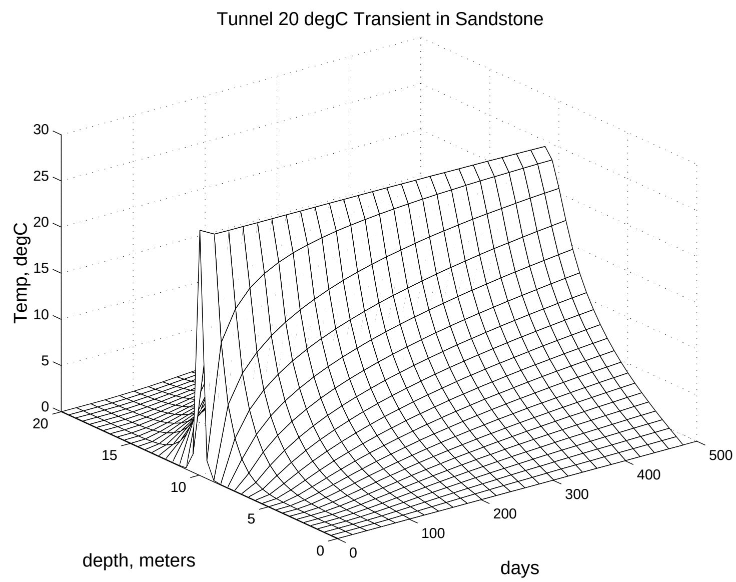

Figure 2: Transient conduction from a tunnel 10 m deep in Sandstone, $\alpha = 3.8 \times 10^{-3}$ m^2/hr and $\rho * c = 17.84$watt $*$ hr/m^3 °C

In this simplified model, Figure 2 shows the wall of the tunnel immediately jumping up to the final steady state temperature but at distances only a meter into the Sandstone, heat-up is much slower. While the full coupled problem of tunnel air convection and thermal diffusion through the surrounding soil has not been solved, it is clear that thermal equilibrium in the surrounding soil takes 100's of days.

References

[1] E.R.G.Eckert & Robert M. Drake,'Heat and Mass Transfer', 1959. See Appendix of Property Values, Table A-5 for thermal properties of selected non-metals.