



# Linear Collider Collaboration Tech Notes

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## Planar Undulator Considerations

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## Planar Undulator Considerations

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### References:

1. H. Wiedemann, **Particle Accelerator Physics II**, 1995
2. V.V. Balandin, *et al.*, *Conceptual Design of a Positron Injector for the TESLA Linear Collider*, Moscow-Hamburg 2000, DESY-TESLA-00-12.
3. **TESLA TDR II Accelerators**, chapters 4 and 5.

### Abstract

This note consists of informal working notes that document an effort to understand the TESLA baseline, unpolarized, undulator based positron source. This is the first step in the design process of an undulator based positron system for the NLC. The expressions and methodologies developed herein are used in subsequent memos that reference this note. In regards to the TESLA design, it is found that a 135 m long (versus 100 m length) undulator is consistent with the performance descriptions in the TDR text. And while operation of the TESLA system with a 250 GeV drive beam energy provides a safety margin of a factor of  $\approx 2$  in  $e^+$  yield, operation of the same system at 160 GeV results in a yield of about 0.6.

**Question:** How long is the undulator? Start with how many positrons are required,  $N_p$ , and a multiplicative safety margin,  $f > 1$ :

$$fN_p = N_e Y_{eu} L_u Y_{pu} V_c \quad (1)$$

where  $N_e$  = number of electrons,  $Y_{eu}$  = yield of electrons to photons per meter of undulator,  $L_u$  = the length of the undulator in meters,  $Y_{pu}$  = yield of photons to positrons leaving the target, and  $V_c$  = the capture efficiency. Note that  $Y_{eu}$ ,  $Y_{pu}$ , and  $V_c$  all depend in detail of the undulator parameters, the incident electron energy, the spot size on the target, and details of the collection and capture system.  $L_u$  is extracted from (1) to give:

$$L_u = \frac{fN_p}{N_e Y_{eu} Y_{pu} V_c} \quad (2)$$

For the TESLA design,  $N_p = N_e$  and  $f = 2$  for  $E_e = 250$  GeV, the electron energy. For the TESLA undulator,  $B_0 = 7.5$  kG and  $\lambda_p = 1.42$  cm (page II-122)<sup>[3]</sup>. From Wiedemann<sup>[1]</sup>,

$$K = 9.344 B_0(kG) I_p(m) = 0.995 \quad (3)$$

The total radiated energy per electron per meter is given as **DE**:

$$\Delta E(eV) = 725 \frac{E_e^2 (GeV) K^2}{I_p^2 (cm)} = 2.23 \times 10^7 eV / m = 22.3 MeV / m \quad (4)$$

The TESLA design report says that on average 3 GeV is lost, this implies an undulator length of  $\frac{3GeV}{22.3MeV/m} = 134.8m$ . This is at odds with the text of the report which indicates that the undulator is 100 m long. Similarly, the report indicates that 135 kW of power is generated by the wiggler. From (4) the average power output is given as  $P_{avg}$ :

$$P_{avg} = qN_e \Delta E(eV) / m \times L_u = 135kW \quad (5)$$

Plugging in the numbers,  $q = 1.602 \times 10^{-19} \frac{J}{eV}$ , and  $N_e = 2.8 \times 10^{14} / s$  and **DE**=22.3 MeV/m, then gives a value for  $L_u$  of

$$L_u = \frac{135kW}{qN_e \Delta E(eV) / m} = 135.2m \quad (6)$$

This is at least consistent with the 3 GeV average loss. Next a quick check of the cutoff energy of the undulator first harmonic,  $E_{c10}$ :

$$E_{c10} = \hbar \omega_{10} = \hbar \frac{4pg^2 c / I_p}{(1 + K^2/2)} \quad (7)$$

For  $\lambda_p = 1.42$  cm ,  $E_{c10} = 28$  MeV (27.96 MeV , actually) which is essentially the same as in Table 4.3.2 (II-120).

So next figure out yields for this particular undulator. The number of undulator photons is found by equating the integral of "universal" undulator spectrum,  $hsum(ww10)$ [note: see z:/positrons/polarized positrons/1149df.m and teslsa150\_250wrkspc.m], to the total energy radiated to find the normalization constant which in turn is used with the integral of the photon number spectrum. So here goes (Matlab is used through out for the numerical results).

$$a \int dw hsum = a dw \sum hsum = \frac{P_{avg}}{N_e} \quad (8)$$

which gives

$$a = \frac{P_{avg} / N_e}{dw \sum hsum} (J / e^-) \quad (9)$$

Similar thinking leads to the ratio of the number of photons radiated to the number of electrons,  ${}_e Y_{hu}$  :

$${}_e Y_{hu} = \frac{adw}{L_u} \sum hsum./w = \frac{P_{avg}(W)/N_e(e^-/s)}{qE_{c10}(eV)L_u(m)} \times \frac{\sum hsum./ww10}{\sum hsum} \quad (10)$$

Note, a plot of the photon energy spectrum,  $hsum(ww10)$ , is shown below;  $ww10$  is the photon energy (frequency) normalized by the cutoff energy of the first harmonic,  $E_{c10}$ :

$$ww10 = \frac{w}{w_{10}} \quad (10a)$$

wherein  $w_{10} = E_{c10}/\hbar$ .

Also the quantity  $\frac{\sum hsum./ww10}{\sum hsum}$  depends only on K. For K=1, and summing over the first 4 harmonics

$$\frac{\sum hsum./ww10}{\sum hsum} = 1.2648 \quad (11)$$

So, for the  $L_u = 135$  m,  $K=1$ ,  $\lambda_p = 1.42$  cm, and  $E_{c10} = 28$  MeV ,

$${}_e Y_{hu} = \frac{135.7}{135} = 1.005(\text{photons / meter / electron}) \quad (12)$$

One can also figure out the average photon energy,  ${}_{hu} E_{avg}$  ,

$${}_{hu} E_{avg} = E_{c10} \times \frac{\sum hsum}{\sum hsum./ww10} \quad (13)$$

For the case at hand,

$${}_{hu} E_{avg} = 22.1385\text{MeV} \quad (14)$$

And the product,

$${}_e Y_{hu} \times L_u \times {}_{hu} E_{avg} = 3.0045\text{GeV} \quad (15)$$

as it should.

In order to determine the yield of photons to positrons leaving the target,  $h\nu Y_p$  one needs to use EGS4. A version of EGS4 user code (ucRTZspc.mortran, ucRTZ.data, pncum\_k1\_120.out, ww10c\_k1\_120.out, and Xegs4run\*) has been written (with significant help and guidance from R. Nelson) which generates positrons from an incident photon beam of arbitrary spectrum. The code also allows one to specify an incident rms gaussian beam size in x and y. The programs all reside in

/afs/slac.stanford.edu/u/ad/jcs/egsruns/yieldcalc/phtnspectrm in Unix land.

Figures 3-8 follow the text. Figures 3 and 4 are scatter plots of the  $x$ - $p_x$  phase space of positrons coming out of the back surface of the conversion target. It should be noted that the emittance of the emitted positrons are only a factor of 2-3 time smaller than the phase space of conventionally produced positrons. This is in large part due to the fact that the beam size of the emitted positrons is large because of the 1.4 cm thickness of the 0.4 r.l. length Ti-alloy target. Figure 5 illustrates the energy spectrum of emitted positrons for the conditions noted in the figure. Figures 6-8 show the spectrum and transverse profiles of the incident photon beam. Figures 3-8 are made from files written by EGS4.

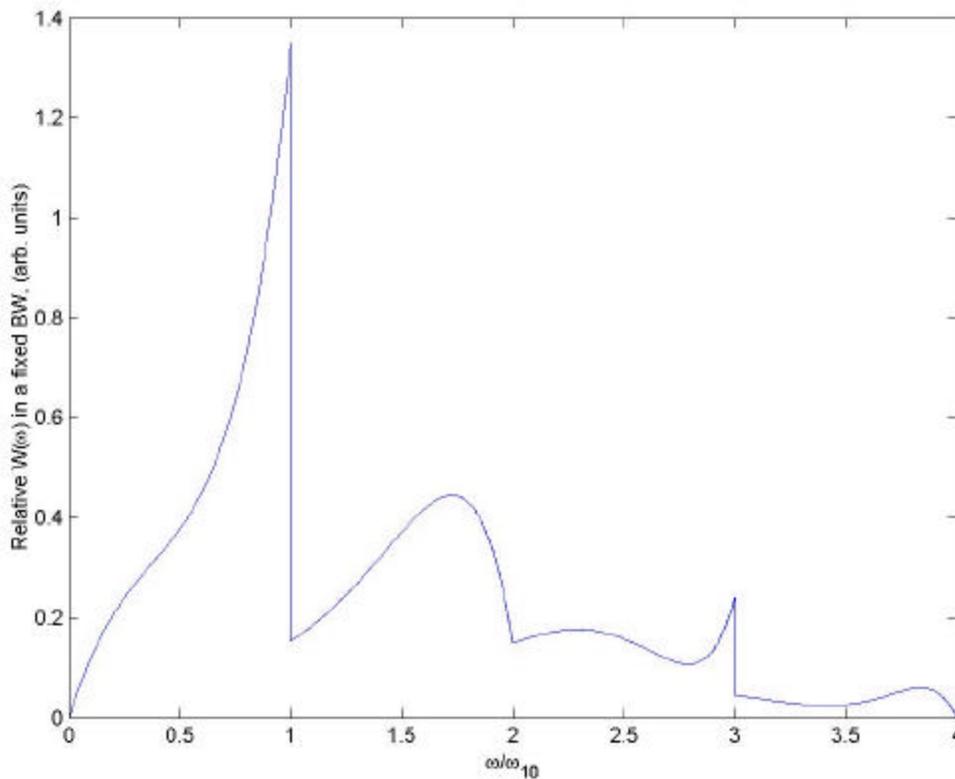


Figure 1: Universal undulator energy spectrum, hsum.

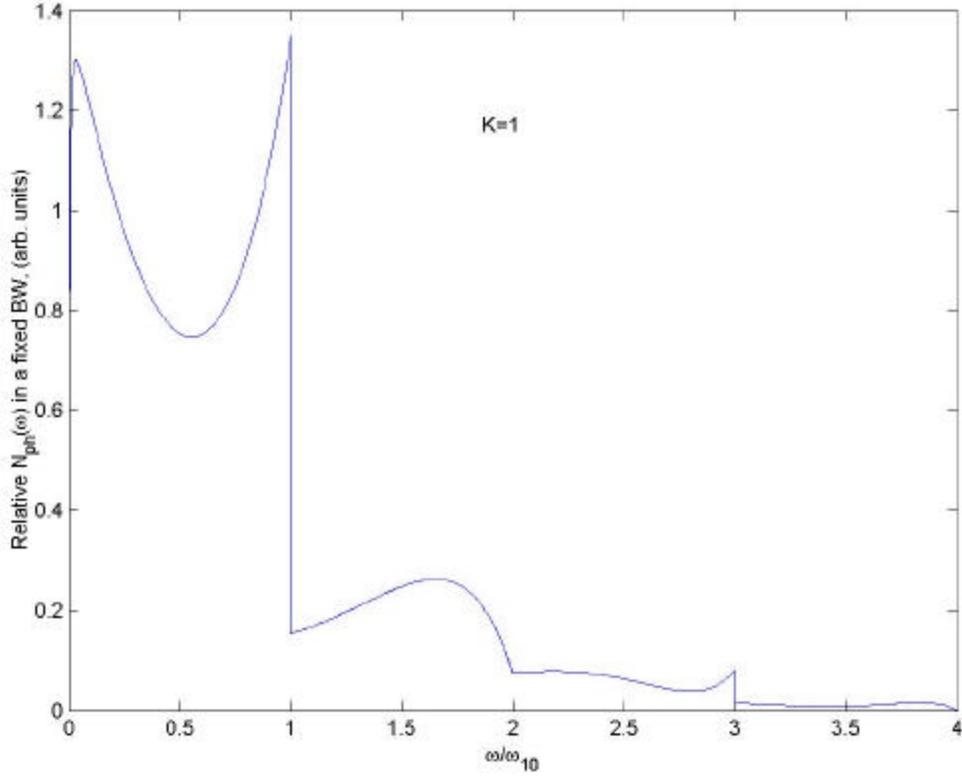


Figure 2: The universal photon number spectrum,  $h\text{sum}/(w\omega_{10})$

At 250 GeV through the undulator,  $E_{c10} = 28 \text{ MeV}$ ,  $\mathbf{s}_x = \mathbf{s}_y = 0.7\text{mm}$ , 100,000 photons into 0.4 R.L. Ti target results in 5404 emitted positrons from the exit face, a yield of  ${}_{hn}Y_p = 5.4\%$ . Similarly, at 160 GeV through the undulator,  $E_{c10} = 11.45 \text{ MeV}$ ,  $\mathbf{s}_x = \mathbf{s}_y = 1.09\text{mm}$ , 100,000 photons into 0.4 R.L. Ti target results in 1750 emitted positrons from the exit face, a yield of  ${}_{hn}Y_p = 1.75\%$ .

The INR report<sup>[2]</sup> states that the capture efficiency into  $\epsilon_x = \epsilon_y = 0.036 \text{ m}$  is  $V_c \square 25\%$ . So, putting this all together, I find

$$f(E_e = 250\text{GeV}) = \frac{N_e}{N_p} Y_{hu} L_u Y_p V_c = 1 \times 1.005 \times 135 \times 0.054 \times 0.25 = 1.83$$

At 160 GeV,

$$f(E_e = 160\text{GeV}) = \frac{N_e}{N_p} Y_{hu} L_u Y_p V_c = 1 \times 1.005 \times 135 \times 0.0175 \times 0.25 = 0.59$$

The ratio

$$f(160\text{GeV})/f(250\text{GeV}) = 0.59/1.83 = 0.32 .$$

This is similar in value to simply counting the number of positrons within the 5-D phase space cuts which are shown in the emitted positron figures. From the figures,

$$N_p(160)/N_p(250) = 1750 \times 0.75 / 5404 \times 0.941 \times 0.721 = 1312 / 3666 = 0.36.$$

On the face of this, it looks as if TESLA has real problems making positrons at 160 GeV. To get up to a capture yield of  $f(160\text{GeV}) = 1$ , the capture efficiency needs to increase by a factor of 1.7 from 0.25 to 0.42. Seems tough;  $V_c \approx 25\%$  also feels optimistic and is quoted at a positron energy of 274 MeV <sup>[2]</sup>, not through the full 5 GeV of acceleration and transport. [Note 11/19/01: Yuri Batygin indicated that  $V_c \approx 25\%$  may be doable but is very sensitive to the relative phasing of the capture rf and also of the linac phases, at the several degree level.]

Time to do our own flux concentrator simulations. Also can figure out the energy cross over point.

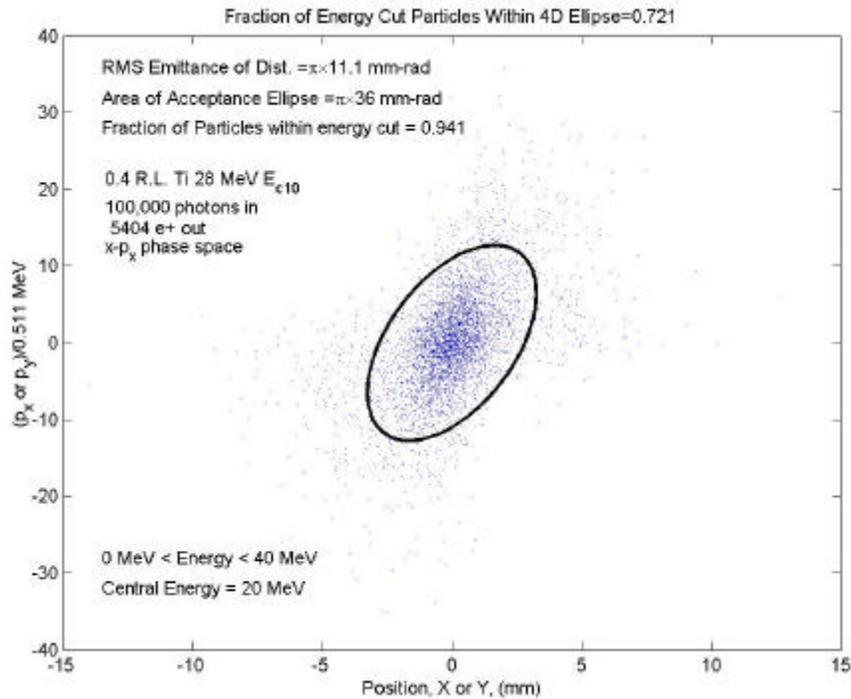


Figure 3:  $x$ - $p_x$  phase space distribution of positrons exiting an 0.4 r.l. thick Ti-alloy conversion target. The incident photon spectrum is that of a  $K=1$  undulator with  $E_{c10} = 28 \text{ MeV}$  and  $s_x = s_y = 0.7 \text{ mm}$ .

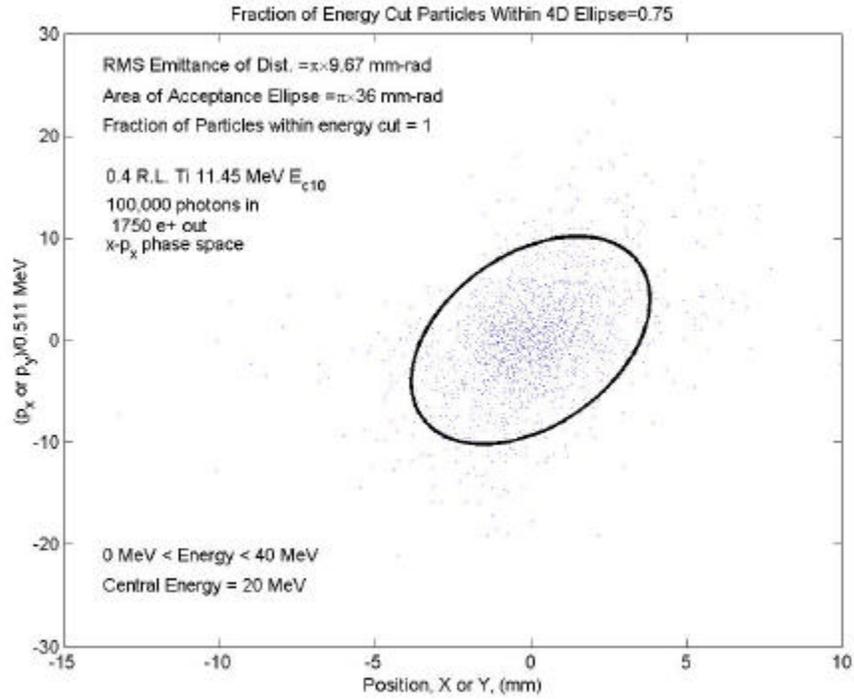


Figure 4:  $x$ - $p_x$  phase space distribution of positrons exiting an 0.4 r.l. thick Ti-alloy conversion target. The incident photon spectrum is that of a K=1 undulator with  $E_{c10} = 11.45 \text{ MeV}$  and  $s_x = s_y = 1.09 \text{ mm}$ .

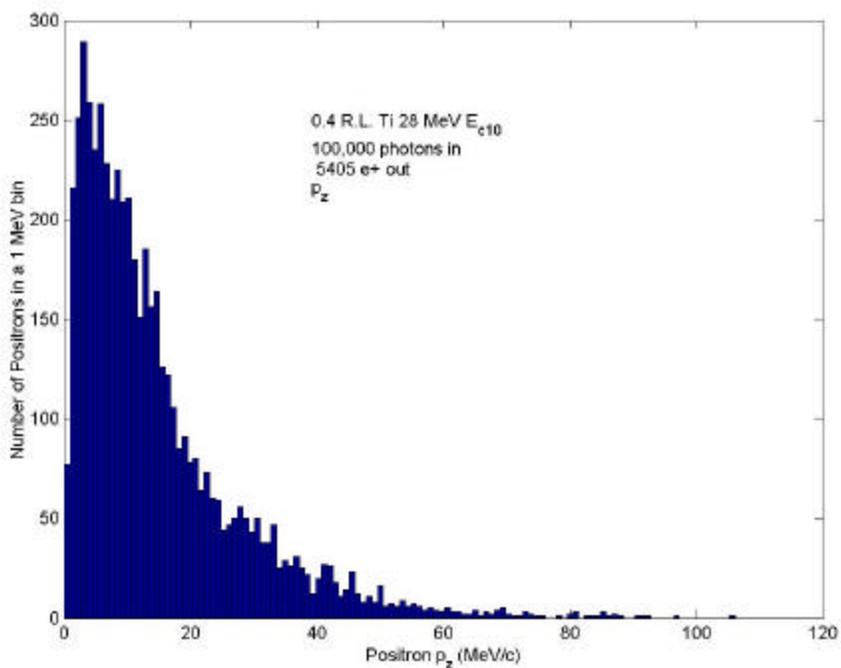


Figure 5.: Emitted positron energy spectrum for K=1 undulator with  $E_{c10} = 28 \text{ MeV}$ , 0.4 r.l. thick Ti-alloy target.

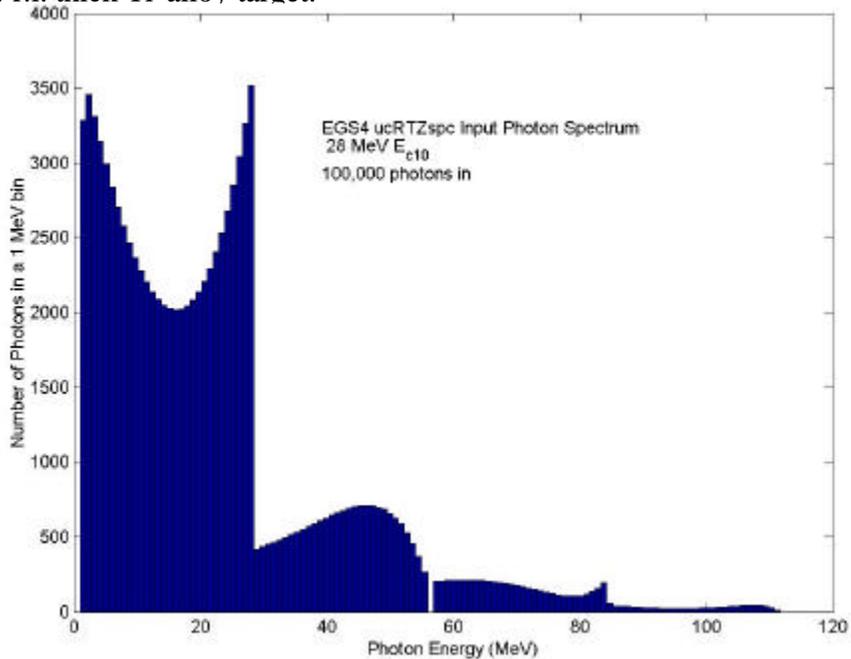


Figure 6.: Incident photon energy spectrum for K=1 undulator with  $E_{c10} = 28 \text{ MeV}$ .

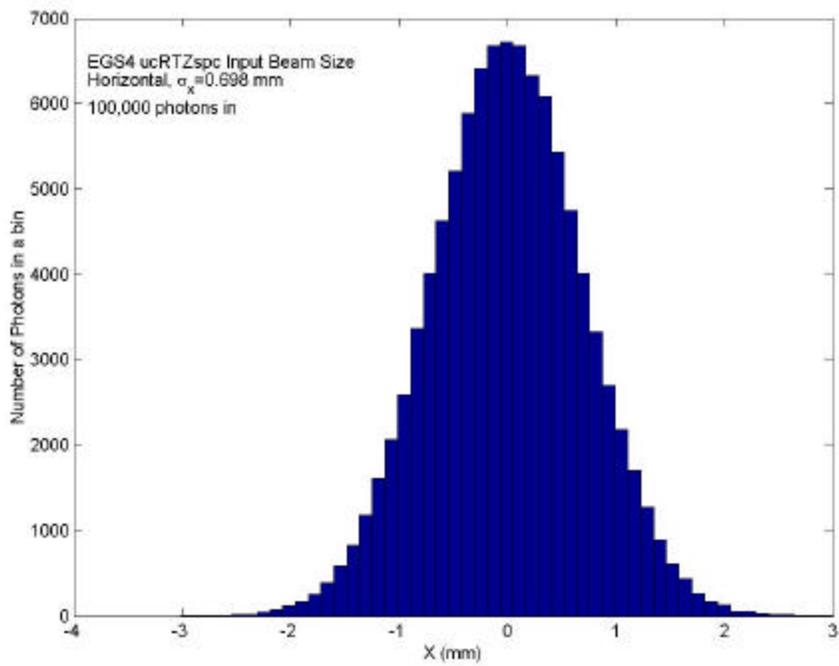


Figure 7.: Incident photon beam transverse distribution, horizontal plane.

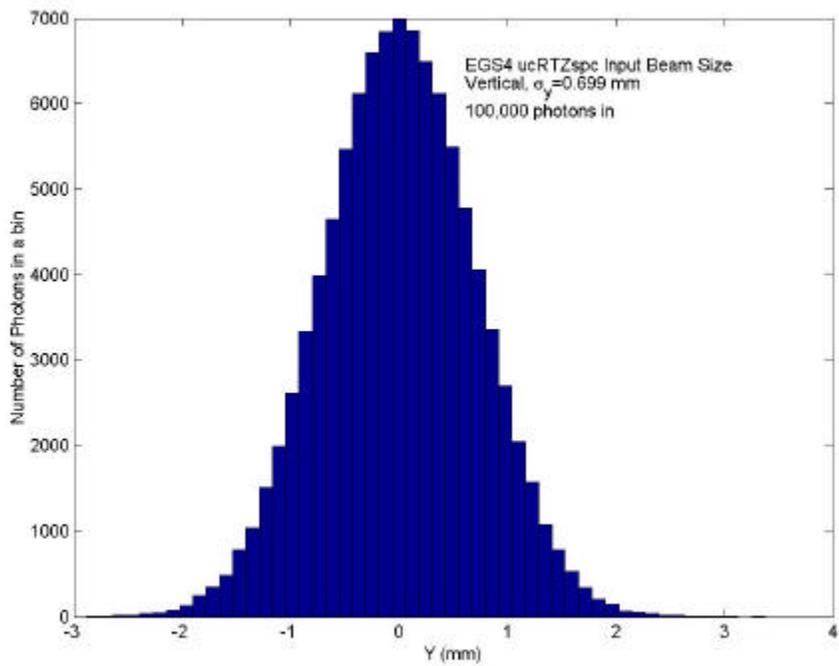


Figure 7.: Incident photon beam transverse distribution, vertical plane.