



Coupled Bunch Instabilities in the NLC Damping Rings

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We evaluate the growth rates of the longitudinal and transverse coupled mode instabilities in the NLC main damping rings. The two options with 1.4 and 2.8 ns spacing between bunches are both investigated. Since the unevenness of the fill makes the analytical problem extremely complex, a number of simplifying assumptions have been made to reach approximated solutions which are, nonetheless, still fairly representative.

1. Introduction

The analytical treatment of coupled bunch instabilities doesn't present any major difficulties only in the case of even fills, that is when each (or, more in general, every n -th) RF bucket is filled with the same number of particles.

Practical situations may differ only slightly from these ideal conditions, for example in a completely filled ring where the current per bunch is not exactly uniform, due to fluctuations in the injection process, or in rings where a small gap has to be allowed in the fill for ion clearing purposes. In such cases the standard theory still provides a good evaluation of the growth rates requiring, at most, only minor modifications.

When, like in our case, the fill shape is highly asymmetric, the theory becomes much more complicated. The uneven fill dynamics can be explained in terms of two physical phenomena: modulation coupling and Landau damping from fill-induced interbunch tune spread, the latter being limited to the longitudinal plane [1].

In general, the oscillation modes are not anymore orthogonal and it is possible to derive an $h \times h$ (h being the harmonic number) matrix describing the coupling of each bunch to every other.

Under a few not excessively restricting hypothesis it is still possible to relate the growth rates of the actual fill to those of a corresponding even fill. In the case of a fractional fill (i.e. N_b equal bunches, with one gap) it can be shown that any damper system keeping the

corresponding symmetric fill (i.e. h equal bunches, with the gap removed, and total current therefore increased by h/N_b) stable can also damp instabilities for the original fill [2].

2. Parameters used in the growth rates calculation

The damping ring lattice parameters are reported in Appendix A [3]. We consider 3 identically tuned RF cavities in each damping ring; their high order mode parameters have been calculated in [4]. In practice the real values of the cavities HOM's will be somewhat different from cavity to cavity which turns out to have beneficial consequences from the stability point of view. With these parameters, the fundamental mode loaded Q is 2460 and its optimal detuning 117.4 kHz.

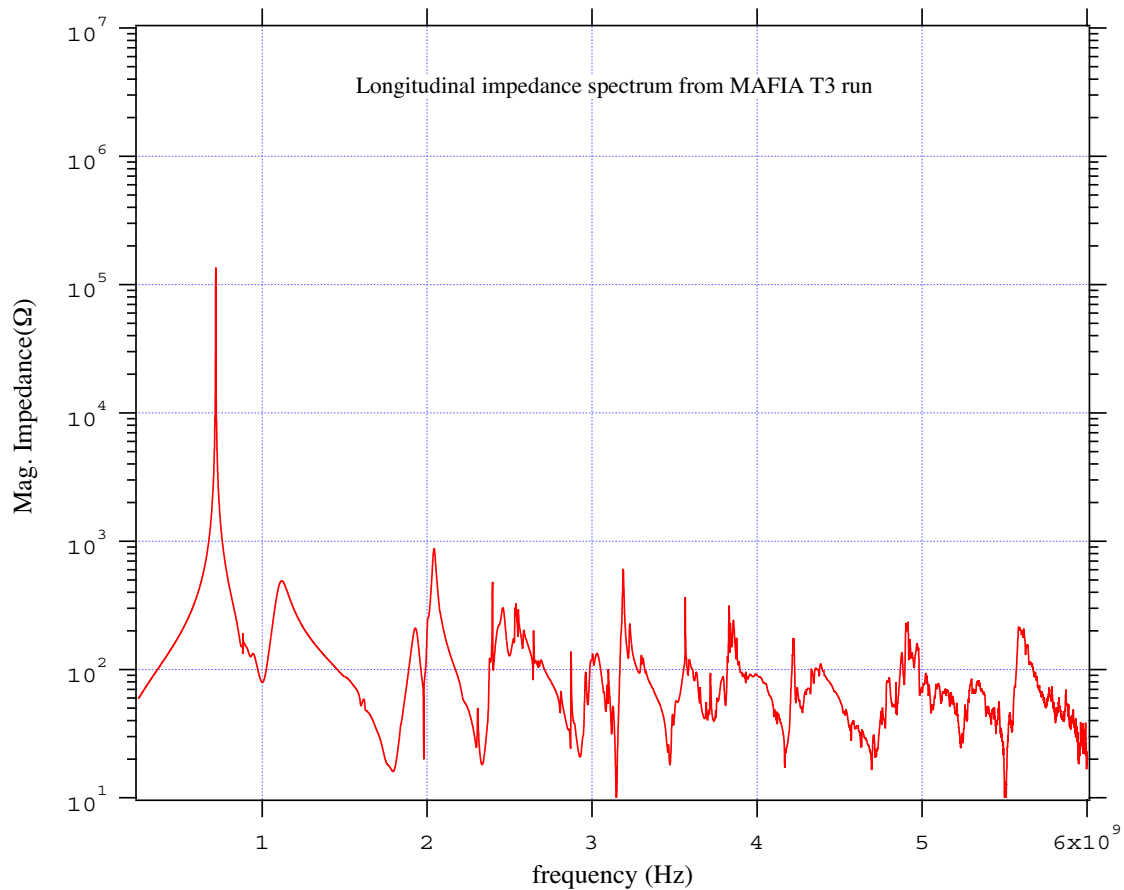


Fig.1 RF cavities longitudinal mode spectrum

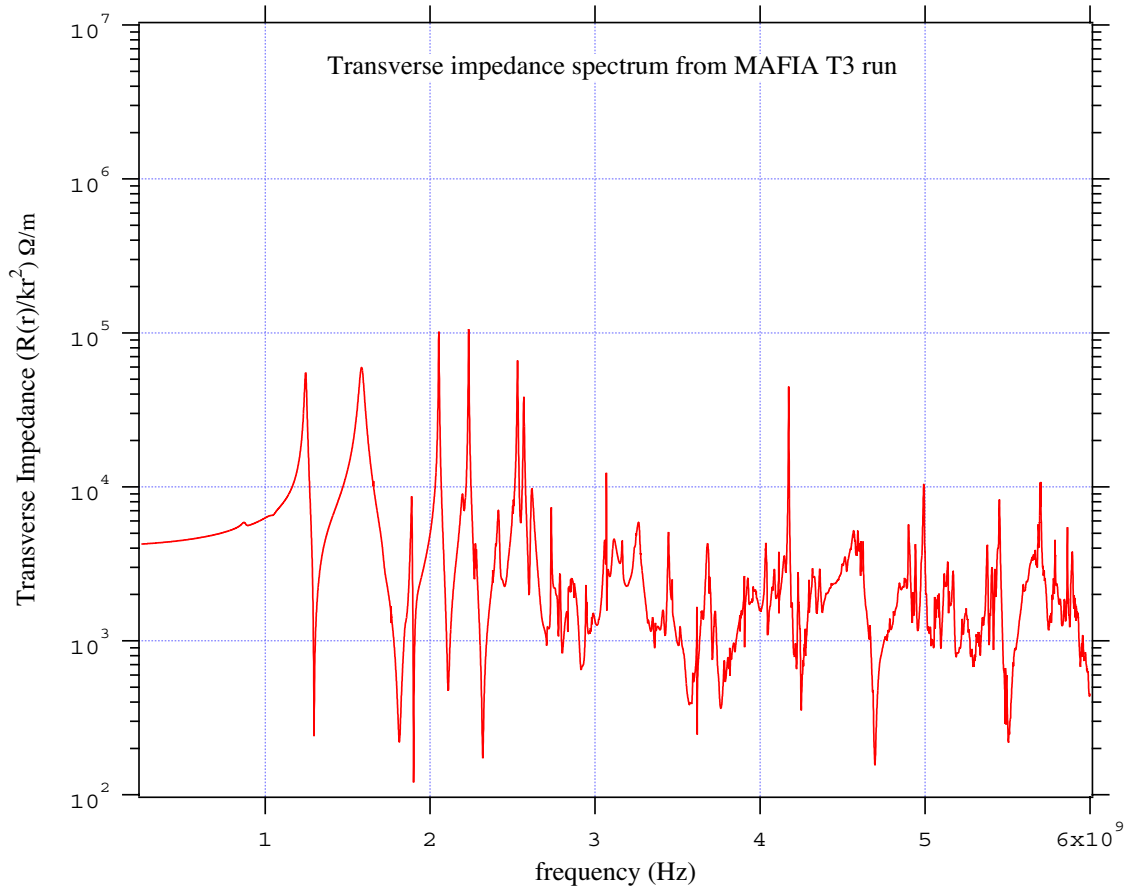


Fig.2 RF cavities transverse mode spectrum

At present, before a prototype of the cavity is available, we cannot separate horizontal and vertical dipole modes. Therefore, using a generic transverse impedance, the only difference between horizontal and vertical growth rates is caused by the difference in the betatron tunes.

We also assume $\beta_{x,y} = 6$ m at the RF cavities location and, to calculate the resistive wall impedance, a beam pipe radius of 16 mm and a wiggler half-gap of 10 mm.

Two different fills have been studied:

Fill A: 570 equal bunches are distributed in 3 trains of 190 bunches each. Individual bunches in the trains are separated by 1.4 ns. The trains are equally spaced around the ring and the total current is 729 mA.

Fill B: As Fill A, but filling only every other bucket in each train. Thus, there are 95 bunches with 2.8 ns spacing in each train, for a total of 285 bunches in the ring. Each bunch has double charge and the total current is therefore the same as in fill A.

3. Beam spectrum

The beam spectrum is calculated applying the Fourier transform to the fill functions described above.

For the 1.4 ns spacing, it consists of lines at every third revolution harmonic ($3n \times \omega_{rev} + \omega_s$, for the longitudinal and $3n \times \omega_{rev} + \omega_\beta$ for the transverse modes) enveloped by a $\sin(x)/x$ function, with a 4 MHz wide main lobe, centered on every RF harmonic.

The 2.8 ns spaced fill differs only in having the $\sin(x)/x$ functions centered on every RF half-harmonic.

Alternatively, the spectral lines amplitude can be obtained from the formula:

$$I_p = \frac{1}{I_0} \sum_{k=0}^{h-1} i_k e^{-2\pi i \frac{pk}{h}} \quad (1)$$

where I_p is the amplitude of the p-th revolution harmonic, I_0 is the beam total current and i_k the current of the k-th bunch.

The coefficient I_p ($p \neq 0$) define the coupling between different modes of the equivalent uniform fill. Specifically, the coupling between modes n, n+p and n-p is proportional to the modulation parameter $N_b |I_p| / h$.

As a simple example let's consider a fill where all bunches except one have been removed. We will have $I_p \equiv 1$ and all modulation parameters equal to $1/h$. In this case it's easy to see that there is only one non null growth rate equal to the sum of all the growth rates for the corresponding symmetric fill divided by h , that is the average growth rate which, by definition, is equal or less than the maximum growth rate. As such, if the symmetric fill is stable the single bunch fill, an extreme case of fractional fill, is also stable as stated in the introduction.

For our fills, it is readily seen that Eq.(1) reaches its maximum value of less than 0.25 for $p=3$ and therefore the modulation parameter is always much less than its maximum value of 1. This means that the growth rates calculated for a uniform fill *with the same total current* are not far off the actual values.

In a practical fill, small variations in the bunch currents would also excite the remaining revolution harmonics. If the current per bunch is almost constant those lines would be of rather small amplitude when compared to the others and we neglect them in the following

analysis. We also neglect the frequency shift in the modes caused by the imaginary part of the impedance.

4. Growth rates

For a uniform fill, the growth time for longitudinal coupled bunch modes is:

$$\tau = \left(\frac{I_{tot} \alpha f_{RF}}{2E_0 Q_s} \text{Re}(Z_{||}^{eff}) \right)^{-1} \quad (2)$$

The effective impedance $Z_{||}^{eff}(p)$ for the p-th mode is the sum of all the RF cavities longitudinal modes at the modal harmonics $f_p(n) = n f_{RF} + p f_{rev} + f_s$:

$$Z_{||}^{eff}(p) = \sum_{n=-\infty}^{\infty} \text{sign}(f_p) \frac{f_p}{f_{RF}} \text{Exp}[-(2\pi f_p \sigma_t)^2] Z_{||}(f_p) \quad (3)$$

The bunch length is taken into account through the exponential roll-off factor $e^{-\omega^2 \sigma_t^2}$. Additionally, the *sign* function in Eq.(3) takes into account that, above transition, negative harmonics have a damping effect on the instability.

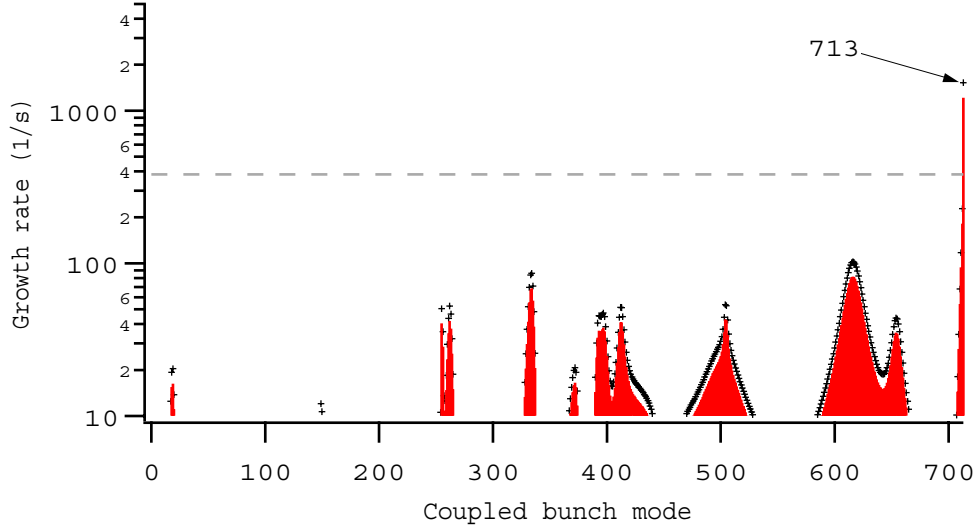


Fig.3 Longitudinal coupled bunch mode growth rates (fill A).

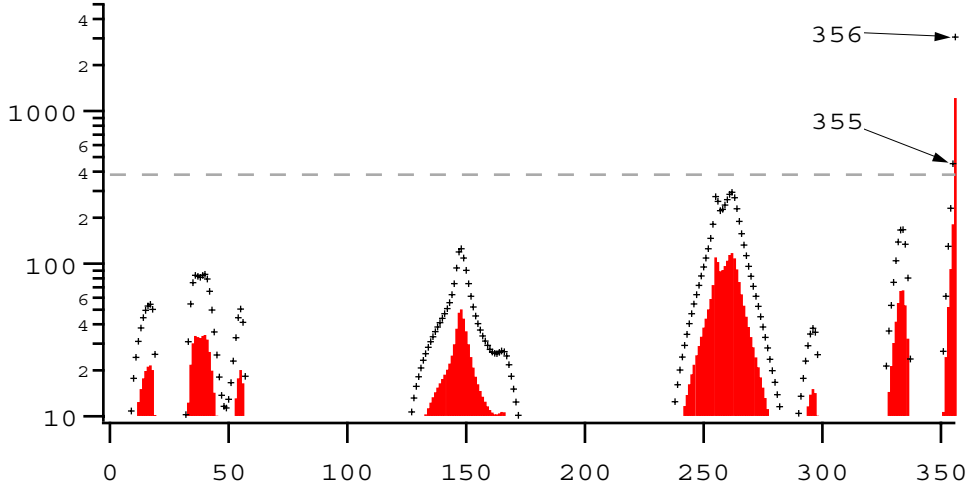


Fig.3 Longitudinal coupled bunch mode growth rates (fill B).

The longitudinal growth (positive values only) rates are reported in Figs.3 and 4 together with the radiation damping (dashed gray line). The black crosses and the red area give an estimate of upper and lower boundaries for the actual growth rates. If the modulation parameters were identically null the growth rates would be on the red (solid) graph and if all the bunches were grouped in a single train the black points would be the theoretically maximum growth rates. Fills A and B are somewhat different from this ideal cases, but in any case, we can say that the actual growth rates for those modes which, excited by the accelerating mode, appear to be above threshold are not higher than indicated since they are coupled to damped (negative growth rate) modes.

The Landau damping, which would reduce the growth rates, caused by the uneven fill seem to be small and is not taken into account.

For the transverse coupled bunch modes we made the additional assumption that the RF cavities HOM's are equal in the horizontal and vertical planes. Therefore, the only differences between the growth rates arise from the different value of the betatron tunes.

The transverse growth times are:

$$\tau = \left(\frac{I_0 f_{rev} \beta}{2E_0} \text{Re}(Z_{\perp}^{eff}) \right)^{-1} \quad (4)$$

The modal harmonics at which the effective transverse impedance has to be calculated are $f_p(n) = n f_{RF} + p f_{rev} + f_{\beta}$, where f_{β} is the horizontal or vertical betatron frequency.

The transverse effective impedance for the p-th mode is:

$$Z_{\perp}^{eff}(p) = \sum_{n=-\infty}^{\infty} -sign(f_p) Exp[-(2\pi f_p \sigma_t)^2] (Z_{\perp,HOM}(f_p) + Z_{\perp,RW}(f_p)) \quad (5)$$

In this case positive frequency harmonics are damping. The resistive wall transverse impedance for an aluminium vacuum chamber is given by:

$$Z_{\perp,RW}(f) = (1 - i) \frac{0.48}{\sqrt{f/f_{rev}}} \text{M}\Omega/\text{m} \quad (6)$$

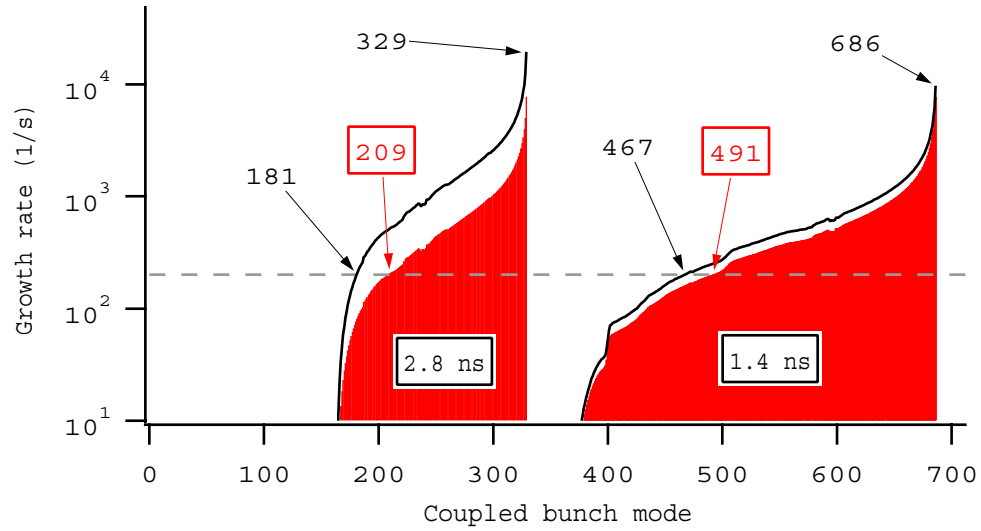


Fig. 4 Horizontal coupled bunch mode growth rates

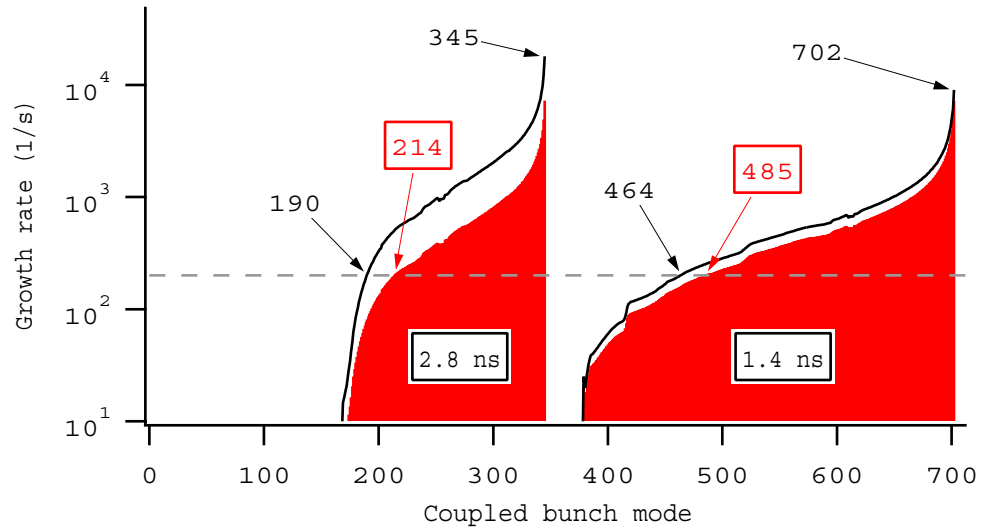


Fig. 5 Vertical coupled bunch mode growth rates

Figures 4 and 5 report the estimated upper and lower boundaries for the horizontal and vertical growth rates. It can be seen that the main contribution comes from the resistive wall

impedance, while the cavities high order modes have little effect. The range of modes above radiation damping is also indicated and the overall vertical stability looks worse than the horizontal, as expected given the fractional tune closer to the integer.

Also the 2.4 ns bunch spacing fill looks less favourable than the 1.4 ns, since the upper growth rates, which are proportional to the current per bunch, are higher by a factor 2.

4. Conclusions

The growth time of longitudinal and transverse coupled bunch modes for rings asymmetrically filled can be approximately estimated using the more tractable techniques developed for symmetric fills.

For the NLC main damping rings, the two different fills proposed do not present dramatic differences from the multi-bunch stability point of view, even though the 1.4 ns bunch spacing fill has slower growth rates in general.

The longitudinal modes appear to be well below the radiation damping threshold except for one or two modes driven by the fundamental mode detuning, while there is a number of transverse modes, both horizontal and vertical, that need to be damped using a feedback system. The source of their growth rates can be traced back to the resistive wall impedance and, thus, to the vacuum chamber material and dimensions.

References

- [1] S. Prabhakar, “New Diagnostics and Cures for Coupled-Bunch Instabilities”, PhD dissertation. Stanford University (2000).
- [2] R. D. Kohaupt, “On Multi-Bunch Instabilities for Fractionally Filled Rings”, DESY 85-139 (1985).
- [3] A. Wolski, “Lattice Description for NLC Main Damping Rings at 120 Hz”, CBP Tech Note-227 (2001).
- [4] R. A. Rimmer, J. N. Corlett, D. Li, et al. “An RF Cavity for the NLC Damping Rings”, PAC 2001 (to be published).

Appendix A – Table of parameters

Circumference	299.792 m
Beap pipe radius	16 mm
Wiggler total length	46.238 m
Wiggler half-gap	8 mm
Energy (E_0)	1.98 GeV
Current (I_0)	729 mA
Bunch length (σ_l)	12 ps
Bunches per train (N_b , fill A/fill B)	190/95
Bunch spacing (fill A/fill B)	1.4/2.8 ns
Bunch trains stored	3
Betatron tune (horiz./vert.)	27.2616/11.1357
Synchrotron tune (Q_s)	0.003496
Momentum compaction (α)	0.000295
Damping times (horiz./vert./long.)	4.85/5.09/2.61 ms
RF voltage	1.07 MV
RF frequency (f_{RF})	714 MHz
Number of RF cavities	3
Total energy loss/turn	777 keV
β at RF cavities location (horiz. and vert.)	6 m