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Correct Account of RF Deflections in Linac Acceleration

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G.V. Stupakov
Stanford Linear Accelerator Center
Stanford, California

Abstract:
During acceleration in the linac structure, the beam not only increases its longitudinal momentum, but also experiences a transverse kick from the accelerating mode which is linear in accelerating gradient. This effect is neglected in such computer codes as LIAR and TRANSPORT. We derived the Hamiltonian equations that describe the effect of RF deflection into the acceleration process and included it into the computational engine of LIAR. By comparing orbits for the NLC main linac, we found that the difference between the two algorithms is about 10\%. The effect will be more pronounced at smaller beam energy.

Correct account of RF deflections in linac acceleration

G. V. Stupakov

SLAC, Stanford University, Stanford, CA 94309

Abstract

During acceleration in the linac structure, the beam not only increases its longitudinal momentum, but also experiences a transverse kick from the accelerating mode which is linear in accelerating gradient. This effect is neglected in computer codes LIAR and TRANSPORT. We derived the Hamiltonian equations that describe the effect of RF deflections during acceleration and included it into the computational engine of LIAR. By comparing orbits for the NLC main linac, we found that the difference between the two algorithms is about 10%. The effect will be more pronounced at smaller beam energy.

1 Introduction

When a particle is accelerated in a RF structure, it not only increases the longitudinal energy, but also changes its transverse momentum. This effect is often neglected in the simulation codes. In this note we derive the equations that take into account the transverse deflections during acceleration and demonstrate their effect in application to the NLC beam dynamics.

2 Equations

It is usually assumed that during acceleration, in an axisymmetric cavity or RF structure, only the longitudinal momentum of the particle increases, $p_{z0} \to p_{z0}+dp_z$, where p_{z0} is the initial longitudinal momentum, and dp_z is the momentum change (considered here as an infinitesimally small quantity) due to the acceleration. With this assumption, the transverse component of the momentum, p_x, remains constant, $p_x = p_{x0}$. This means that the angle $x' \equiv dx/ds$ after passing the cavity is $x' = p_x/(p_{z0}+dp_z) \approx x'_0(1-dp_z/p_{z0})$, where x'_0 is the value of the angle before the acceleration. For a short structure of length dl, one finds the following relation between the initial and final values of the coordinate and the angle:

$$dx = x - x_0 = x'_0 dl, \qquad dx' = x' - x'_0 = -dl \frac{x'_0}{p_{z0}} \frac{dp_z}{ds}. \tag{1}$$

Note that these equations can be obtained with the help of a simple Hamiltonian,

$$H(p_x, s) = \frac{p_x^2}{2p_z\,(s)}, \tag{2}$$

where the conjugate variables are x and p_x, and the dependence $p_z(s)$ takes into account the acceleration in the cavity. Indeed, from Eq. (2) it follows that $p'_x = -\partial H/\partial x = 0$, that is the assumed conservation of p_x, and $x' = \partial H/\partial p_x = p_x/p_z$.

If the change of the longitudinal momentum Δp_z in the acceleration is not infinitesimally small, one should integrate the infinitesimal equations (1) to find the transformation from x_0, x'_0 to x, x'. Assuming a constant acceleration gradient, $dp_z/ds = \text{const}$, one finds

$$\begin{aligned} x &= x_0 + x'_0 l \frac{p_{z0}}{\Delta p_z} \ln\left(1 + \frac{\Delta p_z}{p_{z0}}\right), \\ x' &= \frac{x'_0}{1 + \Delta p_z/p_{z0}}, \end{aligned} \tag{3}$$

where l is the length of the structure. These equations are used in computer codes such as TRANSPORT [1] and LIAR [2] for particle tracking through accelerating structures.

It has been pointed out long time ago [3] that the assumption $p_x = \text{const}$ during acceleration is not correct. Due to the fringe fields in the cavity, the particle gets a kick in the transverse direction that is proportional to the product of the increase of the longitudinal momentum dp_z and p_x,

$$dp_x = \frac{1}{2} p_x \frac{dp_z}{p_z(s)}. \tag{4}$$

It is interesting to note, that with this correction, instead of Eq. (1), we obtain the following formula for dx'

$$dx' = \frac{p_{x0} + dp_x}{p_{z0} + dp_z} - \frac{p_{x0}}{p_{z0}} = -dl \frac{x'_0}{2p_{z0}} \frac{dp_z}{ds}, \tag{5}$$

which is exactly two times smaller then predicted by Eq. (1). Fortunately, as we will see below, the effect of RF deflections on the beam dynamics is not so dramatic, because it is compensated by a particle offset in the acceleration.

The RF deflection can be included into the Hamiltonian (2) by adding a second term

$$H(p_x, s) = \frac{p_x^2}{2p_z(s)} - \frac{1}{2} x p_x \frac{p'_z(s)}{p_z(s)}. \tag{6}$$

Indeed, now $p'_x = -\partial H/\partial x = p_x p'_z(s)/2p_z$, which is equivalent to Eq. (4). Note however that, as follows from the Hamiltonian (6), after the passage of the cavity, the particle will be offset in the transverse direction by dx,

$$dx = dl \frac{\partial H}{\partial p_x} = -\frac{1}{2} x \frac{p'_z}{p_z} dl = -\frac{1}{2} x \frac{dp_z}{p_z}. \tag{7}$$

This offset is due to the radial drift in the cavity under the influence of transverse fields of the accelerating mode.

With equations (4) and (7), the relation between x_0, x'_0 and x, x', for infinitesimally short cavity of length dz that increases the longitudinal momentum by dp_z, becomes

$$\begin{aligned} x &= x_0 + x'_0 dz - \frac{1}{2} x \frac{dp_z}{p_z}, \\ x' &= x'_0 - \frac{1}{2} x' \frac{dp_z}{p_z}. \end{aligned} \tag{8}$$

Again, for a finite value of Δp_z one has to integrate the differential equations (8) to obtain

$$\begin{aligned} x &= \sqrt{\frac{p_0}{p}} (x_0 + l x'_0), \\ x' &= x'_0 \sqrt{\frac{p_0}{p}}. \end{aligned} \tag{9}$$

These equations constitute a Hamiltonian map that describes the acceleration in the cavity with effect of the RF deflections.

3 Acknowledgements

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