WA.5.

# $TE_{111}$ - Mode Oscillation in the Output Cavity of the 75MW PPM Klystron

Perry Wilson

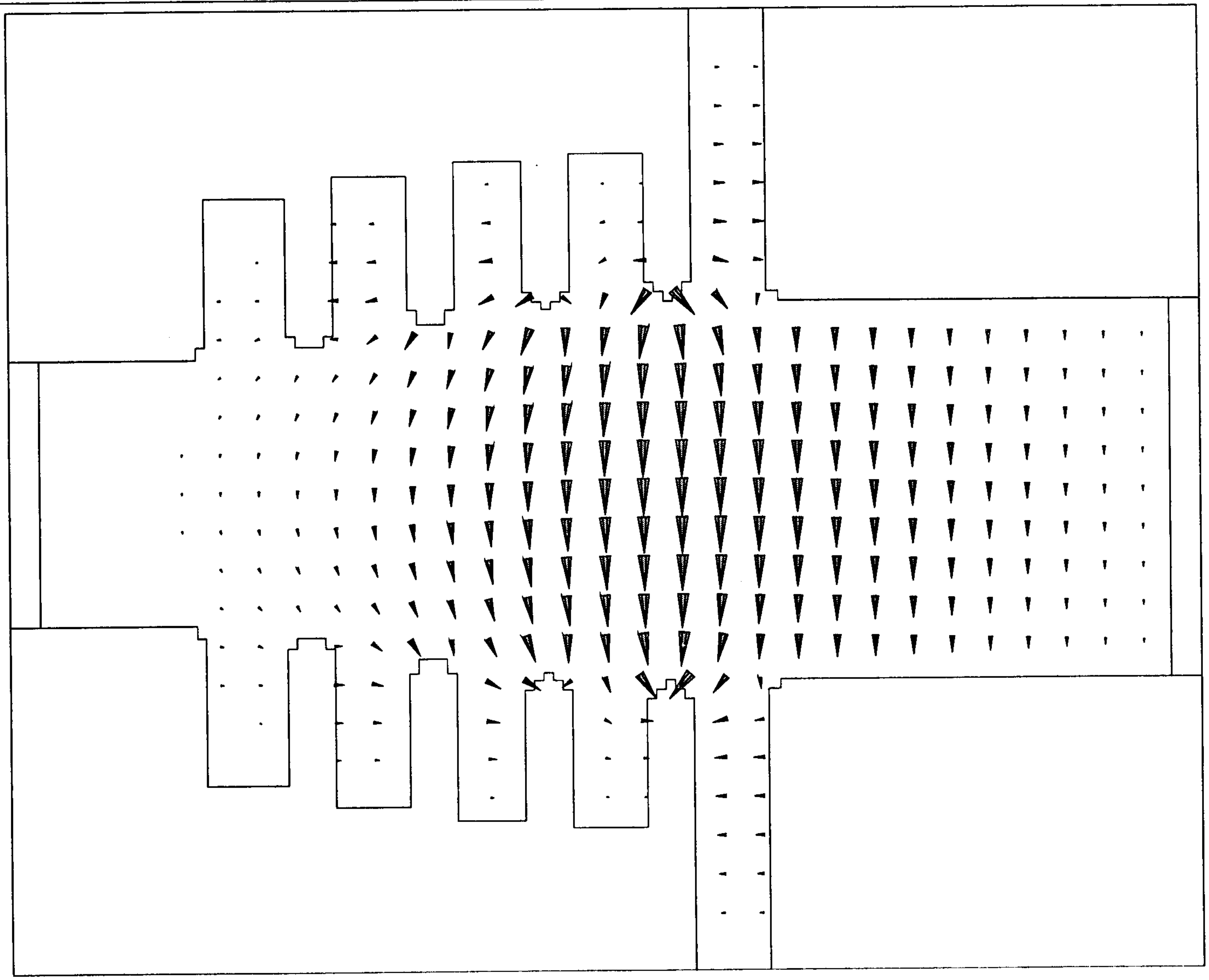
ISG3 - 26 Jan '99

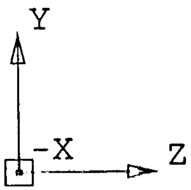
Figures (Kwok Ko + Cho Ng)

| | |
|---|---|
| T-1 | MAFIA plot of $E_y$ |
| T-2 | MAFIA plot of H-field |
| T-3 | Plot of $H_x$ and $E_y$ along axis |
| T-4 | Plot showing beam deflection vs. z |

K

```
MAFIA

FRAME:   1          07/01/99 - 14:06:23          VERSION[V4.016]     E.DRC

FREQUENCY/HZ                           1.1624730624000E+10    PPM75 OUTPUT CAVITIES WITH TAPER
MAXIMUM ERROR OF   CURLCURL-E          4.9055676208809E-04
MEAN    ERROR OF   CURLCURL-E          4.4154494389659E-05
MAXIMUM ERROR OF DIVERGENCE-D          1.1539409570105E-06    TIME HARMONIC ELECTRIC FIELD IN V/M

P--:4016

#ARROW

COORDINATES/M
FULL RANGE / WINDOW
X[  -.014256,   .014256]
 [     .0000,     .0000]
Y[  -.019496,   .019496]
 [  -.019496,   .019496]
Z[     .0000,   .047432]
 [     .0000,   .047432]

SYMBOL = E_5
X-MESHLINE:    35
CUT AT X/M:   6.04464E-11
INTERPOLATE.=       1
LOGSCALE....=   0.00000E+00
MAX ARROW =    754.39
```





1-1

B

**MAFIA**

FRAME: 13 07/01/99 - 14:06:23 VERSION[V4.016]

E.DRC

| | |
|---|---|
| FREQUENCY/HZ | 1.1624730624000E+10 |
| MAXIMUM ERROR OF CURLCURL-E | 4.9055676208809E-04 |
| MEAN ERROR OF CURLCURL-E | 4.4154494389659E-05 |
| MAXIMUM ERROR OF DIVERGENCE-D | 1.1539409570105E-06 |

PPM75 OUTPUT CAVITIES WITH TAPER

TIME HARMONIC MAGNETIC FLUX DENSITY IN VS/M**2

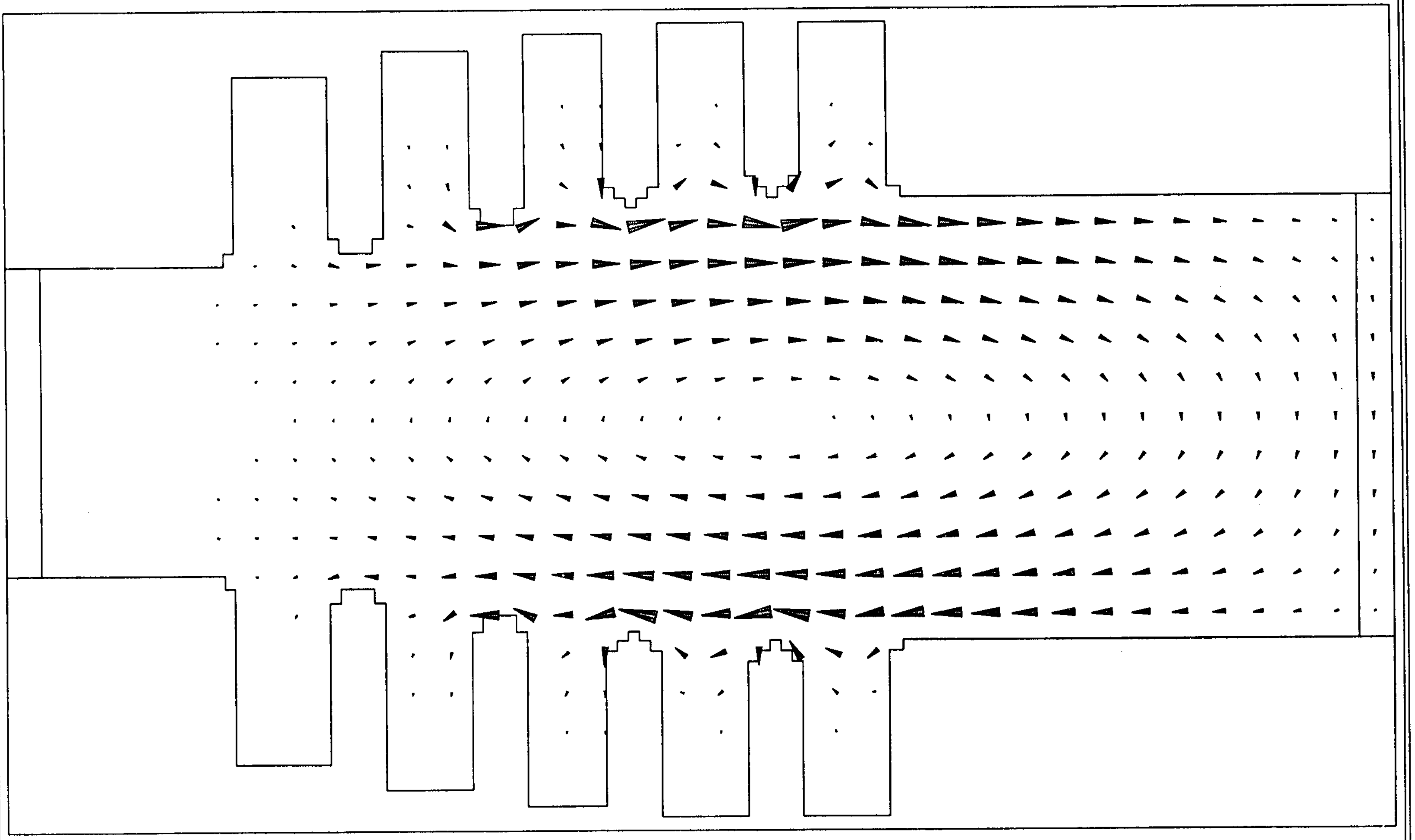
P--:4016

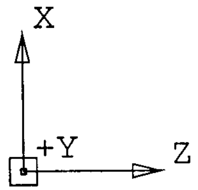
#ARROW

COORDINATES/M
FULL RANGE / WINDOW

| | | |
|---|---|---|
| X[ | -.014256, | .014256] |
| [ | -.014256, | .014256] |
| Y[ | -.019496, | .019496] |
| [ | -.0000, | -.0000] |
| Z[ | .0000, | .047432] |
| [ | .0000, | .047432] |

SYMBOL = B_5
Y-MESHLINE: 41
CUT AT Y/M: -4.05215E-10
INTERPOLATE.= 1
LOGSCALE....= 0.00000E+00
MAX ARROW = 3.02168E-06





7-2

Ey on axis

**MAFIA**

FRAME: 6 07/01/99 - 14:06:23 VERSION(V4.016)

E.DRC

| | |
|---|---|
| FREQUENCY/HZ | 1.1624730624000E+10 |
| MAXIMUM ERROR OF CURLCURL-E | 4.9055676208809E-04 |
| MEAN ERROR OF CURLCURL-E | 4.4154494389659E-05 |
| MAXIMUM ERROR OF DIVERGENCE-D | 1.1539409570105E-06 |

PPM75 OUTPUT CAVITIES WITH TAPER

TIME HARMONIC ELECTRIC FIELD IN V/M

P--:4016

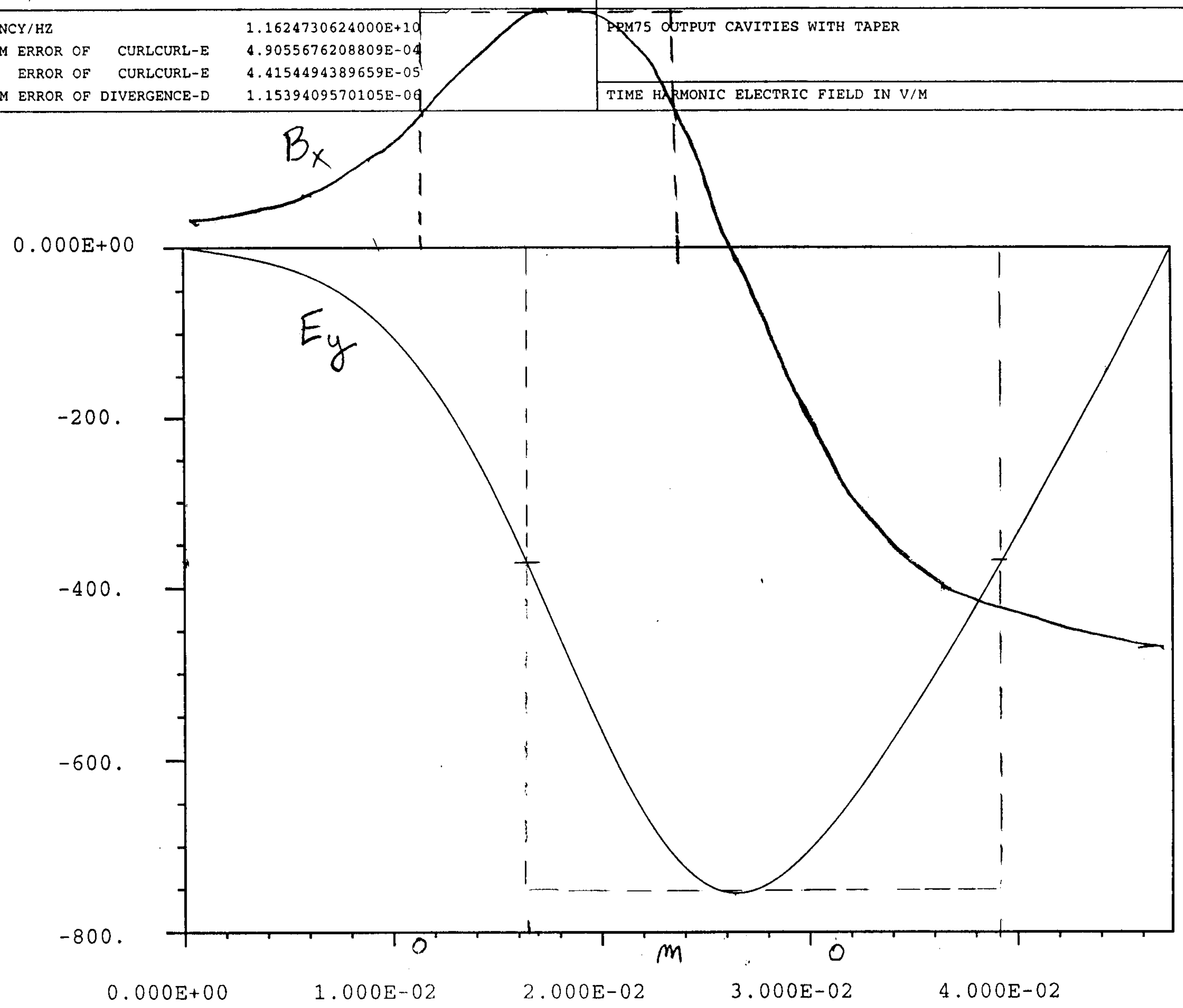
#1DGRAPH

ORDINATE: E_5
COMPONENT: Y

FIXED COORDINATES:
DIM...........MESHLINE
X 35
Y 41

ABSCISSA: GEOMETRY
(BASE OF E_5)

REFERENCE COORDINATE: Z
VARY..........MESHLINE
FROM 1
TO 100



o $v_y = 0$

MAFIA

FRAME: 1 01/12/98 - 09:00:34 VERSION[V4.016]

TS3.DRC

PPM75 OUTPUT CAVITIES WITH TAPER

ELECTRIC FIELD IN V/M

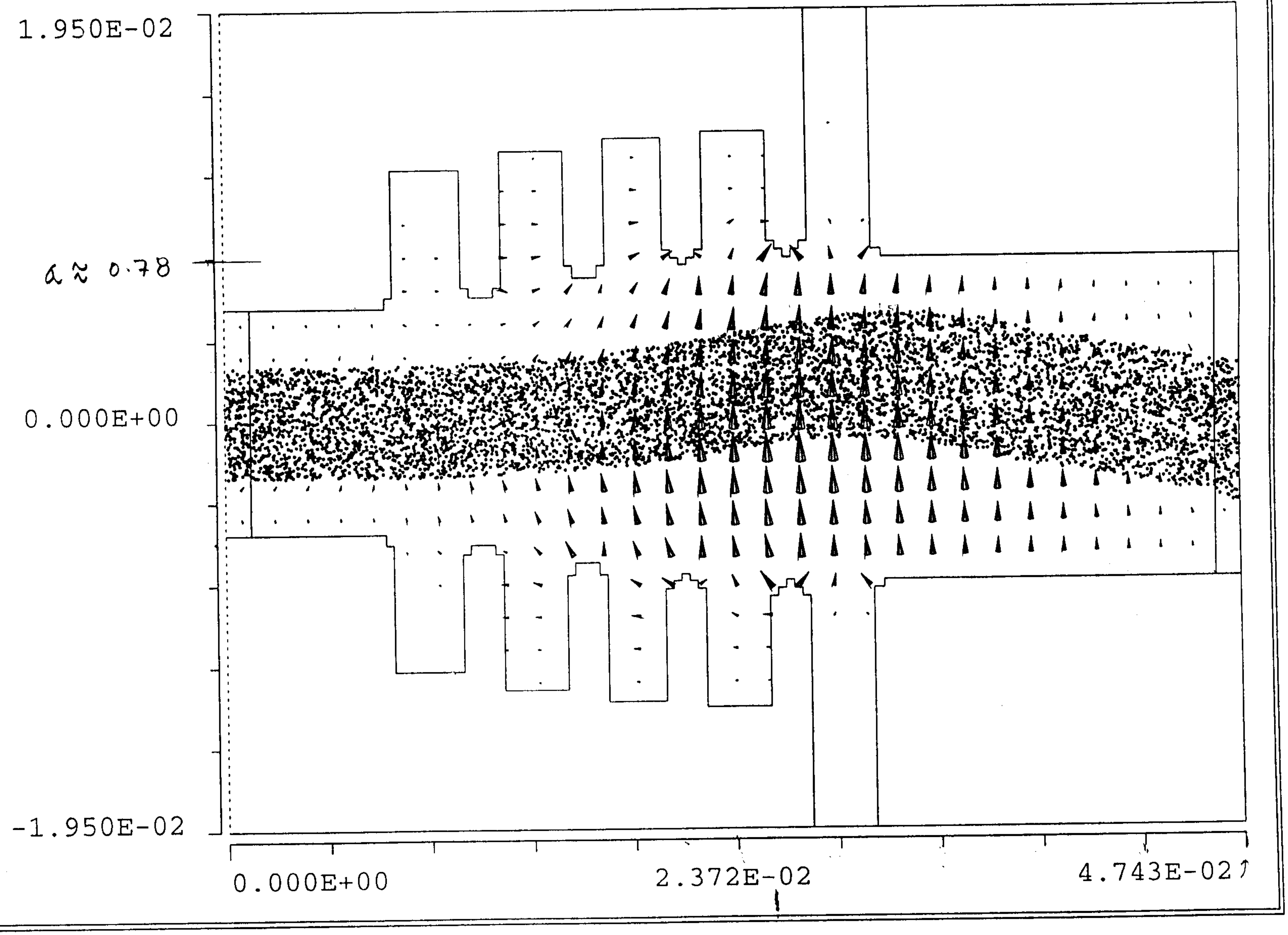
P--:4016

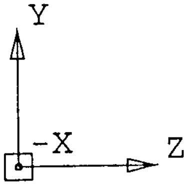
#ARROW

COORDINATES/M
FULL RANGE / WINDOW

| | | |
|---|---|---|
| X[ | -.014256, | .014256] |
| [ | .0000, | .0000] |
| Y[ | -.019496, | .019496] |
| [ | -.019496, | .019496] |
| Z[ | .0000, | .047432] |
| [ | .0000, | .047432] |

| | |
|---|---|
| SYMBOL = EALL | |
| X-MESHLINE: | 35 |
| CUT AT X/M: | 6.04464E-11 |
| INTERPOLATE.= | 1 |
| LOGSCALE....= | 0.00000E+00 |
| ITIME.......: | 2400 |
| TIME......: | 8.75350E-10 |
| MAX ARROW = | 5.39432E+07 |





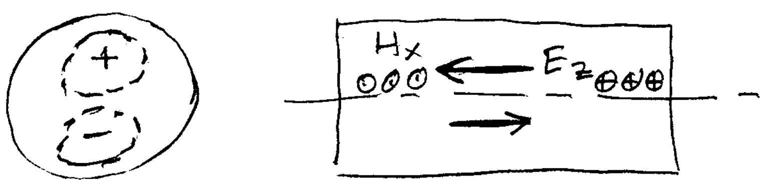
T-4

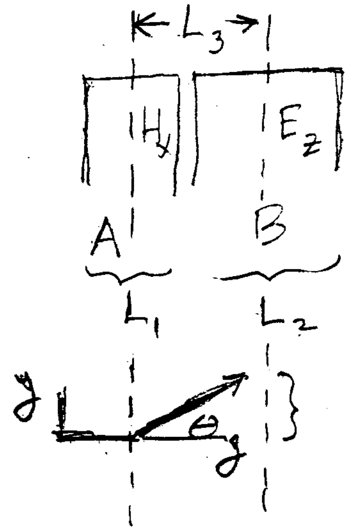
P. Wilson
26 Jan 1999

# $TE_{110}$-Mode Oscillation in the 75 MW Klystron Output Cavity (A BOE calculation)

Compare $TM_{110}$ (BBU) and $TE_{111}$ oscillations

$TM_{110}$ :





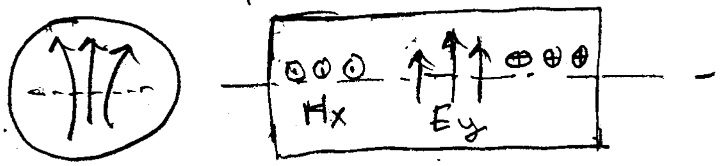
$E_z \sim \Delta y \sim v_y$ (small)

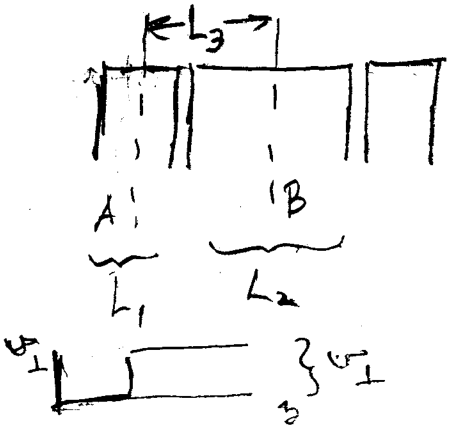
Interaction power:

$P_I \sim I v_z E_z$ ↖ small

↖ large

$TE_{111}$ :





$P_I \sim I v_y E_y$

↑ small ↑ large

Three Transit time Factors

$T_1 = $ fcn of $H_x(z)$, $L_1$

$T_2 = $ fcn of $E_r(z)$, $L_2$

$T_3$ depends on $L_3$ and cavity tuning angle $\psi$

Start Oscillation Condition

$$P_I = P_D$$

$$P_I \sim I v_y E_y \qquad P_D = \frac{\omega U}{Q} \sim \frac{E_y^2}{Q} \times \left\{ \begin{matrix}\text{mode}\\ \text{geometry}\\ \text{factor}\end{matrix} \right.$$

$$v_y \sim H_x \sim E_y$$

$P_I \sim I E_y^2 \times$ mode geometry factor

Equate $P_I = P_D$ $\quad I_s = $ current for start oscillation

$$I_s E_y^2 = E_y^2 / Q \times \text{factor}$$

$I_s Q = $ fcn of mode geometry

* Note: $I_s$ is for infinite pulse length
Starting current is higher for finite pulse length
A separate calculation

## cal culate $v_y$

$$F_y = e v_e \mu_0 H_{x0} = e \beta_e c \mu_0 H_{x0} \leftarrow H_{max}$$

$$\Delta P_y = F_y \Delta t = \frac{F_y L_1}{\beta_e c} T_1 = \gamma m v_y$$

$$v_y = \frac{F_y L_1 T_1}{\beta_e \gamma m c} = (e \beta_e c \mu_0 H_{x0}) L_1 T_1 / \beta_e \gamma m c$$

$$v_y = e \mu_0 H_{x0} L_1 T_1 / \gamma m$$

## cal culate Interaction Power

$$P_e = e \frac{dV_r}{dt} = e E_{y0} \frac{dy}{dt} = e E_{r0} v_y$$

Max Instantaneous Power/electron

$$U_e = P_e \Delta t = \frac{P_e L_2 T_2}{\beta_e c} =$$

Max energy extracted per electron

$$= \frac{e E_{y0} v_y L_2 T_2}{\beta_e c}$$

$$\overline{P}_I = \frac{1}{2} \frac{I}{e} U_e = \frac{I E_{y0} v_y L_2 T_2}{2 \beta_e c}$$

Ave Int. power

Factor of two because $E_y \sim \sin \omega t$
$v_y \sim \sin \omega t$

For maximum $P_I$, transit angle $L_3$ should be 90° (for cavity on resonance)
$T_3$ accounts for actual angle, off resonance

Also, let $H_{x0} = E_{y0}/Z_H$

$$P_I = \frac{E_{y0} L_2 T_2}{2\beta_e c} \cdot \frac{e \mu_0 (E_y/Z_H) L_1 T_1}{\gamma m} \cdot T_3$$

$$\mu_0 = Z_0/c \qquad \frac{mc^2}{e} \equiv V_e = 511\ kV$$

$$P_I = \frac{I E_{y0}^2 L_1 L_2 T_1 T_2 Z_0}{2\beta_e \gamma V_e} \cdot T_3$$

## Oscillation threshold

$$P_I = P_0 = \frac{\omega U}{Q} = \frac{\omega \xi E_{y0}^2}{Q} \qquad U = \xi E_{y0}^2$$

$$\frac{\omega \xi E_{y0}^2}{Q} = \frac{I E_{y0}^2 L_1 L_2 T_1 T_2 T_3 Z_0}{2\beta_e \gamma V_e}$$

$$\boxed{I_s Q = \frac{2\omega \beta_e \gamma V_e \xi Z_H}{Z_0 L_1 L_2 T_1 T_2 T_3}}$$

## Parameter Values

$$\left.\begin{array}{l}\gamma = 1 + \frac{450\ \text{kV}}{511\ \text{kV}} = 1.88 \\ \beta_e = 1 - \frac{1}{\gamma^2} = 0.847\end{array}\right\} \quad \beta_e\gamma = 1.59$$

$$\left.\begin{array}{l}Z_H = 2500\ \Omega \\ \xi = 7.88\times10^{-18}\ \frac{\text{m}^2\,\text{sec}}{\Omega}\end{array}\right\} \quad \text{From MAFIA (Cho Ng)}$$

$V_e = 5.1\times10^5\ \text{V}$, $Z_0 = 377\ \Omega$

$\omega = 2\pi\times11.6\ \text{GHz} = 7.3\times10^{10}\ 1/\text{s}$

$I_0 = K_\mu V_0^{3/2} = 240\ \text{A}$

$L_1 = 1.25\ \text{cm}$ $\quad T_1 = 0.51$

$L_2 \approx 2.0\ \text{cm}$ $\quad T_2 \approx 0.20$

$L_3 = 0.80\ \text{cm}$ $\quad T_3 = 0.88$

$$I_s Q = \frac{(2)\overset{\omega}{(7.3\times10^{10})}\overset{\beta_e\gamma}{(1.59)}\overset{V_e}{(5.1\times105)}\overset{\xi}{(7.88\times10^{-18})}\overset{Z_H}{(2500)}}{\underset{Z_0}{(377)}\underset{L_1}{(1.25\times10^{-2})}\underset{L_2}{(2.0\times10^{-2})}\,T_1T_2T_3}$$

$$I_s Q\, T_1T_2T_3 = 2.15\times10^4$$

$$Q_s = \frac{2.47\times10^4}{I_0 T_1T_2T_3} = \frac{103}{T_1T_2T_3} \quad \text{at } I_0 = 240\ \text{A}$$

$$T_1T_2T_3 = (0.51)(0.20)(0.88) = 0.09$$

$\boxed{Q_s \approx 1150}$ This is for a cw pulse!

Calculation of transit time Functions

$$T_1, T_2 = \frac{1}{L}\int_{-L/2}^{L/2} \cos\beta z \cos kz \, dz$$

$k = \omega/c = \frac{2\pi}{\lambda}$

$\beta = \frac{2\pi}{\lambda_g}$, $k_e = \frac{k}{\beta_e}$

$$= \frac{1}{2}\left[\frac{\sin[(\beta - k_e)L/2]}{(\beta - k_e)L/2} + \frac{\sin[(\beta + k_e)L/2]}{(\beta + k_e)L/2}\right]$$

$L_1, L_2$ are 1/2 ht. pts (30° points)

$\lambda_1 = 3\times 1.25 = 3.75$ cm, $\beta_1 = 2\pi/\lambda_1 = 1.675$ 1/cm

$\lambda_2 = 3\times 2.0 = 6.0$ cm, $\beta_2 = 2\pi/\lambda_2 = 1.05$ 1/cm

$k_e = \frac{2\pi}{\lambda\beta_e} = \frac{2\pi}{(2.6\text{ cm})(.847)} = 2.85$ 1/cm

$T_1 = \underline{0.51}$  $T_2 = \underline{0.20}$ *  * Very uncertain

$T_3 = \cos\psi \cos(\delta - \psi)$

optimize with respect to $\psi$ gives

$\psi_{opt} = \delta/2$

$T_3(\text{opt}) = \cos^2(\delta/2)$

$= \underline{0.88}$

where $\delta = \theta_3 - \pi/2$

$\theta_3 = k_e L_3$

$= (2.85)(0.80) = 2.28$

$\delta = 0.71$