HIGH BRIGHTNESS MUON BEAMS AND LUMINOSITY ESTIMATE IN A
MULTI-TEV MUON COLLIDER USING $e^+e^- \rightarrow \mu^+\mu^-$ AS THE MUON SOURCE

L. P. Keller†, J. P. Delahaye, T. Markiewicz, U. Wienands
SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA

Abstract

Direct annihilation of high energy positrons and atomic electrons in the reaction, $e^+e^- \rightarrow \mu^+\mu^-$ can be used to produce low emittance muon beams for injection into a multi-TeV muon collider. Two material targets are considered. A luminosity estimate is made and compared to the design luminosity of a 6 TeV muon collider using a proton source.

INTRODUCTION

Direct annihilation of positrons with atomic electrons, producing two muons, is a source of high brightness muon beams and a potential source for a muon collider. The electron target can be an electron plasma or a thin material. Even though the muon rate from direct positron annihilation is 5-7 orders of magnitude smaller than the muon rate from a proton source, the emittance of the muon beam is lower and the much higher energy muon beam at the source saves 6D cooling and the early acceleration stages of a muon collider using a proton source. This study began as an option for a Higgs Factory but it became clear that the Higgs Factory requirement of a very small energy spread at the IP severely limited the number of muons which could be collected at the source.

The fixed target threshold to produce two muons at rest in the center of mass is a 43.7 GeV positron beam. To achieve the smallest possible muon beam emittance it is important, as much as possible, that the emittance will only be determined by the positron beam size and the maximum muon production angle. In general this determines the length of the target to limit multiple coulomb scattering (MCS) of the incoming positron beam and outgoing muon beam.

For a positron beam near the muon production threshold the target length is also limited by beam degradation from bremsstrahlung and radiative Bhabha scattering, $e^+e^- \rightarrow e^+e^-\gamma$. Even for relatively thin targets with equivalent thickness $\rho et \approx 10^{24}e^-/cm^2$ and positron beam energy near production threshold, a substantial fraction of the beam positrons begin to fall below the muon production threshold.

For targets greater than about one radiation length, the muon production rate is dominated by the two-stage Bethe-Heitler process, $e^+ \rightarrow \gamma \rightarrow \mu^+\mu^-$, but which gives a much larger angular spread as well as more MCS. For a given length, short material targets have a much larger equivalent thickness than an electron plasma, so only material targets are considered here.

CHOICE OF POSITRON BEAM ENERGY

Previously D. Kaplan et. al. [1] looked at ways to get the required muon rate of $10^{15}$–$10^{16}$/sec by emphasizing a very high intensity positron beam but not the brightness of the muon beam. They also considered colliding beams, high-power targets, $e^+$ storage rings, $e^-$ guns, and a $e^+$ ERL; and concluded that “given the extraordinary beam and target parameters required, the cost effectiveness is far from clear”. M. Antonelli and P. Raimondi [2] considered an electron plasma of density of $10^{20}e^-/cm^3$ and 10 m length. They discussed the muon production cross section and beam degradation due to radiative Bhabha scattering. With an SLC–type machine they estimated a muon production rate near threshold of $\sim 0.5 \times 10^9$/sec and suggested trying for very low emittance.

CHOICE OF TARGET

Fig. 1 shows the muon production angle versus the muon kinetic energy for direct annihilation of a 44.5 GeV positron beam. The central energy of 22.1 GeV occurs when the $\mu^+$ and $\mu^-$ are produced at exactly $90^\circ$ in the center of mass.

The total fixed target annihilation cross section versus positron energy from threshold to 250 GeV is shown in Fig. 2 [4]. Beyond the peak at 60 GeV the cross section falls inversely as the square of the center-of-mass energy. The annihilation cross section at 44.5 GeV is $0.42 \mu$b, and choosing beam energies above 44.5 GeV only results in producing muons outside the usable energy range.

CHOICE OF TARGET

Fig. 3 is a FLUKA [5] simulation of degradation of a 44.5 GeV positron beam as it passes through two material targets which both have an equivalent thickness of $\rho et = 10^{24}e^-/cm^2$. The simulation includes bremsstrahlung and radiative Bhabha scattering. It shows the fraction of positrons exiting the target above the muon production threshold of 43.7 GeV in 0.1 GeV steps. These are the

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keller@slac.stanford.edu
Figure 1: Muon production angle vs. muon energy for a 44.5 GeV positron beam. The muon energies range from 19.1 to 25.1 GeV (±13.6%) with a central energy of 22.1 GeV.

Figure 2: Fixed target cross section for muon production by positron annihilation. The point on the curve is for a positron energy of 44.5 GeV where the cross section is 0.42 μb.

Figure 3: FLUKA simulation showing the fraction of positrons exiting carbon (solid curve) and copper (dashed curve) targets in 0.1 GeV energy intervals vs. positron energy above production threshold. The equivalent thickness of each target is $10^{24} \text{e}^{-/\text{cm}^2}$. The integral for carbon is 69% and only 25% for copper. These are the positrons that are available for muon production.

where $N = \text{electron density}$ and $43.7 \leq E(\text{GeV}) \leq E_{\text{beam}}$.

Fig. 4 shows a FLUKA simulation of the positron track length as a function of positron energy for a 44.5 GeV positron beam incident on 1.5 (0.4) cm carbon (copper) targets. The sum of track lengths above the muon production threshold for a 1.5 cm carbon target is 1.25 cm, 83% of the target length, while for a 0.4 cm copper target the sum of track lengths is only 0.22 cm because of more bremsstrahlung in the copper.

Figure 4: FLUKA simulation of the positron track length in 0.1 GeV bins vs. positron energy for a 44.5 GeV positron beam incident on 1.5 cm carbon (solid curve) and 0.4 cm copper (dashed curve). The sum of track lengths above the muon production threshold in the carbon is 1.25 cm, 83% of the target length, while for a 0.4 cm copper target the larger radiation length results in a track length sum of only 55% of the target length.

**ESTIMATE OF MUON PRODUCTION RATE**

Because of beam degradation the target has an effective length that is a function of the track length (TL) of positrons which remain above threshold before they exit the target. The units of TL(E) are cm/GeV. The number of muons/positron is given by

$$\frac{\mu}{e^+} = N \int_{E=43.7}^{E=44.5} TL(E) \sigma(E) dE,$$

where $N = \text{electron density}$ and $43.7 \leq E(\text{GeV}) \leq E_{\text{beam}}$.

The integral for carbon is 69% and only 25% for copper. These are the positrons that are available for muon production.

The difference is due to more bremsstrahlung in the higher Z copper target. If bremsstrahlung is turned off in the simulation, the integral becomes 83% for both targets.
Fig. 5 is a MUCARLO [6] calculation of the angular distribution of muon production in a 1.5 cm carbon from two processes: direct annihilation and Bethe-Heitler. It is seen that for a relatively thin target like 1.5 cm carbon, muon production from direct annihilation is much more forward than from Bethe-Heitler.

![Figure 5: Program MUCARLO calculation of the muon angular distribution from direct annihilation and Bethe-Heitler processes in a 1.5 cm carbon target. Muons from direct annihilation are much more forward than from Bethe-Heitler production.](image)

Integrating the product of track length and cross section, Eq. (1), in 0.1 GeV energy intervals from 43.7 to 44.5 GeV (see also Figs. 2 and 4) gives the total number of muons/positron,

\[ \mu/e^+ = 3.4 \times 10^{-7} \]

and

\[ \mu/\text{bunch} = (2.6 \times 10^{13} e^+/\text{bunch}) \times (3.4 \times 10^{-7} \mu/e^+) = 8.8 \times 10^{6}. \]

Assume \( \beta^* \) at the muon IP can be set equal to the incoming positron beam bunch length, \( \sigma_L = 0.25 \text{ mm} \). Therefore,

\[ \sigma_x^* = \sigma_y^* = 0.022 \mu\text{m} \quad \text{and} \quad \sigma_{x'}^* = \sigma_{y'}^* = 90 \mu\text{rad}. \]

In a 9 km ring (scaled from 3 TeV CoM in [3]) the luminosity \( L \) at \( t = 0 \) is then,

\[ L(t=0) = 3.6 \times 10^{28} \text{cm}^{-2}\text{s}^{-1}. \]

Averaging over the muon lifetime reduces this by a factor of 0.33 so,

\[ L_{\text{ave}} = 1.2 \times 10^{28} \text{cm}^{-2}\text{s}^{-1}. \]

Table 1 summarizes the results for a 6 TeV collider.

<table>
<thead>
<tr>
<th>Source</th>
<th>Muon Rate</th>
<th>e^+e^- Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>2.4 \times 10^{13}/sec</td>
<td>9.5 \times 10^{13}/sec</td>
</tr>
<tr>
<td>Muons/bunch</td>
<td>2 \times 10^{12}</td>
<td>8.8 \times 10^{6}</td>
</tr>
<tr>
<td>( \gamma \times N )</td>
<td>0.07πm</td>
<td>0.007πm</td>
</tr>
<tr>
<td>( \gamma \times L )</td>
<td>0.005 m</td>
<td>0.00025 m</td>
</tr>
<tr>
<td>( \sigma_x^* = \sigma_y^* )</td>
<td>1.5μm</td>
<td>0.022 μm</td>
</tr>
<tr>
<td>( \sigma_{x'}^* = \sigma_{y'}^* )</td>
<td>420μrad</td>
<td>90 μrad</td>
</tr>
<tr>
<td>( \sigma_L )</td>
<td>0.005 m</td>
<td>0.00025 m</td>
</tr>
<tr>
<td>Luminosity</td>
<td>8.8 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}</td>
<td>1.2 \times 10^{39} \text{cm}^{-2}\text{s}^{-1}</td>
</tr>
</tbody>
</table>

**DISCUSSION AND SUMMARY**

- Because of the relatively large divergence in muon production angles, even near threshold, could only achieve a factor of about 450 reduction in normalized emittance compared to the proton source design, and cutting on the production angle reduces the muon production cross section linearly.
• If compared to the MAP Higgs Factory design, the emittance reduction factor becomes about 7,000, but the requirement of a very small energy spread severely limits the fraction of usable muon production cross section.

• The obvious way to achieve a luminosity comparable to the proton source design is to increase the positron beam current by a factor \( \left( \frac{8.8 \times 10^{34}}{1.2 \times 10^{28}} \right)^{1/2} \approx 2,700 \), so 41 \( \mu \text{A} \rightarrow 110 \text{ mA} \). An ERL might be an option, but is beyond current technology.

• To achieve \( \rho t = 10^{24} \text{e}^-/\text{cm}^2 \) in plasma, either a very long plasma or a series of short plasmas with optics in between to control the effective source size would be needed [8].

• Survival of the 15 mm carbon target has not been addressed here, but simple dE/dx shows that at 110 mA, 550 kW is deposited in the carbon alone.

• A Higgs Factory starting with the SLC positron beam parameters at 44.5 GeV and 0.1% energy spread at the IP give a rough luminosity estimate of \( 1.8 \times 10^{16} \text{cm}^{-2}\text{s}^{-1} \).

We conclude that, even with the substantial emittance reduction estimated here, significant developments would be needed to reach the proton source luminosity design in a TeV class muon collider.

REFERENCES


